Crustal deformation
at the Hengill triple junction, Iceland
2001-2006

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Abstract

The Hengill area in south-west Iceland is the junction between two rift zones and a transform zone, within the Icelandic divergent plate boundary between North-America and Eurasia. It is an area of significant tectonic, geothermal and magmatic activity. In this study, GPS data from the Icelandic Geosurvey (ISOR), collected in 2001, 2003 and 2005 as well as GPS campaign data from the Nordic Volcanological Center (University of Iceland) and continuous GPS measurements for the same period of observation are processed together using the Bernese GPS software Version 5.0 in order to get a velocity field at the Hengill triple junction. InSAR ENVISAT data acquired between 2005 and 2006 were processed using the Stanford Method for Persistent Scatterer (StaMPS) developed by Andy Hooper (Hooper et al., 2007), in order to get an idea of the vertical motion at the triple junction. Using the horizontal results of GPS, rate of strain over the area was calculated using the method described by Beaven et al., 2001. The deformation pattern over the area between 2001 and 2006 is mainly due to spreading within a divergent plate boundary and local deformation due to fluid extraction.

Résumé

Au niveau de la région du volcan Hengill dans le sud ouest de l’Islande, deux zones de rift et une zone transformante se rejoignent pour former un point triple dans la zone de divergence entre les plaques Nord Américaine et Eurasiatique. Cette région présente une activité tectonique, magmatique et géothermique significative. Au cours de ce travail, des mesures GPS effectuées par la société “Icelandic Geosurvey” en 2001, 2003 et 2005, des données de campagnes GPS effectuées par le “Nordic Volcanological Center” de l’université d’Islande ainsi que des enregistrements continus de GPS au cours de ces années ont été rassemblées afin d’obtenir un champs de déformation dans la région de Hengill. Des interferogrammes InSAR formés à partir d’images ENVISAT enregistrées entre 2005 et 2006 ont été analysés suivant la méthode “Stanford Method for Persistent Scatterer” (StaMPS) développée par Hooper et al., 2007 afin de comprendre les mouvements verticaux dans cette zone. Enfin, le taux de déformation a été calculé suivant la méthode développée par Beaven et al., 2001. Ces mesures géodésiques ont permis d’établir un champs de déformation dans cette région de point triple, dominé par le mouvement de divergence des plaques et par des subsidences locales dues à l’extraction d’eau chaude.
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Introduction

In Iceland, the divergent plate boundary between North America and Eurasia is constituted by several N-S rift zones that are the onshore part of the Mid-Atlantic ridge. The rift zones are separated in two major segments, one located in the eastern part of the island, and the other in the western part. In the south of Iceland, those two segments are linked by an E-W transform zone. The Hengill triple junction is located in the south-west part of the Icelandic plate boundary, where the south Iceland transform zone joins two rift zones of the western part of the Iceland (figure 1). This is therefore a ridge ridge transform triple junction. This area is tectonically active: a large amount of faults were mapped or inferred by interpretation of seismicity (Clifton et al., 2001), three volcanic systems are present and several postglacial eruptions occurred. Between 1993 and 1998, the area experienced seismic swarms and an inflation that was interpreted in terms of magma incom in a shallow magma chamber (Sigmundsson et al., 1997). Furthermore, an important geothermal activity exists in the Hengill area, fluids circulate through the fissures, and hot water is extracted from the upper 2 km of the crust.

In order to study crustal deformation associated with plate spreading, magmatic processes and geothermal activity in the area, geodetic methods are an efficient tool since it allows to monitor deformation over one year, which is the time scale of those phenomena. In this study, I used GPS and InSAR data to look at crustal deformation at the Hengill triple junction for a period between 2001 and 2006, following the period of inflation studied by Sigmundsson et al., 1997 and Feigl et al., 2000. The first objective of this work is to see the deformation pattern after the inflation event, in order to constrain the behaviour of this kind of triple junction. Iceland is one of the few places on Earth where such a structure comes onshore, and where GPS and InSAR measurements are possible. Therefore, this place is of great importance concerning the understanding of divergent plate boundaries. The other objectives are to understand the effects of geothermal activity on crustal deformation, especially the effects of fluid pumping, and to monitor eventual magmatic processes. The Hengill triple junction is also a place where relations between seismic and volcanic activity can be studied.

In the following sections, the tectonic setting of the Hengill triple junction is developed, as well as the main aspects of the 1993-1998 events. Then, some details about GPS data, measurements and processing are provided, and results from 2001-2005 are presented. InSAR results for 2005-2006 are also presented. In a last part, an interpretation of those observations is provided.
1 Tectonic setting and recent activity

1.1 The triple junction within the plate boundary

Iceland is located within a divergent plate boundary between North American plate and Eurasian plate, on the Mid-Atlantic ridge. The relative plate motion has been studied by GPS campaigns measurements in 1986, 1989 and 1992 (Sigmundsson et al., 1995), and between 1992 and 2004 (Árnadóttir et al., 2006). It has also been studied through continuous GPS measurements between 1999 and 2004 (Geirsson et al. 2006). Continuous GPS network is shown on figure 1. GPS results are consistent with the geological model NUVEL-1A, therefore no significant changes in plate spreading occurred during the last million of years. The spreading direction is WNW-ESE, with an average velocity of 19.4 mm/yr.

The plate boundary consists of several rift zones which accommodate extension (WVZ, EVZ, NVZ and Reykjanes Peninsula), and two transform zones which accommodate lateral shear (SISZ and TFZ). Interpretation of GPS measurements between 1992 and 2006 is that both extension and left lateral shear strain occur along the Reykjanes Peninsula (Arnadottir et al., 2006 and Keiding et al., 2008).

