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# Conditions for Earth-like geodynamo models

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ABSTRACT

For many published dynamo models an Earth-like magnetic field has been claimed. However, it has also been noted that as the Ekman number (viscosity) is lowered to less unrealistic values, the magnetic field tends to become less Earth-like. Here we define quantitative criteria for the degree of semblance of a model field with the geomagnetic field, based on the field morphology at the core–mantle boundary. We consider the ratio of the power in the axial dipole component to that in the rest if the field, the ratios between equatorially symmetric and antisymmetric and between zonal and non-zonal non-dipole components, and a measure for the degree of spatial concentration of magnetic flux at the core surface. We also briefly discuss shortcomings of possible other criteria for an Earth-like model. We test the compliance with our criteria for a large number of dynamo models driven by imposed temperatures at their inner and outer boundaries that cover the accessible parameter space. We order models according to their magnetic Reynolds number *Rm* (ratio of advection to diffusion of magnetic field) and magnetic Ekman number $E_\eta$ (ratio between rotation period and magnetic diffusion time). Requirements for an Earth-like field morphology are that $E_\eta < 10^{-4}$ and that *Rm* falls into a limited range that depends on $E_\eta$. Higher values of *Rm* are required at low values of $E_\eta$. Extrapolating the boundaries of compliant dynamos in this parameter space to the Earth's value of $E_\eta$ suggests that Earth-like dynamos exist all the way between present model values and parameter values of the geodynamo. We also study a more limited set of dynamo models with flux boundary conditions. The nature of the boundary condition and the distribution of sources and sinks of buoyancy have a secondary influence on the field morphology.



## 1. Introduction

In the past 15 years many geodynamo models based on direct numerical simulations of convection-driven magnetohydrodynamic flow in a rotating spherical shell have been published, following on several early seminal papers (Glatzmaier and Roberts, 1995; Kageyama and Sato, 1995; Kuang and Bloxham, 1997). Some models reproduce the properties of the Earth's magnetic field remarkably well. The model fields are dominated by the axial dipole and show power spectra and a morphology at the outer boundary of the dynamo that resemble those of the geomagnetic field at the core–mantle boundary (CMB). Some models show stochastic dipole reversals similar to those inferred from the paleomagnetic record. Referring to the semblance in the field properties, often the term 'Earth-like' has been used for a model. However, the models are not Earth-like in some physical conditions. The viscosity $\nu$ and thermal diffusivity $\kappa$ are far too large in the models and the rotation rate $\Omega$ is often too low. In terms of non-dimensional control parameters, the Ekman number $E = \nu/(\Omega D^2)$ and the magnetic Prandtl number $Pm = \nu/\eta$ are too large and the Rayleigh number $Ra = \alpha g \Delta T D^3/(\kappa\nu)$ is too low (here $D$ is the thickness of the convecting shell, $\alpha$ thermal expansivity, $\Delta T$ driving temperature contrast, $g$ gravity and $\eta$ magnetic diffusivity). In geodynamo models the choice of parameter values is restricted by what is numerically feasible, but it is possible to vary parameters within a certain range. For many published models the rationale for the precise choice of control parameters is not very clear.

Making use of the increase in computer power, more recently attempts have been made to push parameters towards somewhat more realistic values, in particular by lowering the Ekman number and magnetic Prandtl number. However, some authors noted that this seems to make the magnetic field less Earth-like rather than more Earth-like. A non-dipolar field has been reported in one case (Kageyama et al., 2008), but usually it was found that at low Ekman number of order $10^{-6}$ the field becomes dipole-dominated to a degree that is unrealistic for the Earth (Jones, 2007; Sakuraba and Roberts, 2009; Takahashi et al., 2008), whereas at more moderate Ekman number of order $10^{-4}$ the magnetic field morphology better resembles that of the present geomagnetic field. One may thus question if Earth-like dynamos exist in a contiguous region of parameter space that encompasses both the high Ekman number models and the true geodynamo. If this happens to be the case, our present models may be generically similar to the geodynamo. In

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contrast, if Earth-like dynamos reside in isolated islands of parameter space, say at Ekman numbers around $10^{-4}$ and again at around $10^{-15}$, these two classes may be dynamically very distinct and the agreement in magnetic field properties could be fortuitous.

Aside from the values of control parameters, boundary conditions and the distribution of sources and sinks of buoyancy in the fluid core also influence the magnetic field properties. Sakuraba and Roberts (2009)find that a heat flux condition at the CMB results in a more Earth-like magnetic field than a condition of fixed constant temperature. Driving convection by heating from within (or secular cooling, which is equivalent) tends to generate less dipolar fields than are found in models that are driven from below (Kutzner and Christensen, 2002; Olson and Christensen, 2006), although the choice of a fixed flux condition instead of a fixed temperature condition at the CMB reduces this trend (Aubert et al., 2009). For models with stress-free boundaries and partially internal heating, a variety of magnetic field morphologies depending on control parameter values have been found, which encompass dipolar, quadrupolar, hemispheric and small-scale magnetic fields (e.g. Simitev and Busse, 2005).

This paper follows the spirit of the reviews by (Dormy et al. (2000) and Kono and Roberts, 2002), who extensively discussed which properties of the geomagnetic field could be considered well-established (at the time of writing) and compared the limited number of geodynamo models then available regarding their compliance with these properties. Here we make use of a now much larger data basis of numerical dynamo models and concentrate in particular on the combinations of control parameters that make a model 'Earth-like'. Earth-likeness is measured in a quantitative manner by relying on a few well-established structural properties of the geomagnetic field. We also address the influence of different boundary conditions and different modes of driving convection.

In Section 2 we critically discuss possible criteria that might be applied to determine if a model is Earth-like. For various reasons we settle for a set of criteria that quantify several traits of the morphology of the historical geomagnetic field. Having introduced the dynamo models we use in this study (Section 3), we next apply our criteria to models with fixed temperature conditions that cover a substantial range in all four basic control parameters (Section 4.1). For a more limited number of models we determine the effect of different boundary conditions and driving modes (Section 4.2). Our findings are summarized and discussed in Section 5.

## 2. Criteria for an Earth-like dynamo model

In order to be useful, a criterion for judging dynamo models must satisfy several requirements. It should be objective and if possible quantifiable, it should address a well-established property of the geomagnetic field or of the Earth's core, and it should be practical to apply to the results of current dynamo simulations. Sometimes a model has been claimed Earth-like based on criteria that do not agree with some of these requirements or that are too fuzzy. Most criteria address a property of the observable part of the magnetic field. Ideally the representative Earth value for such a property should be supported both by historical measurements and by paleomagnetic data. However, we prefer not to use criteria based on magnetic field properties that may depend critically on complex conditions that are not normally employed in dynamo models. For example, a particular inhomogeneous heat flux pattern at the core–mantle boundary may be essential for causing non-zonal stationary features in the field (e.g. Aubert et al., 2008; Gubbins et al., 2007; Olson and Christensen, 2002) or it may strongly affect reversal frequencies (Courtillot and Olson, 2007; Glatzmaier et al., 1999; Kutzner and Christensen, 2004). Such properties are indeed difficult to quantify in an objective way and are anyway bound to be variable through geological times (see Hulot et al. (2010) for a recent review).

