

TRANSIENT MAGMATIC CONVECTION PROLONGED BY SOLIDIFICATION

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Abstract. Magma chambers cool from all sides, with expected vigorous convection. Strong changes in viscosity with crystallization, however, lock up within an inwardly propagating crust a great deal of buoyancy otherwise available to drive convection. With sufficiently small degrees of superheat, the Rayleigh number (Ra) is small and convection is transient and never becomes fully developed. In order to understand this transient stage of convection, we have performed some fluid dynamic experiments on the solidification of a paraffin layer. Except in the growing crust and its thin forward boundary layer, the interior remains isothermal and the temperature decreases uniformly until it is locked at the convective liquidus. The crystals are fine and hair-like dendrites and no major differentiation of the fluid composition is observed. The time-scale of the convective stage is predictable and is very short relative to the time of complete solidification. We also report measurements over time of crust thickness, convective velocity, and heat transfer. When the interior temperature reaches the convective liquidus, convection wanes, although the calculated Ra , based on an arbitrary small temperature contrast of 0.2°C , is still large ($\sim 10^7$). A small ($\pm 0.2^\circ\text{C}$) uncertainty in ΔT (i.e., $0 \leq \Delta T \leq 0.2^\circ\text{C}$) produces a large uncertainty in Ra (i.e., $0 \leq Ra \leq 10^7$). This uncertainty in Nature may preclude an accurate determination of Ra in a magma chamber residing at a temperature very close to the convective liquidus.

Introduction

In 1905 L. V. Pirsson described in some detail how crystallization-induced convection at the margins of a magma chamber could greatly facilitate crystal settling, fractionation, and differentiation. Ordinarily slowly settling crystals could be swiftly carried downward by descending convective currents to be deposited as the current approaches and spreads across the chamber floor. Later, F. F. Grout (1918) again clearly enunciated the process with the additional concept of a strongly variable magmatic viscosity due to large changes in crystallinity at the margins. This form of convection and crystal transfer has many times since been invoked to explain magmatic differentiation (e.g., Wager and Deer, 1939; Brandeis and Jaupart, 1986; Marsh, 1989).

These considerations and results often suggest that most any body of basaltic magma will reveal the effects of this process through a systematic variation in composition downward through a sheetlike body. The overall process involving actual crystallization and convection in aqueous solutions has been experimentally modelled by Kerr et al. (1989). They observe strong differentiation and suggest that this might explain the large compositional change observed in sills such as the Palisades sill (Shirley, 1987). However, there are also numerous thick sills and lava flows that show no or very little effect of internal differentiation (e.g., Stott

Mountain gabbro, Columbia River basalt flows, etc; Mangan and Marsh, 1990). We have thus tried to investigate experimentally how this might occur by studying the cotectic solidification and convection by cooling from above of a fluid that may dynamically more closely resemble magma, paraffin. We have previously (Brandeis and Marsh, 1989; henceforth designated as (ref. I)) shown how the overall experimental thermal history resembles that observed for Hawaiian lava lakes. In contrast, here we concentrate on the early vigorous or transient period of convection, for it is the redistribution of crystals during this stage that may influence the final state of the body. We emphasize the differences between this cotectic paraffin system, with fine or hair-like dendritic crystals, and the binary eutectic aqueous system of coarse crystals studied by Kerr et al. (1989). We feel it is particularly important to understand fully these two systems as they may well approximate endmember analogues to actual magmatic systems.

Experimental set-up

The experimental set-up has been described in ref. I. The fluid is a mixture of pure paraffins (40% n-pentadecane $\text{C}_{15}\text{H}_{32}$ and 60% n-nonadecane $\text{C}_{19}\text{H}_{40}$) and forms a binary solid-solution in the system $\text{C}_{15}\text{H}_{32}$ - $\text{C}_{19}\text{H}_{40}$ (Turner, 1970). It has a melting range of $\sim 5^\circ\text{C}$ and no eutectic. The liquidus temperature is $T_L \sim 22.7^\circ\text{C}$. The residual liquid is lighter and the crystals denser than the initial liquid. Physical properties are listed in ref. I and more details can be found in Humphries and Griggs (1977). We used a $20 \times 20 \times 20$ cm Plexiglass tank with 2 cm thick walls and bottom. The upper boundary is a metal plate in which thermostated water circulates. Temperatures are monitored by a vertical array of 7 thermocouples, and at the upper boundary by a thermocouple buried in the cooling plate. The accuracy of the temperature measurements is $\pm 0.2^\circ\text{C}$. Distances between the thermocouples and the cold boundary have been measured with a caliper to an accuracy of ± 0.05 cm. At the onset of cooling, the initial temperature in the fluid is uniform and equal to T_0 , and only slightly ($\sim 3.5^\circ\text{C}$) above the liquidus T_L . At time $t=0$, when the metal plate is switched to the cold bath and begins cooling to T_f . The Rayleigh number (Ra) is $\sim 6 \times 10^8$ and the Stefan number (Ste) is ~ 0.2 .

