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Time-reversal seismic-source imaging and moment-tensor inversion

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SUMMARY

The time reversal operation in seismic source estimation is considered. We show that the time reversal operation, equally the adjoint operation, for seismic source imaging gives an approximate solution to more conventional seismic source inverse problem through the 'happy approximation' underlined by Claerbout. Practical applications of such methods in a long-period range to monitor earth's activities in realtime are also discussed.

Key words: Theoretical seismology.

GJI Seismology

1 INTRODUCTION

Availability of high-performance computing and numerical techniques have brought a new class of analysis methods in seismology, and so-called adjoint or time reversal operations are getting great interests in global seismology (Tromp *et al.* 2005; Larmat *et al.* 2006). For example, Tromp *et al.* (2005) drew connections among different techniques, such as seismic tomography, adjoint methods, and 'banana-doughnut' kernels, and demonstrated how waveform tomography for 3-D earth structure may be performed based on numerically simulated waveform synthetic techniques (Komatitsch & Tromp 2002; Tsuboi *et al.* 2003). By analyzing broadband seismic recording of the global network for the 2004 great Sumatra earthquake, Larmat *et al.* (2006) showed the time reversal source imaging may be feasible on global scale. This short note is motivated by the work of Larmat *et al.* (2006), and draws connections between the time reversal source imaging and more conventional moment tensor inversion techniques. For the purpose, we focus on the distinction between 'inverse' and 'adjoint' methods introduced by Claerbout (2001).

2 SEISMIC SOURCE ESTIMATION

Green's function

For simplicity, we start with the excitation problem by a single force.

The displacement at $\boldsymbol{x}$ excited by a single force point source located at $\boldsymbol{\xi}$ may be given as

$$u_n(\boldsymbol{x}, t) = G_{ni}(\boldsymbol{x}, t; \boldsymbol{\xi}, 0) * f_i(t) \tag{1}$$

where G_{ni} and f_i are the Green's function and single force source time function respectively, and the multiplication * denotes convolution (Aki & Richards 1980).

Equivalently in frequency domain, we have

$$u_n(\boldsymbol{x}, \omega) = G_{ni}(\boldsymbol{x}, \omega; \boldsymbol{\xi}) f_i(\omega) \tag{2}$$

and in vector form,

$$\boldsymbol{u} = \boldsymbol{G} \cdot \boldsymbol{f} \tag{3}$$

Time reversal operation

For the observation waveform at a point $\boldsymbol{x}$, $d_i(t)$, the time reversal (TR) operation for 'wavefield' at $\boldsymbol{\xi}$ may be written as

$$TR_n(t) = G_{ni}(\boldsymbol{\xi}, t; \boldsymbol{x}, 0) * d_i(t_0 - t), \tag{4}$$

where $TR_n(t)$ denotes the n-th component of the time-reversed seismogram at $\boldsymbol{\xi}$ and t_0 is an arbitrary reference time; in frequency domain it becomes

$$TR_n(\omega) = G_{ni}(\boldsymbol{\xi}, \omega; \boldsymbol{x}) \bar{d}_i(\omega) e^{i\omega t_0}, \tag{5}$$

where $\bar{d}$ denotes complex conjugate of d, and the sign convention of the Fourier transform is the same as in Aki & Richards (1980).

Source inversion

In source inversion, if the source location is assumed to be known, we search $\boldsymbol{f}(t)$ such that the difference between predicted waveforms and observed ones are minimized. If we use the ordinary least-squares criterion, the solution $\hat{\boldsymbol{f}}$ in frequency domain may be given as

$$\hat{\boldsymbol{f}}(\omega) = (\boldsymbol{G}^* \boldsymbol{G})^{-1} \boldsymbol{G}^* \boldsymbol{d} \tag{6}$$

where $\boldsymbol{G}^*$ is the conjugate transpose (or adjoint transpose) of $\boldsymbol{G}$. Also as the intermediate equation, the normal equation, we have

$$\boldsymbol{G}^* \boldsymbol{G} \cdot \boldsymbol{f}(\omega) = \boldsymbol{G}^* \boldsymbol{d} \tag{7}$$

The right hand side of this equation may be written as

$$rhs = \bar{G}_{in}(\boldsymbol{x}, \omega; \boldsymbol{\xi}) d_i(\omega) = \bar{G}_{ni}(\boldsymbol{\xi}, \omega; \boldsymbol{x}) d_i(\omega) \tag{8}$$

where we used the spatial reciprocity of Green's function. Comparing with (5), this is a phase-shifted complex conjugate of $TR_n(\omega)$ in (5). In other words, the time reversal operation is nothing different

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from the phase-shifted cross-correlation between the observed displacement and the Green's function (e.g. Draeger & Fink 1997). As we employ only the spatial reciprocity of Green's function to draw the connection between (5) and (8), it should hold for anelastic case as well (Dahlen & Tromp 1998).