In the following we will first introduce four criteria based on the magnetic field morphology that we chose for judging the Earth-likeness of a dynamo model. We proceed by defining a quantitative measure for the 'compliance' of the magnetic field of a dynamo model with the geomagnetic field. We close this section by discussing other possible criteria that have been suggested or used in the literature, but which we do not find suitable or practical for our study.

### 2.1. Criteria based on magnetic field morphology

We use structural and spectral properties of the magnetic field at the core–mantle boundary (CMB) up to harmonic degree and order eight. The CMB field is well resolved up to this degree back to at least 1840. We use the gufm1 model of the CMB field (Jackson et al., 2000) from 1690 to 1990 in ten-year time intervals and the IGRF11 model (www.ngdc.noaa.gov/IAGA/vmod/igrf.html) for 2000 and 2010. The time interval 1690−2010 corresponds to no more than roughly two overturn times of the flow in the fluid core. This is a short averaging time and the historical field might be unusual in some property or other. Therefore we also consider supporting evidence from archeomagnetic and paleomagnetic data.

We use the model CALS7K.2 for the geomagnetic field structure during the past 7000 yr (Korte and Constable, 2005), which is based on archeomagnetic and lake sediment data. Its accuracy is certainly lower than that of the historical field models based on direct measurements, in particular for higher spherical harmonic degrees. These degrees are also significantly damped in CALS7K.2. We try to assess the impact of the reduced resolution by comparing the properties of CALS7K.2 with those of gufm1 for the time period 1690−1950, where both models overlap.

For earlier periods deterministic models of the field structure at a given instant in time are not available. We use here statistical models for the paleofield and its secular variation (Constable and Parker, 1988; Constable and Johnson, 1999; Hatakeyama and Kono, 2002; Quidelleur and Courtillot, 1996; Tauxe and Kent, 2004). These models, summarized in (Khokhlov et al., 2006), give estimates for mean values of the Gauss coefficients $\overline{g}_{nm}$ and $\overline{h}_{nm}$ (usually zero except for $\overline{g}_{10}$ and $\overline{g}_{20}$) and for their variances $\sigma^g_{nm}$ and $\sigma^h_{nm}$ in the paleofield of the past few million years. In this respect our criteria may thus be viewed as characterising the regime of the geomagnetic field in the recent geological past, controlled by planetary parameters that remain unchanged on time scales of a few million years.

#### 2.1.1. Relative axial dipole power

The dominance of the axial dipole component is a primary property of the geomagnetic field. However, at the CMB non-dipole components and the equatorial dipole make a substantial contribution to the total field. The ratio of power in the axial dipole field to that in the rest of the field up to degree and order eight at the core–mantle boundary is given by

$$AD/NAD = P_{10} \Big/ \left( P_{11} + \sum_{n=2}^{8} \left(\frac{a}{c}\right)^{(2n-2)} \sum_{m=0}^{n} P_{nm} \right), \tag{1}$$

where $a$ is Earth's radius, $c$ the core radius, and

$$P_{nm} = (n+1)\left(g_{nm}^2 + h_{nm}^2\right) \tag{2}$$

is the power in a component of degree $n$ and order $m$ at the Earth's surface.

In the historical field the AD/NAD ratio has been steadily declining (Fig. 1a) to a value slightly less than one in 2010. Values prior to 1840 are perhaps overestimated because harmonics beyond $n=5$ are less resolved at earlier times and more damped in the gufm1 model. The

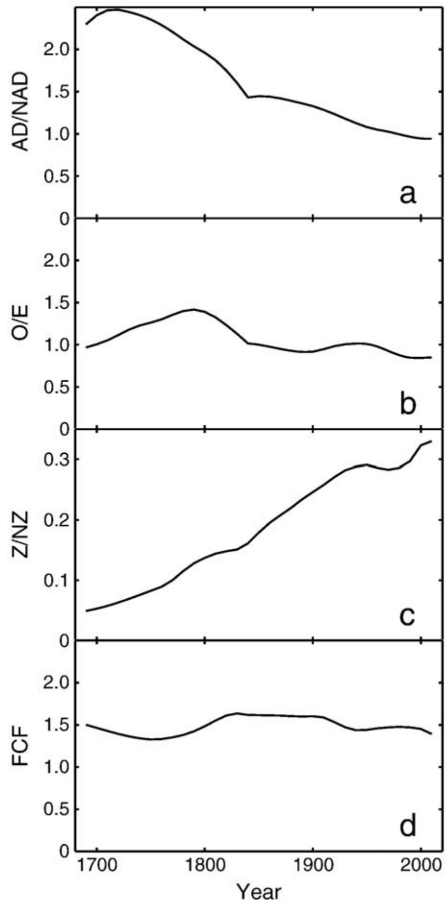


**Fig. 1.** Variation of (a) the axial dipole to non-axial dipole ratio, (b) odd–even ratio, (c) zonal–non-zonal ratio, and (d) flux concentration in the historical geomagnetic field.

time-average AD/NAD is 1.20 for the interval 1840–2010 and 1.58 for the time span 1690–2010.

For the time interval 1690–1950 the AD/NAD ratio in CALS7K.2 is 3.0 times higher than its value in the gufm1 model, reflecting the lower resolution and stronger damping of higher multipole components (Table 1). Applying a factor of three to renormalize the mean AD/NAD ratio of the past 7 kyr in CALS7K.2, we estimate that its value for the fully resolved field has been around 1.5.

The AD/NAD ratio can be calculated for statistical paleofield models, which assume that the various field components (their Gauss coefficients) vary in an uncorrelated manner with a Gaussian distribution. The time-average power in a specific field component is then

$$\overline{P}_{nm} = (n+1)\left( \overline{g}_{nm}^{2} + \left[\sigma_{nm}^{g}\right]^2 + \overline{h}_{nm}^{2} + \left[\sigma_{nm}^{h}\right]^2 \right), \tag{3}$$

which replaces $P_{nm}$ in Eq. (1) to calculate the mean AD/NAD ratio.

Three of the paleomagnetic models (Constable and Parker, 1988; Quidelleur and Courtillot, 1996; Tauxe and Kent, 2004) have AD/NAD

**Table 1**
Properties of historic and archeomagnetic models.

| Model | Time span | AD/NAD | O/E | Z/NZ | FCF |
|---|---|---|---|---|---|
| gufm1 + igrf | 1840 to 2010 | 1.20 | 0.94 | 0.235 | 1.52 |
| gufm1 + igrf | 1690 to 2010 | 1.58 | 1.06 | 0.156 | 1.48 |
| gufm1 | 1690 to 1950 | 1.76 | 1.09 | 0.116 | 1.49 |
| CALS7K.2 | 1690 to 1950 | 5.28 | 0.75 | 0.156 | 0.91 |
| CALS7K.2 | −5000 to 1950 | 4.48 | 0.78 | 0.202 | 1.25 |

values near one (from 0.88 to 1.09) and two of them (Constable and Johnson, 1999; Hatakeyama and Kono, 2002) have higher values of 2.6 and 2.8, respectively. Interestingly, those models leading to values closest to the historical average AD/NAD value are also found to be most compatible with the paleomagnetic data (Khokhlov et al., 2006).