Interior temperature during convective cooling

As cooling proceeds, convection sets in almost immediately at the roof and spreads quickly throughout the tank. It is 3-D and unsteady, however, it is not turbulent in the usual sense as will subsequently be seen, for the governing Reynolds number is always relatively small (< 350). Below the crust and a thin boundary layer, the interior is always isothermal (ref. I). The interior temperature T_i decreases uniformly with time to the liquidus, convection ceases and further cooling is by conduction (Figure 1). The cooling is partitioned into the two distinct stages: Stage I, defined by active convection and a rapidly decreasing interior temperature; and Stage II when the interior temperature is locked to a temperature which we have called the convective liquidus and all cooling is dominated by conduction. We are interested in this paper in the transient period or Stage I.

The evolution of the interior temperature can be calculated using the model of Jaupart and Brandeis (1986; henceforth ref. II) for the cooling by convection of a constant viscosity

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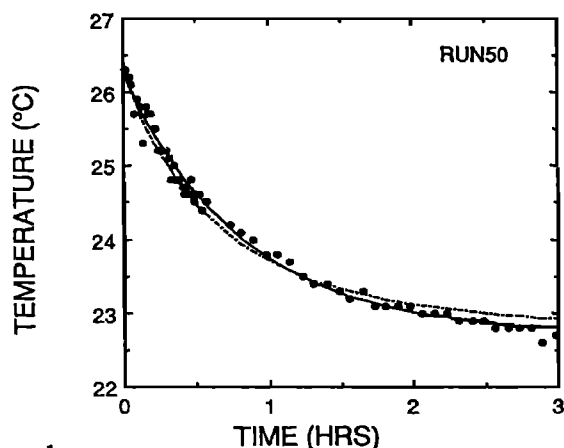


Figure 1

Evolution of the interior temperature for experiment 50 (roof cooling with $\Delta T=3.6^\circ\text{C}$ and $T_f=8.6^\circ\text{C}$). Also shown are the theoretical curve derived from equation (1) (dashed line) and the curve described by equation (4) (solid line).

fluid cooled simultaneously from the top and the bottom. Although cooling in the present experiments is only from the top, cooling from the bottom (as in ref. II) is due to conduction only and is thus always relatively small. Therefore, the cooling of a layer of thickness d by convection induced by top cooling can be adequately described by a layer of the same thickness d cooling from both top and bottom. Furthermore, since the viscosity changes in the superheated paraffin are small over the temperature interval considered here ($<10\%$; Humphries and Griggs, 1977), the constant viscosity theory of ref. II is also appropriate here, where the temperature contrast $\Delta T = T_0 - T_L$. For a layer of thickness d , cooled by ΔT , the interior temperature decreases as (ref. II)

$$T_i - T_L = \Delta T [1 + t/\tau]^{-3} \quad (1)$$

where the characteristic time-scale of convection (τ) is defined by

$$\tau = 14.3 (d^2 / \kappa) \text{Ra}^{-1/3} \quad (2)$$

the Rayleigh number (Ra) is $\alpha g \Delta T d^3 / \kappa$ and α is thermal expansion, g is gravity, κ is thermal diffusivity and ν is kinematic viscosity. The curve derived from (1) is shown in Figure 1 along with the experimental data for experiment 50 where $\Delta T=3.6^\circ\text{C}$. Note that the agreement is best in the early part of Stage I when convection is vigorous. The theory departs from the data after ~ 1.5 hrs, when convection is dying. This theory, nevertheless, is still useful as it allows prediction of the overall characteristic time scale of the active convection of Stage I. A similar fit was also not very good at large times in the experiments of ref. II. This is expected because the theoretical relationship between Nusselt number and Rayleigh number, which depends on the assumption of large Ra , is no longer true when convection is waning. For these data, it has been found that the best representation of temperature during the whole of Stage I is given instead by the empirical function

$$T_i - T_L = \Delta T \exp - [bt] \quad (3)$$

where for experiment 50, $b = 3.41 \times 10^{-4} \text{ s}^{-1}$, and the curve obtained from this expression is also shown by Figure 1. The overall convective heat flux (Q_{cv}) closely follows a parametric

law of $Q_{cv} \sim T$, and not $T^{4/3}$ as is often assumed in large Ra flows. This is possibly another indication of the nonturbulent nature of the convection.