Imaging *vs.* inversion

Claerbout (2001) introduces a clear definition of the difference between inverse and adjoint operators as follows: In many geophysical inverse problems (e.g. Tarantola 1984), $\boldsymbol{G}^* \boldsymbol{G}$ can be often well approximated by the identity matrix as

$$\boldsymbol{G}^* \boldsymbol{G} \approx \alpha\, \boldsymbol{I} \tag{9}$$

where α is some real proportionality constant. In this case, the least-squares solution (6) can be also approximated as

$$\hat{\boldsymbol{f}}_{TR}(\omega) \approx \alpha^{-1}\, \boldsymbol{G}^* \boldsymbol{d}. \tag{10}$$

Here following the idea of Claerbout, we define this type of solution as 'imaging', while the conventional least squares solution as 'inversion'.

Multi-station case

We can easily extend the above argument for observations with multiple stations. In many seismological source estimation problems, (9) appears to be a good approximation (e.g. Kawakatsu 1996, for the moment tensor of deep earthquakes with a sufficient station coverage), and 'source imaging' using (10) may be possible. As (10) is just a phase-shifted complex conjugate of the time reversal, now we understand why the time reversal appears to work (e.g. Larmat *et al.* 2006).

In case the point source approximation is appropriate, the convergence (goodness of fit) can be measured by the ratio of the cross correlation of observed ($\boldsymbol{d}$) and predicted ($\boldsymbol{G}\hat{\boldsymbol{f}}$) waveforms to the observed waveform energy ($\boldsymbol{d}^2$),

$$R = \frac{\boldsymbol{d}^* \boldsymbol{G} \hat{\boldsymbol{f}}}{\boldsymbol{d}^2} \tag{11}$$

which is equal to the variance reduction for the least-squares solution. For source imaging, this may be approximated by

$$R \approx \alpha\, \frac{\hat{\boldsymbol{f}}_{TR}^2}{\boldsymbol{d}^2}, \tag{12}$$

and a TR-image which gives the largest energy may be considered as the best one. For a spatially extended source such as the 2004 Sumatra event, more sophisticated measure may be used; nevertheless, (12) should still give some idea of the spatial extent of the seismic source (e.g. Larmat *et al.* 2006).

Moment tensor case

In case of a more general seismic source represented by the moment tensor $M_{ij}(t)$, the displacement is now given as

$$u_n(\boldsymbol{x}, t) = G_{ni,j}(\boldsymbol{x}, t; \boldsymbol{\xi}, 0) * M_{ij}(t), \tag{13}$$

but the whole argument above can be repeated if we use $G_{\mathrm{n}i,j}$ instead of the Green's function itself and if we treat six independent components of symmetric M_{ij} as a vector. So $G_{\mathrm{n}i,j}$ should be used rather than $G_{\mathrm{n}i}$ to do TR source imaging. It should be noted that the spatial derivative $_{,j}$ is applied at the source location $\boldsymbol{\xi}$; i.e. in case of TR imaging, it is the strain field which will be used for back-propagation: i.e.

$$TR_{ij}(t) = E^n_{ij}(\boldsymbol{\xi}, t; \boldsymbol{x}, 0) * d_n(t_0 - t) \tag{14}$$

$$TR_{ij}(\omega) = E^n_{ij}(\boldsymbol{\xi}, \omega; \boldsymbol{x})\, \bar{d}_n(\omega)\, e^{i\omega t_0}. \tag{15}$$