We adopt a value of AD/NAD = 1.4 as being representative for the geomagnetic field. This is slightly lower than the average in the historical field (1690–2010) and in the archeomagnetic field of the past 7 kyr. The dipole moment in the recent field has been larger than its long-term time-average (Olson and Amit, 2006), which may imply a slightly higher than average AD/NAD ratio as well. With respect to the paleomagnetic field, our adopted value represents a compromise between the various models.

### 2.1.2. Equatorial symmetry

The dominance of the axial dipole makes the geomagnetic field (its radial component) predominantly antisymmetric with respect to the equator. For the non-dipole field there is less pronounced symmetry. Components with odd values of $(n+m)$ are antisymmetric and those with even values are symmetric. We use the odd–even ratio $O/E$ (Coe and Glatzmaier, 2006), which we define here as the ratio of power at the CMB of components that have odd values of $(n+m)$ for harmonic degrees between two and eight to the analogous power in components with $(n+m)$ even. In the historical field $O/E$ has varied between 0.84 and 1.42 with a mean value of 1.06 (Fig. 1b). For a purely random equipartioned non-dipole field $O/E = 0.833$. The value is less than one because there are more even-valued combinations than odd ones. Hence the historical non-dipole field shows a weak preference for antisymmetry.

The average $O/E$ ratio in the CALS7K.2 model for the past 7 kyr is 0.78, significantly lower than in the historical field. However, this is also the case for the time period of overlap with the historical field model (Table 1). Hence the archeomagnetic field model may be biased towards low $O/E$ values, which to some part is due to the strong damping of higher harmonics (for a random field the $O/E$ ratio drops to 0.75 when it is truncated at degree four). We conclude that there is no evidence for a strong preference of either odd over even components in the archeomagnetic non-dipole field.

Paleomagnetic data suggest a more pronounced dominance of odd terms in the long-term field. The odd–even ratio is related to the observed dependence of the dispersion of virtual geomagnetic pole positions (VGPs) on latitude (odd harmonics do not contribute to the VGP dispersion at the equator (Hulot and Gallet, 1996; Kono and Tanaka, 1995)). Most paleosecular variation models account for the increase of the VGP dispersion with latitude by assigning a larger variance to the Gauss coefficients $g_{21}$ and $h_{21}$ than to other quadrupole terms. The $O/E$ ratios are in the range 1.2–1.9 in most paleosecular variation models (Constable and Johnson, 1999; Hatakeyama and Kono, 2002; Quidelleur and Courtillot, 1996). Only the model by (Tauxe and Kent (2004), who assume that all coefficients with odd $(n+m)$ have much higher variance than even coefficients, arrives at $O/E = 11$. However, this is far above the values for the historical and archeomagnetic field and we see no plausible reason why the field should have been so different over the past few millenia compared to the past few million years. Besides, (Khokhlov et al. (2006) found this model to be less compatible with the paleomagnetic directional data than the model by Quidelleur and Courtillot (1996), which displays a much smaller value of $O/E = 1.2$. We therefore adopt a value $O/E = 1.0$ for our study, close to that in the historical field, which implies a slight preponderance for odd field components.

### 2.1.3. Zonality

We now consider the relative power of axisymmetric components in the non-dipole field. The zonal-to-non-zonal ratio ($Z/NZ$) is defined by the power in all zonal components from degree two to eight at the CMB to the power in the non-zonal components. This ratio has increased almost monotonically in the historical field from 0.05 to 0.33 (Fig. 1c) with a (geometric) mean of 0.156. This is slightly larger than the value for a purely random equipartioned field $Z/NZ = 0.100$ (there are 7 zonal and 70 non-zonal coefficients from $n = 2$ to 8). The mean $Z/NZ$ value for the 7 kyr archeomagnetic field is 0.20 and in the various statistical paleomagnetic models it ranges between 0.08 and 0.14, roughly consistent with the historical values. In particular, there is no indication for a dominance of zonal components in the non-dipole field, although a weak preference may exist. We adopt therefore $Z/NZ = 0.15$ as the characteristic value.

#### 2.1.4. Flux concentration

The magnetic flux at the core–mantle boundary is concentrated to some degree into patches of strong radial field $B_r$. As a quantitative measure we use the relative variance in the squared radial field and call it the flux concentration factor FCF:

$$FCF = \left[<B_r^4> - <B_r^2>^2\right] / <B_r^2>^2, \tag{4}$$

where <...> stands for the mean value taken over the spherical surface. A pure dipole field has FCF = 0.8. The flux concentration factor is sensitive to spatial resolution and becomes potentially larger with increased resolution. For example, its value in the GUFM1 field truncated at degree and order four is 1.16 compared to 1.48 for a truncation at eight. For the latter resolution, which we use here, the maximum possible flux concentration factor is of order thirty when all the flux emerges at a single very concentrated spot and re-enters the core uniformly over the rest of the sphere. In the other extreme, when the flux emerges uniformly in one hemisphere and re-enters uniformly in the other hemisphere, FCF approaches zero. In the historical field FCF has been around 1.5 without strong variations (Fig. 1d). The slightly low values prior to 1840 may be due to the insufficient resolution of the highest harmonics in the field. The mean value in CALS7K.2 of 1.25 is in reasonable agreement with the historical mean, given the reduced spatial resolution of that model. Statistical paleomagnetic field models are of limited value for estimating the average flux concentration in the paleofield, because they lack information on the phase relation between different spectral components. To test if the flux concentration factor in the historical field is typical for an arbitrary field with similar spectral properties as the present geomagnetic field, we have generated 5000 random field models with an AD/NAD ratio near 1.4 and a nearly white power spectrum at the CMB in degrees 2–8. For the random fields we find a (geometric) mean FCF value of 1.49 with a variation within a factor of 1.74 at the $3\sigma$ level. Hence the flux concentration of the geomagnetic field is neither unusually high nor low for the present degree of dipolarity. We adopt the mean value of the historical field, FCF = 1.5, as characteristic for the geomagnetic field.

### 2.2. Rating of compliance with geomagnetic field

In order to rate the morphological compliance of a model magnetic field with the geomagnetic field, we calculate time-average values of the four properties AD/NAD, $O/E$, $Z/NZ$ and FCF. For each property $\Pi_i$ ($i = 1$–4) we determine the squared logarithmic deviation from the geomagnetic field value $\Pi_i^E$ and normalize it with the acceptable standard deviation $\sigma_i$:

$$\chi_i^2 = \left[(ln(\Pi_i) - ln(\Pi_i^E)) / ln(\sigma_i)\right]^2. \tag{5}$$

The summary rating is then done on the basis of the sum of the individual $\chi_i^2$ deviations, which we denote by $\chi^2$ without index.