Interior temperature during conductive cooling

To establish a reference state free from convection, the same layer was cooled solely from below (Figure 2; experiment 51); the initial conditions are otherwise identical to experiment 50. The only possible convection here would be due to compositional effects. Now the interior temperature does not decrease uniformly to the liquidus. The cold front slowly propagates upwards and the temperature at the highest portion of the tank does not start to decrease before 1 hr. The temperature at 11.8 cm decreases to the convective liquidus in ~ 9 hrs. by conduction (Run 51) compared to 3 hrs. by convection (Run 50). A comparison of Figures 1 and 2 shows the relatively poor efficiency of bottom cooling. This supports our previous approximation in using the theory of ref. II to describe these slightly different experimental conditions. This is interesting in light of the possibility that the lighter rejected fluid may rise and induce some convective cooling. No fluid motion was detected, although some motion can not be wholly discounted because at the close of the experiment we observed the accumulation of some low melting point fluid at the top of the tank. Similar results, both in terms of heat transfer and no noticeable fluid motion, were found by Webb and Viskanta (1986).

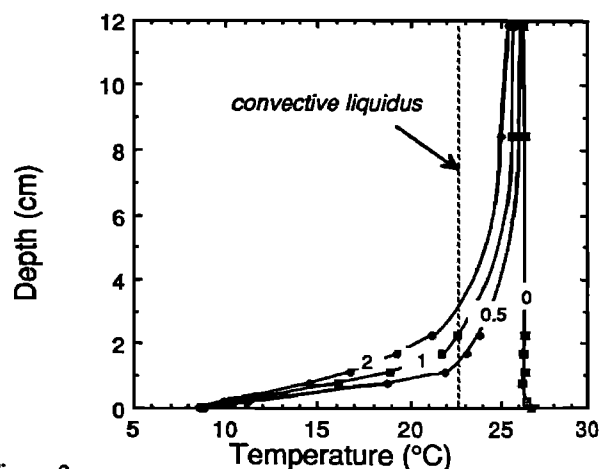


Figure 2

Temperature profile versus depth for experiment 51 (basal cooling with $\Delta T=3.6^\circ\text{C}$ and $T_f=8.6^\circ\text{C}$) at different times (indicated in hrs along the curves).

Crust growth rate

The crust thickness has been measured during experiments 50 and 51 (Figure 3). During Stage I, convection slows crystallization by maintaining a hot - superliquidus - isothermal reservoir in front of the advancing solidification front. When convection ceases, crust growth returns to a curve parallel to that obtained when the layer is stagnant. The larger the amount of superheat, the larger the delay in the most rapid (i.e. conductive) rate of crust growth. If the amount of superheat were made much larger than that used here, it is conceivable that the crust might not form at all for a certain period of time. This is, however, only a transient stage; for convection diminishes rapidly with time, eventually ceasing altogether, and formation of a crust must always follow.

Similar conclusions were reached by Viskanta and Gau (1982) in the cylindrical solidification of pure paraffins; convection diminished rapidly and cooling and crust growth was controlled by conduction.

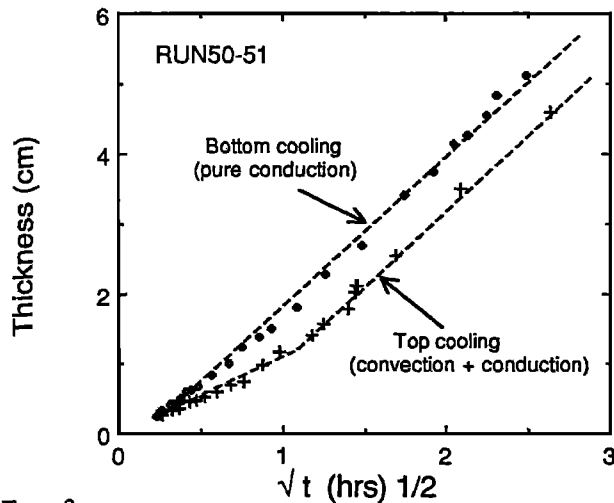


Figure 3

Growth of the solidifying crust as a function of time in experiments 50 (convection due to roof cooling) and 51 (conduction due to basal cooling). Notice that the loss of convection after about 1 hour is clearly reflected in the rate of crust growth.