Here E^n_{ij} is the strain field due to an unit force directed to nth direction, and defined as

$$E^n_{ij}(\boldsymbol{\xi}, t; \boldsymbol{x}, 0) = c_{ij}\left(\frac{\partial G_{in}(\boldsymbol{\xi}, t; \boldsymbol{x}, 0)}{\partial \xi_j} + \frac{\partial G_{jn}(\boldsymbol{\xi}, t; \boldsymbol{x}, 0)}{\partial \xi_i}\right), \tag{16}$$

where the repeated index summation rule does not apply, and we choose $c_{ij} = 1/2$ for $i = j$, and $c_{ij} = 1$ for the rest to be consistent with (13). The 'stress-glut field' will be back-propagated in this case, and will converge at the source location as the moment tensor. The time reversal imaging of the moment tensor may be obtained as

$$\hat{M}_{TRij}(\omega) \approx \alpha^{-1}\, \bar{E}^n_{ij}(\boldsymbol{\xi}, \omega; \boldsymbol{x})\, d_n(\omega) \tag{17}$$

Similar to (12), the convergence may be measured by

$$R \approx \alpha\, \frac{\hat{\boldsymbol{M}}_{TR}^2}{\boldsymbol{d}^2} \equiv \alpha\, \frac{\sum_{i\le j} \hat{M}^2_{TRij}}{\boldsymbol{d}^2}, \tag{18}$$

where $\sum_{i\le j}$ denotes a summation over six independent components of $\hat{M}^2_{TRij}$.

It should be noted that the imaging solution (10) or (17) does not scale properly when the parameterization is changed. For example, if we solve for $M'_{11} = 2 \times M_{11}$ instead of M_{11} while keeping other parametrization the same, the resulting solution is not guaranteed to be consistent with that solved for M_{11} (this is not the case for the least-squares solution). Also the convergence criteria (18) should be modified accordingly.

3 DISCUSSION

Imaging *or* inversion

We have shown that the time reversal (adjoint) source imaging gives an approximation to the seismic source inversion by way of (9). Claerbout (2001) notes that *'the adjoint is the first step and a part of all subsequent steps in this inversion process. Surprisingly, in practice the adjoint sometimes does a better job than the inverse! This is because the adjoint operator tolerates imperfections in the data and does not demand that the data provide full information.'* The question left is then whether we should use imaging or inversion for the seismic source estimation problems.

There are cases when certain components of the moment tensor are not well resolved by data (e.g. Kanamori & Given 1981; Kawakatsu 1996). There may also exist cases that the knowledge of the Green's function is largely poor. In these cases, spurious solution may be obtained by the inverse and the imaging may be preferred. Other than those cases, considering the easiness of calculating the inverse for source estimation, the conventional moment tensor inversion may be preferred to the time-reversal source imaging. The non-uniqueness of the solution mentioned above may also limit the usage for earthquake mechanism estimation purposes. On the other hand, once the parameterization scheme is fixed, (12) or (18) can be used for estimating strength of the source, and the spatial extent of seismic source may be imaged effectively.

Realtime monitoring applications

Although (9) may give a good approximation, calculation of the actual inverse is not so difficult for the source inversion; for the moment tensor source we need to solve for 6 × 6-matrix, and for the single force source only for 3 × 3-matrix. So there appears no practical difficulty in solving the inverse problem. Indeed, in GRiD MT (Grid-based Realtime Determination of Moment Tensors) algorithm (Kawakatsu 1998; Tsuruoka *et al.* 2008), moment tensor inversions are performed in realtime continuously for every one second for limited source areas. GRiD MT is a grid-based earthquake analysis system that continuously monitors long-period seismic wavefield of 20 to 50 seconds recorded by broadband seismometers, and automatically and simultaneously determines the origin time, location and seismic moment tensor of seismic events within three minutes of their occurrence. This system has been in operation since 2003 at the Earthquake Research Institute (http://wwweic.eri.utokyo.ac.jp/GRiD MT/), and has been shown to be successful (Tsuruoka *et al.* 2008). The success of GRiD MT proves that it is now practical to perform the seismic source inversion in realtime, consequently the time reversal source imaging as well.

Either performing inversion or imaging, such a realtime operation in a long-period range is now possible as mentioned above for regional scale monitoring, and may be for global scale as well (e.g. Larmat *et al.* 2006; Ekström 2006). This type of source inversion/imaging should also help monitoring glacial quakes (Ekström *et al.* 2003) and sources of background seismic microseism and HUM (e.g. Nishida & Fukao 2007), as well as some other exotic seismic sources (e.g. Kumagai *et al.* 2001; Ito *et al.* 2007). The realtime monitoring of Earth's activity field may soon become available.

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