In Table 2 we list the parameter values used for the rating. Given the degree of variation in the historical field and the differences between historical mean, archeomagnetic mean and the values of the various paleomagnetic models, we basically consider a deviation by a factor of order two in each of the properties as acceptable. For the zonal-to-non-zonal ratio in the non-dipole field, whose historical value has varied over a wide range, we allow a larger deviation by a factor 2.5. The flux concentration factor has remained fairly constant in the historical field. Assuming a value of $\sigma = 2$ would mean that a flux concentration as weak as that of a pure dipole field would be considered acceptable. This does not seem reasonable and we set $\sigma = 1.75$ in the case of FCF.

**Table 2**
Rating parameters.

| | AD/NAD | O/E | Z/NZ | FCF |
|---|---|---|---|---|
| $\Pi^E$ | 1.4 | 1.0 | 0.15 | 1.5 |
| $\sigma$ | 2.0 | 2.0 | 2.5 | 1.75 |

Finally, we classify the compliance of a model with the Earth's field as excellent when the (total) $\chi^2$ is less than two and as good when $\chi^2 \leq 4$. The compliance is considered marginal when $\chi^2$ lies between four and eight and the model is called non-compliant for $\chi^2 > 8$. For example, a model is still rated as good when it deviates by a factor of one sigma in each of the four properties, or, when it fits perfectly in three of them and deviates by less than $2\sigma$ in the fourth.

### 2.3. Criteria not applied in this study

Often it has been noted that a particular dynamo model reproduces approximately the correct strength of the geomagnetic field. While at first glance this seems to be a very natural requirement for an Earth-like model, the comparison of the model field strength with that of the Earth is ambiguous. Simulations are typically performed in terms of non-dimensional variables and the re-scaling of the non-dimensional field strength to physical units is not unique. A more specific requirement would be that a non-dimensional measure of the field $B$, such as the Elsasser number $\Lambda = B^2/(\mu\eta\rho\Omega)$ or the Lorentz number $Lo = B/[(\mu\rho)^{1/2}\Omega D]$ matches Earth values ($\mu$ is magnetic permeability and $\rho$ is density). The two numbers are related by $\Lambda = E_\eta^{-1} Lo^2$, where $E_\eta = E/Pm$ is the magnetic Ekman number. It is not possible that $\Lambda$ and $Lo$ can simultaneously match their respective Earth values when $E_\eta$ is not Earth-like. This is the case in almost all dynamo models (an exception is the model by (Glatzmaier and Roberts (1995), but at the expense of an extremely large value of $Pm \gg 1$ and of using hyperdiffusivities, which means that the low value of $E_\eta$ applies only to the largest scales). It is not clear whether it would be more desirable to match the Earth value of the Elsasser number or that of the Lorentz number or of any other conceivable measure of the field strength.

Another plausible requirement seems to be that the model exhibits a secular variation behaviour that is characteristic for the Earth, for example, that the time scales of field variation (e.g. as defined in (Hulot and Le Mouël, 1994)) are the same. Here again the question of re-scaling non-dimensional time to real time comes in. Two possible ways to scale time are by using the rotation period or the magnetic diffusion time. Again, these two options of re-scaling will give the same result only if the magnetic Ekman number in the model matches the core value. Most authors have used magnetic diffusion time for scaling. In this case the condition of obtaining Earth-like values of secular variation time scales is linked to having an Earth-like value of the magnetic Reynolds number $Rm = UD/\eta \approx 1000$ (Christensen and Tilgner, 2004), where $U$ is the characteristic flow velocity.

The shape of the power spectrum of the non-dipole field is nearly white at the core mantle boundary, except for the quadrupole, which is low by a factor of two to three in the historical field and by a similar factor in CALS7K.2. We did not use the whiteness of the spectrum as one of our criteria, because most dynamo models show approximately white power spectra. Some but not all dynamo models have low quadrupole power. This suggests that the requirement for low quadrupole power, whose persistence in the historical and archeomagnetic field carries some statistical significance (Hongre et al., 1998), could possibly be used as additional criterion. But the compatibility of such criterion with the paleomagnetic data remains to be assessed, and for the time being we do not use this criterion either.

Often westward drift of magnetic structures has been observed in geodynamo models and taken as an indicator of Earth-like behaviour (e.g. Sakuraba and Roberts, 2009). However, in the historical field westward drift is restricted to the Atlantic hemisphere. Such hemispherical properties may well reflect the influence of inhomogeneous heat flux pattern at the core–mantle boundary (Aubert et al., 2007; Christensen and Olson, 2003).

An Earth-like dynamo model should exhibit occasional dipole reversals at a frequency comparable to what is known from the paleomagnetic record. Aside from the problem of scaling time, this is a relevant and objective criterion. The occurrence of reversals is sometimes considered as being essential for an Earth-like dynamo model. However, as noted, the reversal frequency appears to be sensitive to inhomogeneous heat flux pattern at the core mantle boundary. In addition, to obtain a meaningful statistics on reversals, a dynamo model must be run for many magnetic diffusion times. While this can be done at high Ekman numbers of order $10^{-3}$ (e.g. Driscoll and Olson, 2009), it is not possible at $E \leq 10^{-4}$. To employ this criterion is therefore impractical when one aims at a comparison of models that cover the full range of the accessible control parameter space.

Sometimes the existence of magnetic flux bundles (also called lobes or patches) at high latitudes has been used to claim semblance with the geomagnetic field (e.g. Christensen et al., 1998; Gubbins et al., 2007). These flux lobes have been persistent in the historical geomagnetic field, but the evidence for their existence in the paleofield is still debated (Hulot et al., 2010). Furthermore, it is difficult to define a quantitative measure for the presence or absence of flux lobes. Finally, their number (two in each hemisphere) and possible stationarity would likely again reflect a possible degree and order two heat flow heterogeneity at the CMB (Gubbins et al., 2007; Olson and Christensen, 2002), which is not accounted for in simple dynamo models. Other conceivable criteria that rely on local properties of the historical CMB field, such as weak flux at the rotation poles or specific field structures at low latitudes (e.g. Finlay and Jackson, 2003), likewise suffer from uncertainties as to how representative they are for the long-term geomagnetic field and how to appropriately quantify them.

Occasionally a dynamo model has been claimed Earth-like not (only) based on the properties of its magnetic field, but in the sense that it satisfies a particular dynamical constraint which is thought to be relevant for Earth's core, for example that it is in the so-called Taylor state (e.g. Kuang and Bloxham, 1997; Takahashi et al., 2005). While the compliance with relevant physical conditions is very desirable for any model, there is no unambiguous observational evidence that the Earth's core is close to a Taylor state and there is also some ambiguity in proving this state for a dynamo model (see Wicht and Christensen (2010) for Taylor states in numerical dynamos).