Velocity measurements

The flow is visualized with small aluminum flakes ($<100\mu\text{m}$) illuminated from the side by a vertical sheet of light. Velocities are measured directly from a time-lapse movie and also from streak lengths on long-exposure photographs. Figure 4 represents the envelope of the maximum velocity measurements versus the Rayleigh number where now $\Delta T (= T_i - T_L)$ is time varying. The velocity is 0 at time $t=0$ when Ra is largest, and increases as the first convection develops. Plumes start to reach the bottom of the tank (ref.1) and convective vigor increases. However, as there is no additional supply of heat to the system, convection wanes with time and in this respect is transient. With time the velocity decreases, eventually vanishing altogether as convection itself ceases. Note the rapid increase to the maximum velocity followed by a slow decay. The error bar on the velocity is large when the velocity is large, and small when the velocity is small. On the other hand, the error bar on Ra becomes very large as convection dies. It is especially important to realize that, as the temperature difference driving convection becomes

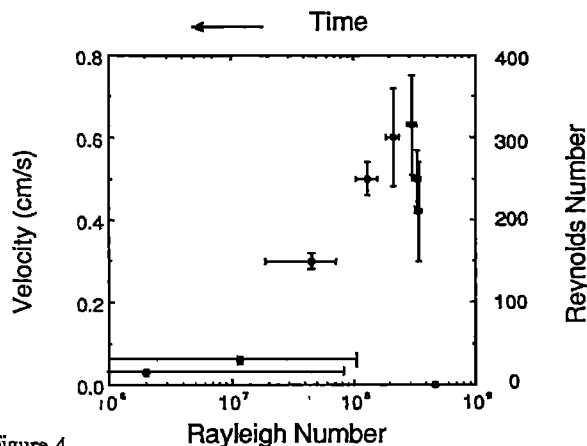


Figure 4

Maximum velocities versus Rayleigh number for experiment 47 ($\Delta T=2.5^\circ\text{C}$, $T_f=10.8^\circ\text{C}$), and along the right axis are the associated Reynolds numbers based on a length scale of 20 cm, the full tank size.

increasingly small ($<1^\circ\text{C}$), the relative error made on the measurement of the interior temperature becomes increasingly large such that the accuracy of these measurements $\pm 0.2^\circ\text{C}$ does not allow definition of a Rayleigh number smaller than 2×10^7 . That is, near the close of Stage I, the fact that $0 \leq \Delta T \leq 0.2^\circ\text{C}$ imposes an uncertainty on Ra of $0 \leq Ra \leq 2 \times 10^7$.

Heat Transfer

We define a somewhat special Nusselt (Nu^*) number, following Marsh (1989), as

$$Nu^* = \frac{Q_{cv}}{Q_{cd}(t)} \quad (4)$$

where Q_{cv} is the convective heat flux and $Q_{cd}(t)$ is the time-dependent conductive flux from the same but stagnant layer cooled under the same thermal conditions. The convective heat flux is equal to:

$$Q_{cv} = -\rho c_p H \frac{dT_i}{dt} \quad (5)$$

where H is the thickness of the convective layer, ρ is density, and c_p is specific heat. Because the thermal boundary layer is very small, this is well approximated by the thickness of the liquid layer, $d-h = H$, where h is the crust thickness. The conductive flux in a stagnant layer cooling under the same initial conditions as in the convective experiments, is calculated from:

$$Q_{cd}(t) = -k \left. \frac{\partial T}{\partial z} \right|_{z=0} \quad (6)$$

The change in Nu^* with time as calculated in this fashion is shown by Figure 5 for experiment 50. The derivative of the interior temperature needed in (5) is obtained from equation (3) and h (cm) is given by:

$$h = 1.46 \times 10^{-2} \sqrt{t} + 9.78 \times 10^{-5} t - 2.38 \times 10^{-2} \quad (7)$$

which has been found to match closely the data of Figure 3. Nu^* is 0 for time $t=0$, increases to a maximum and then decreases as convection slows. This behavior is similar to that