## 3. Dynamo models

We evaluated the degree of compliance with our criteria for 155 MHD models of convection-driven dynamos in rotating spherical shells. Many of them have been reported before (Christensen and Aubert, 2006; Christensen et al., 2009), whereas others are new. In most of our cases convection is driven by imposed constant temperatures at the inner and outer boundaries. These models cover a rather broad part of the accessible parameter space. Each of the basic control parameters is varied by at least two orders of magnitude. In other cases we impose a constant flux on the boundaries, which may represent compositional flux or heat flux or a combination of both, although for simplicity we usually describe it as a heat flux. We also vary the ratio of total flux at the inner boundary to that at the outer boundary in models with volumetric source or sink terms. This represents different proportions of compositional driving plus driving by latent heat of inner core freezing to driving by secular cooling and radiogenic core heating (Aubert et al., 2009).

A general outline of the model concept can be found in Christensen and Aubert, (2006) and Aubert et al., (2009). The ratio of inner boundary radius to outer radius is fixed to the present Earth value of 0.35 and the boundaries are mechanically rigid. When homogeneous flux conditions are used, the integrated superadiabatic heat fluxes $F_i$ at the inner radius $r_i$ and $F_o$ at the outer radius $r_o$ are fixed and an internal source/sink term $S$ is set such that the fluxes are in balance:

$$S = \frac{3(F_o - F_i)}{4\pi(r_o^3 - r_i^3)}. \tag{6}$$

Aside from this, the dynamo is characterized by the three control parameters introduced in section 1, the Rayleigh number $Ra$, Ekman number $E$, magnetic Prandtl number $Pm$, and additionally by the Prandtl number $Pr = \nu/\kappa$. In models driven by a fixed temperature contrast, the Rayleigh number is

$$Ra_T = \frac{\alpha g \Delta T D^3}{\kappa \nu} \tag{7}$$

and when flux conditions are used it is given by

$$Ra_F = \frac{\alpha g F D^2}{\kappa^2 \rho c_p \nu}, \tag{8}$$

where $c_p$ is heat capacity and $F = F_o + F_i$. In order to facilitate the comparison between corresponding fixed flux and fixed temperature models, we calculate for the fixed flux models the value of $Ra_T$ *a posteriori*, by using the difference between the spatially and temporally averaged boundary temperatures for $\Delta T$ in Eq. (7).

Some of the earlier reported models (Christensen and Aubert, 2006; Christensen et al., 2009) employed symmetry conditions in longitude. Because our quantitative measures for the field morphology can be affected by this assumption, we use only full sphere dynamo simulations. In some cases we continued earlier model runs with the symmetry assumption relaxed and the missing modes excited. All models have been run for at least 50 advection times $D/U$, with $D$ the shell thickness and $U$ the rms-velocity of the convective flow in the rotating frame of reference. This had been found to be sufficient for bulk properties, such as the mean kinetic and magnetic energies, to reach a statistical equilibrium. The properties of the CMB field were averaged for at least 30 advection times, rejecting the initial transient of a model run. Taking the advection time in Earth's core to be 150 yr, this corresponds to roughly 5000 yr of averaging. This averaging time interval may be insufficient to determine a long-term mean of some of the morphological measures with high precision. However, the scatter introduced by the short averaging time is balanced by the large number of model cases, which probably ensures a robust statistics.

To determine the variation of the compliance with the geomagnetic field for the full four dimensional parameter space (defined for example by $Ra$, $E$, $Pm$ and $Pr$) is not possible, even when setting aside the influence of different boundary conditions. We must rather try to identify an appropriate lower-dimensional subspace. Here we found it useful to employ the magnetic Ekman number $E_\eta = E/Pm = \eta/(\Omega D^2)$, also called the magnetic Rossby number by some. As second important parameter we need a measure for the convective vigor or the flow velocity. While the core values of $E$ ($E_\eta$), $Pr$, and $Pm$ are known within a factor of a few, the value of the Rayleigh number is very uncertain. Instead of a Rayleigh number we therefore use the magnetic Reynolds number $Rm = UD/\eta$ along with $E_\eta$ to characterize the dynamo model. A disadvantage is that the value of $Rm$ is not known apriori, but is a model result. Scaling relations (Aubert et al., 2009; Christensen and Aubert, 2006; Olson and Christensen, 2006) can be used to approximately link $Rm$ to the Rayleigh number and the other control parameters. The value of $Rm$ in Earth's core is approximately known by the flow velocity inferred from secular variation.

The appeal of using the combination ($Rm$, $E_\eta$) to characterize the dynamo is that these parameters represent the ratios between the potentially most important time scales in rotationally controlled dynamos: the advection time, the magnetic diffusion time, and the time scale of rotation. The two parameters are independent of the viscosity and the thermal diffusivity, which are both very small in the geodynamo and which have been found to have little effect on the scaling of the magnetic field strength and flow velocity (Christensen and Aubert, 2006).

## 4. Results

In Fig. 2 we compare for illustrative purposes snapshots of the radial field at the outer boundary of several dynamo models, filtered to degree eight, with the geomagnetic field at the core surface expanded to the same degree (Fig. 2a). The case in Fig. 2b agrees well in all criteria and is rated excellent. The model in Fig. 2c has a very small axial dipole contribution and is clearly non-compliant. In contrast, in Fig. 2d the axial dipole is too dominant. In addition, the non-dipole field is too antisymmetric with respect to the equator, its zonal components are too strong compared to the non-zonal part, and the (unsigned) flux is too evenly distributed. This case is also rated non-compliant. The case in Fig. 2e is fair in most properties. The most serious deviation from the Earth's field is a too pronounced concentration of the field into a small number of strong flux patches. The overall rating for this case is 'marginal'.

### 4.1. Parameter dependence

Fig. 3 shows for all models with a temperature boundary condition the degree of compliance of the field morphology with that of the geomagnetic field. White symbols (no fill) are for cases that do not agree with the geomagnetic field at all ($\chi^2 > 8$). Earth-like models are shown by dark grey and black fill ($\chi^2 \leq 4$ and $\chi^2 < 2$, respectively). They fall into a wedge-shaped region bounded by broken lines in Fig. 3. A few non-compliant cases also fall into this wedge, but they lie close to the boundaries. For a model to be Earth-like, the magnetic Ekman number must be less than approximately $10^{-4}$. The magnetic Reynolds number must neither be too small nor too large. The range of suitable values of $Rm$ depends on the magnetic Ekman number. Lower values of $E_\eta$ require larger values of $Rm$. Cases with too low $Rm$ are typically too dipolar, too (anti) symmetric with respect to the equator and the non-dipole field contains too much zonal energy. Dynamos with a high $E_\eta$ to the right of the compliant wedge region in Fig. 3 are non-dipolar, except at very low $Rm$. Cases with a very large magnetic Reynolds number on the top left in Fig. 3 show too much flux concentration. Their AD/NAD ratio is too small, but not extremely low as in the case of non-dipolar dynamos.

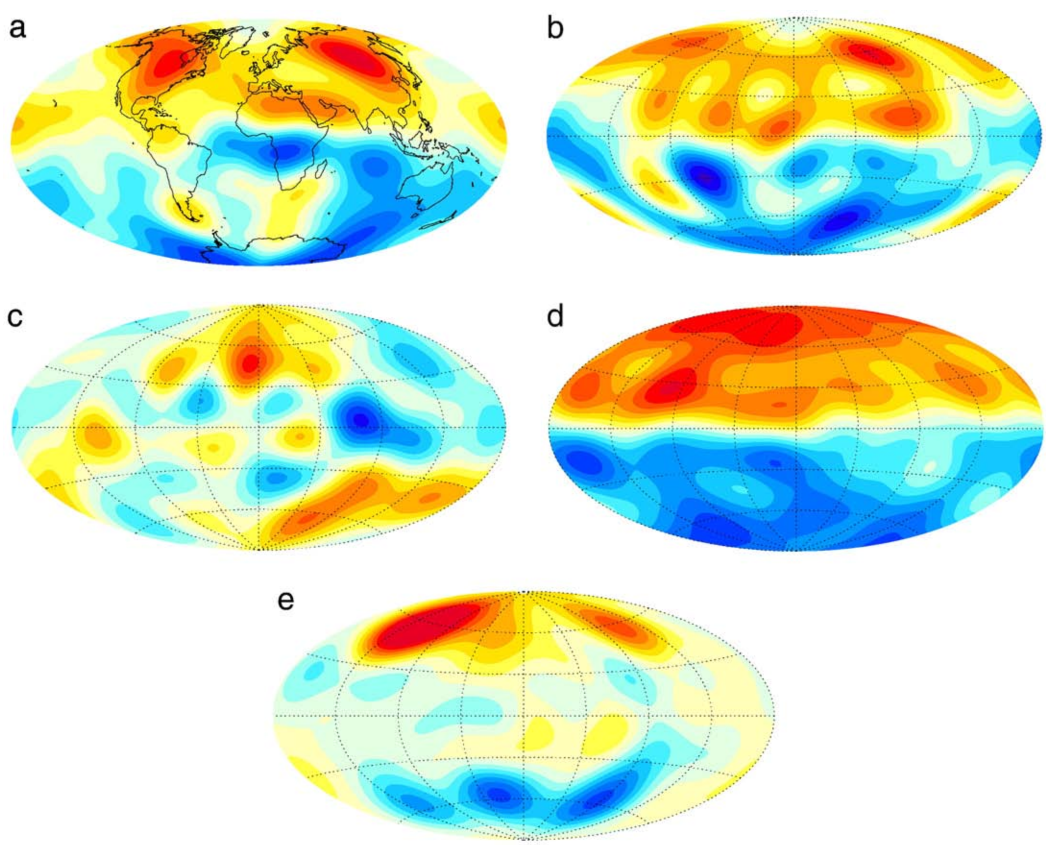


**Fig. 2.** Radial field at the outer boundary of the dynamo expanded up to degree eight. (a) Geomagnetic POMME model for 2005 (Maus et al., 2006), (b)–(e) dynamo models with parameters ($E$, $Ra_T$, $Pm$, $Pr$) and instantaneous morphology properties [AD/NAD, O/E, Z/NZ, FCF] and time-average $\chi^2$ in (b): ($3\times10^{-5}$, $3\times10^{8}$, 2.5, 1), [0.92, 0.96, 0.19, 1.33], $\chi^2 = 0.3$; (c): ($10^{-5}$, $1.7\times10^{9}$, 0.5, 1), [0.01, 0.81, 0.20, 2.18], $\chi^2 = 9.2$; (d): ($3\times10^{-6}$, $4\times10^{8}$, 1, 1), [5.02, 4.54, 1.20, 0.51], $\chi^2 = 20$; (e): ($10^{-3}$, $5.2\times10^{5}$, 12, 1), [0.62, 2.35, 0.45, 5.94], $\chi^2 = 7.7$. Cases c and d are with fixed temperature conditions, cases b and e use fixed zero flux on the outer boundary.

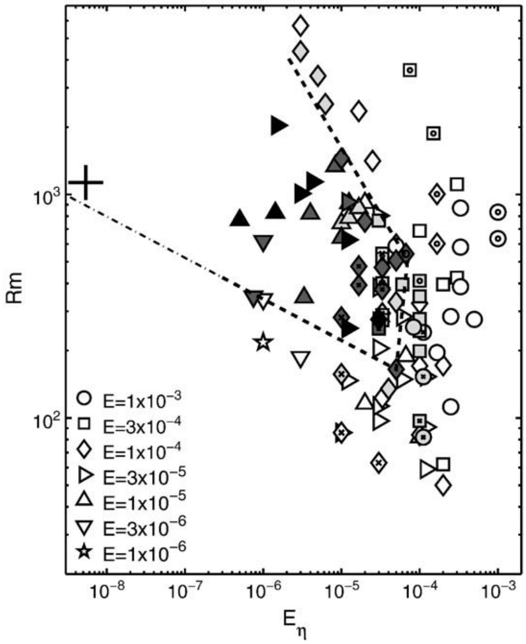


**Fig. 3.** Compliance of field morphology with that of the geomagnetic field for dynamo models with fixed temperature boundary conditions plotted as function of magnetic Reynolds number and magnetic Ekman number. Symbols with black fill show excellent agreement, dark grey good agreement, light grey marginal and white cases are non-compliant. The symbol shape is keyed to the Ekman number, a value of Pr>1 is indicated by a cross inside the main symbol and Pr<1 by a circle, all others have Pr = 1. The region of Earth-like dynamos in the $Rm-E_\eta$ parameter space is bounded approximately by the broken lines. The cross indicates the approximate location of Earth's core.

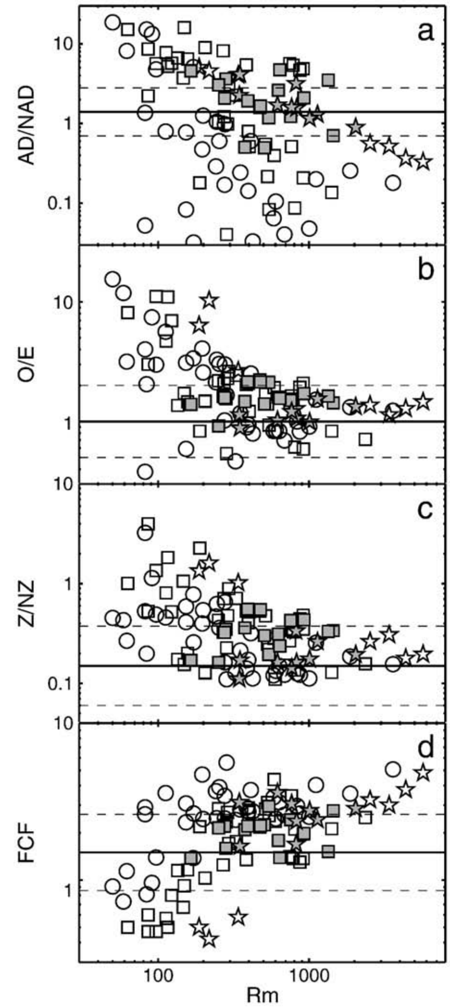


**Fig. 4.** The four morphological field properties vs $Rm$. Stars are for $E_\eta<7\times10^{-6}$, circles $E_\eta>7\times10^{-5}$ and squares for intermediate values. The thick horizontal line is the nominal value for the geomagnetic field with $1\sigma$ tolerance range shown by broken lines. Grey fill is for models with good or excellent overall rating.