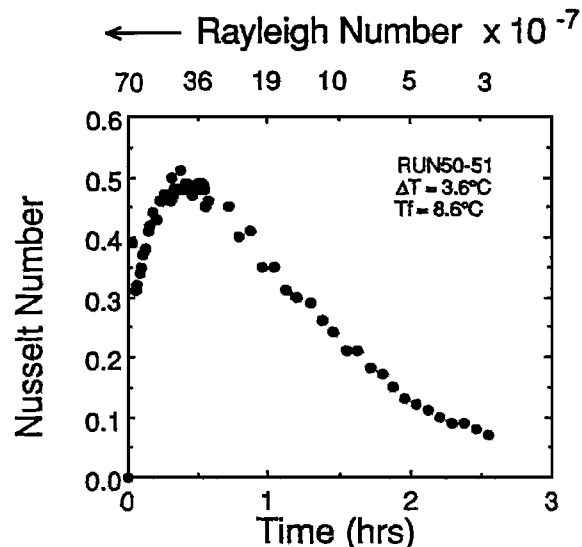


Figure 5

Special Nusselt number versus time for experiment 50. Notice the rise time in establishing vigorous convection and then the steady drop in Nu^* as convection wanes.

observed for the velocity (Figure 4) and reflects the transient nature of convection.

That is, the increasing part of the curve reflects the onset and the development of vigorous convection, but with the increasing loss of convectable heat from the interior, both Nu^* and velocity steadily diminish to the eventual conductive state (Marsh, 1989).

Discussion

Let us thus define the convective superheat as the temperature contrast between the initial temperature and the convective liquidus. Suppose that a magma chamber is intruded with a large amount of convective superheat. Even with the strong effects of variable viscosity induced by solidification, convection is fairly well described by constant viscosity, large Ra theory. Equation (2) shows that τ scales as $Ra^{-1/3}$, and since Ra is large, the transient stage is very short. If on the other hand, a magma is emplaced with a vanishingly small amount of convective superheat, the transient stage is very long, and may be commensurate with solidification to a state of 50% crystals everywhere (i.e., $\kappa t/d^2 \sim 0.1$ or more; see Marsh (1989)). It is important to realize in this latter instance that convection will not contribute significantly to cooling of the interior, but convection instead will be a slow process.

If as suggested by ref. 1 and Marsh (1989) magmas have a convective liquidus near their usual liquidus, then vigorous magmatic convection is associated with superheating. In this light, a key issue at the heart of understanding magmatic convection is the uncertainty in the Rayleigh number for vanishingly small amounts of superheat. That is, an uncertainty in this sense of 1°C in ΔT (i.e., $0 \leq \Delta T \leq 1^\circ\text{C}$) yields a corresponding uncertainty in Ra of $0 \leq Ra \leq 10^{15}$ for a 1 km deep basaltic magma chamber. A reasonable value of ΔT cannot thus be safely assumed a priori from which to calculate Ra , and thus it may be untenable to use without additional evidence the usual formulation of large Ra , turbulent convection. Moreover, because any superheat is so rapidly dispensed by convection, magmatic temperature may always be confined to this state of uncertainty in Ra . It is very clear from our experiments that the undercooling due solely to crystallization cannot itself sustain vigorous convection.

The broadly similar experiments of Kerr et al. (1989) exhibit a similar thermal history. Convection rapidly removes the initial superheat and the temperature closely tracks the liquidus, which itself in their case slowly decreases with time due to a steady change in the internal bulk liquid composition. Convection is nevertheless sustained for long times by the expulsion of large amounts of light fluid from coarse crystals growing on the floor. Although cooling is also only from above, significant floor crystallization occurs because of crystals falling from the roof region and collecting and growing on the floor. The difference between these two systems (i.e., aqueous vs. paraffin) may be due to a difference in the kinetics of crystallization (few nucleation sites and rapid growth in the aqueous system and many nucleation sites and slow crystal growth in paraffin) which may support different undercoolings and thus induce different convective behaviors. These two sets of experiments (i.e., Kerr et al. and ours) are thermally quite similar but differ significantly in the role of compositional convection and crystal size. Together they represent two possible end members between which actual magmatic convection may fall.

The conundrum of uniform composition of some sills and thick lava flows, as mentioned at the onset, seems best explained by the paraffin model whereas other highly differentiated bodies may be better approximated by the aqueous model, or by specific side wall processes (e.g., Trial and Spera, 1990).

Acknowledgments. This work has been supported by grants NSF EAR-86-11362, EAR-88-1734 from the National Science Foundation, NASA grant NAG5-32 and IPGP (France). Reviews and discussions by A. McBirney, F. Spera, D. Yuen, and M. Smith are appreciated.

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(Received: April 9, 1990
Accepted: April 14, 1990)