An Earth-like value of the magnetic Ekman number of about $5\times10^{-9}$ is two orders of magnitude below the lowest model value in Fig. 3. Extrapolating the lower boundary of the compliant region to $E_\eta=5\times10^{-9}$ (dash–dotted line in Fig. 3) results in a minimum value of the magnetic Reynolds number of 900. Estimates of the Earth value of $Rm$ (Christensen and Tilgner, 2004) are of that order. The extrapolation of the compliant wedge region seems to encompass the location of geodynamo, shown by the large cross in Fig. 3, but perhaps only marginally so.

Fig. 4 shows separately the values for the four morphological properties plotted against the magnetic Reynolds number. Models with a large magnetic Ekman number are shown by circles, medium values by squares and small values by stars. Models rated good or excellent are highlighted by grey fill of the symbol. The largest variations occur in the AD/NAD ratio, which contributes most to distinguishing between compliant and non-compliant models. However, there are cases with a very Earth-like AD/NAD value that are downgraded to only marginally compliant because of the misfit in other properties. In general, the dipole/non-dipole ratio, the odd–even ratio and the zonal/non-zonal ratio decrease with increasing $Rm$, whereas the flux concentration factor increases. For models rated as good, the maximum deviation in any one of the four properties is typically less than $1.5\sigma$.

*4.2. Influence of thermal boundary condition*

For a number of cases that cover the full range in magnetic Reynolds number and magnetic Ekman number that we explored so far, we have replaced the fixed temperature condition on the outer boundary by a condition of fixed homogeneous flux, keeping the values of $E$, $Pm$ and $Pr$ unchanged and tuning the flux Rayleigh number $Ra_F$ such that the value of the Rayleigh number $Ra_T$ based on the temperature contrast and the convective power are almost identical between the corresponding models. With very few exceptions we find as a trend upon changing to a flux condition that the AD/NAD, $O/E$ and $Z/NZ$ ratios become smaller whereas FCF becomes larger (Fig. 5). Depending on how well the original fixed temperature model fitted the Earth values, this can either improve or deteriorate the compliance with the geomagnetic field. In many cases an improvement in the power ratios is accompanied by an impairment of the flux concentration factor and the overall rating of the compliance does not change very much.

For two selected cases with $E=10^{-4}$ and $Pm=3$ or $Pm=7$, respectively, we investigated the influence of different distributions of the sources and sinks of buoyancy, varying the ratio between inner core flux and the sum of the fluxes on both boundaries $F_i/(F_o+F_i)$ between zero and one. The former value corresponds to internal heating (secular cooling) and the second to purely compositional convection with a neutrally stable temperature gradient at the CMB. We adjusted the Rayleigh number such that the magnetic Reynolds number stayed constant within 3% at 380 and 850, respectively. Upon increasing the driving from below, the field tends to become more strongly dipolar and the non-dipole field becomes less zonal (Fig. 6). There are no clear trends for the odd–even ratio and the flux concentration factor. The $\chi^2$ values lie in a limited range between 2 and 6, i.e., the compliance of these models is good to marginal. The cases with $Rm=380$ become generally more

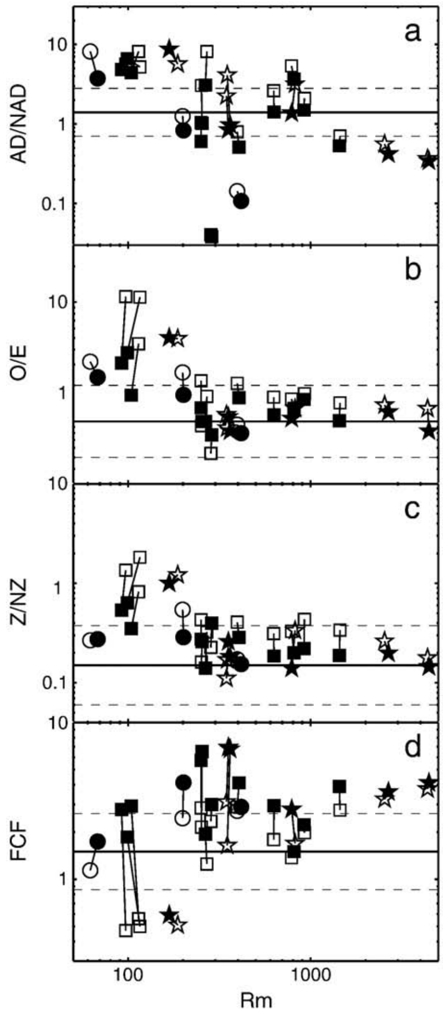


**Fig. 5.** As Fig. 4, but with dynamo models with fixed heat flux condition shown by filled symbols. The corresponding fixed temperature cases are shown by open symbols and linked with a connecting line.

compliant the stronger they are driven from below, whereas at $Rm = 850$ there is no clear trend.

Fig. 7 shows the overall compliance for models with a flux boundary condition, sorted by their values of the magnetic Reynolds number and the magnetic Ekman number. Cases with equal flux on the inner and outer boundaries (symbols with grey rim) and those with zero flux on the outer boundary (compositional convection, black rim) are included. Compliant models lie in roughly the same wedge-shaped region of the parameter space as was found for fixed temperature models. However, Earth-like cases now reside also in the apex part of the wedge region that has been cut out in Fig. 3, but is indicated by dotted lines in Fig. 7. A slightly larger minimum value of $Rm$ may be necessary for a good compliance rating in the case of flux boundary conditions. However, more case studies would be needed to confirm this.

## 5. Discussion and conclusions

Whether a dynamo model is 'Earth-like' in the sense that it reproduces basic morphological properties of the geomagnetic field as defined here is primarily a question of the 'right' combination of control parameters. Such combinations exist in regions of the parameter space that are remote from Earth's core values. Basically the recipe for an Earth-like model is that the magnetic Ekman number

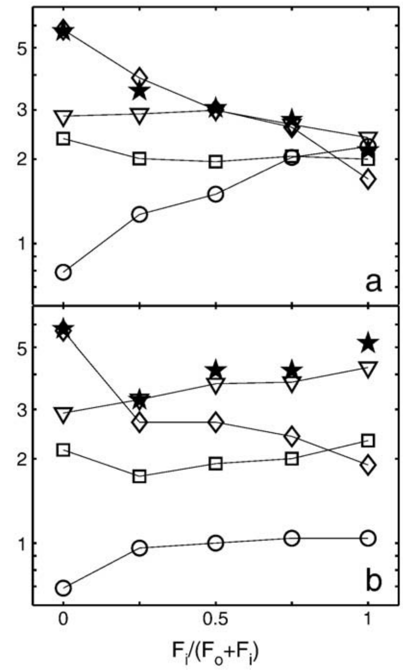


**Fig. 6.** Morphological field properties vs. the buoyancy flux distribution for models with $E = 10^{-4}$. Circles: AD/NAD, squares: O/E, diamonds: $10 \times$ Z/NZ, triangles: FCF. Stars indicate the overall $\chi^2$ value. (a) $Rm \approx 380$, (b) $Rm \approx 850$.

must be less than approximately $10^{-4}$ and that the magnetic Reynolds number must be high enough. The minimum value depends on the magnetic Ekman number. It is not very well defined by our results, but tentatively it can be taken as

$$Rm_{min} \approx 27 E_\eta^{-2/11}. \tag{9}$$

The magnetic Reynolds number must not be too high either. Taking twice the minimum value should usually give a very Earth-like

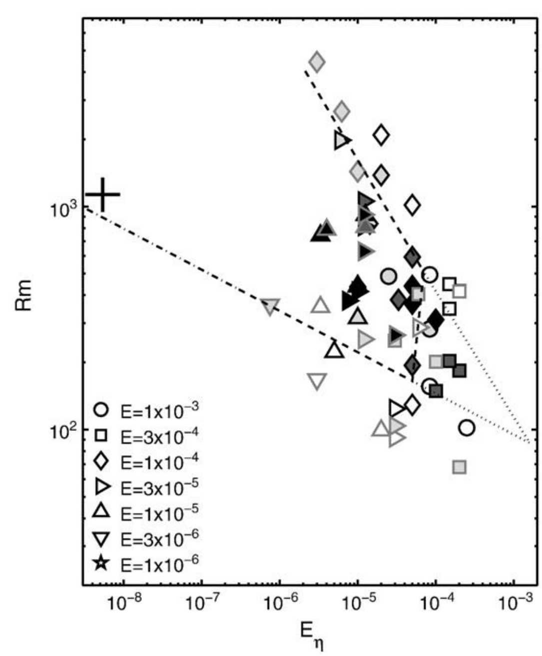


**Fig. 7.** As Fig. 3, but for cases with a fixed flux condition on the outer boundary. Symbols with grey rim have equal flux on the inner and outer boundaries and those with black rim have zero flux on the outer boundary. Broken lines copied from Fig. 3.

magnetic field structure. The magnetic Reynolds number is not a control parameter, but it can be related to them by re-casting the scaling laws in Christensen and Aubert (2006) for cases with fixed temperature boundary conditions:

$$Rm = 0.09(Ra_T / Pr)^{0.87} E^{0.74} Pm. \tag{10}$$

At low magnetic Reynolds number the degree of order in the field is high, which results in a strong dominance of the dipole and too much equatorial symmetry and too much axisymmetry compared to the geomagnetic field. At higher $Rm$ the dynamo becomes magnetically more turbulent, to a degree that seems realistic for the geodynamo. If the magnetic Reynolds number is too high, the field is weakly dipolar. Also, an unrealistic degree of concentration of magnetic flux into a few very intense spots at the core mantle boundary is facilitated by the comparatively small role of magnetic diffusion.

The (magnetic) Ekman number controls the degree of symmetry and order that is imposed on the flow by rotational constraints. For a fixed value of $Rm$ the flow becomes more geostrophic at lower $E_\eta$. If at the same time driving of convection is weak (low $Rm$), the flow can be very geostrophic and the associated order of the flow is a prerequisite for an unrealistic degree of symmetry of the magnetic field. At higher $E_\eta$ geostrophy is weakened, which favours a more chaotic and more small-scaled field. The finding that some models inside the wedge-shaped regions in Fig. 3 still show a marginal or poor fit may indicate that the field morphology depends on all control parameters and that the dependence on just two parameters is only an approximation.

Low values of the magnetic Ekman number can be obtained either by a low value of $E$ or a high value of $Pm$. To generate a dynamo model with a given combination ($E_\eta$, $Rm$) it is computationally less demanding to choose rather larger values of both $E$ and $Pm$ than smaller values. While to first order the magnetic field morphology seems to be controlled by the combination $E/Pm$ ($=E_\eta$), it is not entirely clear within what limits only the ratio of the two parameters matters.

In previous models that aimed at reaching a very low Ekman number (Jones, 2007; Sakuraba and Roberts, 2009; Takahashi et al., 2008) the magnetic Reynolds number has been fairly small because of computational limitations. Our results explain why in these cases the field structure was usually not very Earth-like: at low $E_\eta$ higher values of $Rm$ are needed to obtain a morphology similar to that of Earth's field. A tentative extrapolation of the parameter region of compliant dynamos comprises the Earth values of $E_\eta$ and $Rm$. This suggests that Earth-like dynamos exist in a continuous swath of the parameters space connecting current models with the actual geodynamo. For our estimate of the core value of the magnetic Reynolds number we used a characteristic velocity of 15–20 km $yr^{-1}$ (Hulot et al., 2002) and a value of $6\times10^5$ S $m^{-1}$ for the electrical conductivity (Secco and Schloessin, 1989). Possible lower conductivity values (Stacey and Loper, 2006) would lead to $Rm\approx500$, which lies outside the extrapolated bounds of the compliant region in parameter space. It is desirable to ultimately test our extrapolation by running a model at $E_\eta\approx5\times10^{-9}$ and $Pm\approx1$. However, this goal, which has been called the 'grand challenge' by (Glatzmaier (2002), should be augmented by the requirement that the magnetic Reynolds number is in the range of possible Earth values.

The choice of boundary conditions, fixed temperature or fixed flux at the core–mantle boundary, and the distribution of buoyancy sources have some effect on the magnetic field morphology. In general, it is secondary compared to the influence of the control parameters. But with the condition of fixed zero flux at the core–mantle boundary compliant cases are also found at high values of $E_\eta\geq10^{-4}$. These are the cheapest Earth-like dynamos that can be obtained in the sense of the lowest computational expense needed. Such models have been used, for example, for an extensive study of the dipole reversal frequency (Driscoll and Olson, 2009). However, at high $E_\eta$ Earth-like models exist only in a narrow range for the magnetic Reynolds number (or Rayleigh number).

An important question is if models that have an Earth-like magnetic field morphology according to our criteria display dipole reversals. Near the maximum possible value of the magnetic Ekman number, where long simulations can be performed, this seems often to be the case (Driscoll and Olson, 2009; Wicht et al., 2009). To test this at low $E_\eta$ is desirable, but with current computational resources impractical to do. Because the reversal frequency seems very sensitive to relatively small changes in parameter values or boundary conditions, as shown in numerical simulations (Driscoll and Olson, 2009; Glatzmaier et al., 1999; Kutzner and Christensen, 2004) and by the strong changes of the observed geomagnetic reversal frequency on the 100 million year time scale, an Earth-like field morphology is not necessarily synonymous with an Earth-like reversal frequency. Most of our compliant simulations have not been run long enough to observe reversals. (Coe and Glatzmaier (2006) suggest that the $O/E$ ratio is correlated with the reversal frequency (low $O/E$ associated with frequent reversals). Hence the degree of equatorial symmetry in the field of a model could perhaps serve as a proxy for the likelihood and frequency of dipole reversals on a long time scale.

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