The Icelandic mantle plume is situated beneath the Vatnajökull ice-cap. Because of the westward motion of the plates relative to this thermal anomaly, the plate boundary across Iceland is shifted to the Eastern Volcanic Zone between the Reykjanes ridge and the Kolbeinsey ridge. Eastern Volcanic Zone is connected to the ridges by two transform zones: TFZ and SISZ.

Sigmundsson et al., 1995 estimated that in south Iceland 85 % of the divergence is accommodated through the SISZ, which means that 85 % of the divergence is taken within the Eastern Volcanic Zone, whereas 15 % is taken within the Western Volcanic Zone. Since SISZ seems to have been active for 10 000 years this repartition of spreading was not necessarily the same before and the WVZ has certainly been more active in the past. More recent results obtained by analysis of GPS measurements across south Iceland between 1994 and 2003 shows that the repartition of extension between WVZ and EVZ is latitude dependent: in the north part, EVZ accommodate 19.8 mm/yr of extension and WVZ accommodate 2.6 mm/yr, whereas EVZ accommodate 11 mm/yr and WVZ 7 mm/yr in the south (LaFemina et al., 2005). Furthermore, the EVZ is propagating to the south. The authors modeled the spreading by dike injection in an elastic or layered halfspace (several historical rifting events affected the area, the Laki rifting event for instance in 1783).
Figure 1: Plate boundary in Iceland. The black arrows show the velocities predicted by the REVEL plate motion model. Black solid lines south and north Iceland are the Mid-Atlantic ridge. E-W black solid lines represent transform zones: SISZ: South Iceland Seismic Zone and TFZ: Tjörnes Fracture Zone. Dark grey represent volcanic fissure swarms, light grey correspond to glaciers. Continuous GPS stations are indicated with black dots. The white stars represent epicenters of the Mw $\geq 5$ earthquakes. The red square outlines the Hengill triple junction. RP: Reykjanes Peninsula, E: Eyjafjallajökull volcano, He: Hekla volcano, K: Katla volcano, GL: Grimsey Lineament, HFF: Husavik Flatey Fault, DL: Dalvik Lineament. From Geirsson et al., 2006.
The Hengill triple junction is located in SW Iceland (figure 1). This is the connection between the Reykjanes Peninsula, the WVZ and the SISZ. Tectonic characteristics of SW Iceland are shown on figure 2, top.

Crustal deformation in SW Iceland has been studied by GPS measurements since 1986. Results for 2000-2006 are presented on figure 2, bottom. Keiding et al., 2007 interpreted the velocity field as resulting from the relative motion of the plates across the peninsula. Those results were consistent with what was inferred by Arnadottir et al., 2006 for the period between 1992 and 2000.

In those models three zones are present:

- The Reykjanes Peninsula, where left lateral shear motion and extension occur.
- The SISZ where left lateral shear below \( \sim 10 \text{km} \) occurs.
- The WVZ where mostly extension is present.

Parameters of the best models of dislocation obtained by Keiding et al., 2008 are summarized in figure 3, where Depth is locking depth. The deep motion and azimuth are the magnitude and direction of the vectorial sum of opening and left lateral motion.

Sigmundsson et al., 1995 identified ”bookshelf faulting” as the main way to accomodate left lateral shear strain in the SISZ. This means that within the transform zones, vertical right lateral strike slip North-South faults accomodate the regional strain. This is consistent with focal mechanisms of the June 2000 earthquakes, as well as the one of May 2008 and with surface faulting. This seems to be also the case for historical earthquakes.

Within the RP, NE oriented fissure swarms seem to accomodate extension, whereas left lateral shear may be accomodated in the same way as within the SISZ.

1.2 The Hengill triple junction

1.2.1 Geology and tectonics

The Hengill triple junction shows tectonic characteristics both from the Reykjanes Peninsula and the SISZ. There are three volcanic systems (Hengill, Hromundartindur and Grensdalur), constituted by a central volcano and fissure swarms with a NE-SW trend (figure 4) which is a common structure in the peninsula. The Hromundartindur volcanic system last erupted 10000
Figure 2: Top: Structural map of south-west Iceland. Black solid lines represent fissure swarms and central volcanoes in RP and WVZ. Little black lines in SISZ represent mapped holocene faults (Einarsson and Saemundsson, 1987). Red squares are continuous GPS stations. The green square delimits the Hengill triple junction. Bottom: GPS observations and model prediction of plate boundary deformation (From Keiding et al., 2007). Black lines represent the dislocations used in the model.
1 TECTONIC SETTING AND RECENT ACTIVITY

Figure 3: Best dislocation model parameters for SW Iceland determined by Keiding et al., 2008

years ago. Since then, the activity shifted towards the Hengill system. Several eruptions occurred since, for instance 2000 years ago at Nesjahraun. NS trending faults are also found in this area which corresponds to the influence of SISZ. According to Clifton et al., 2001, most of the topographic lineaments are parallel to these directions and mostly correspond to buried fault planes. Relocation of micro earthquakes that occurred between 1993 and 1998 is consistent with those directions of faulting (Clifton et al., 2001).

1.2.2 Geothermal activity

One important characteristic of the Hengill area is geothermal activity. It is one of the largest geothermal area in Iceland and hot water of Reykjavik

<table>
<thead>
<tr>
<th></th>
<th>RP</th>
<th>SISZ</th>
<th>WVZ</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth (km)</td>
<td>$7^{1+1}_{2-2}$</td>
<td>$6^{1+1}_{2-2}$</td>
<td>$3^{1+1}_{2-2}$</td>
</tr>
<tr>
<td>Left lateral (mm/yr)</td>
<td>$18^{2+2}_{1-1}$</td>
<td>$22^{2+2}_{1-1}$</td>
<td>-</td>
</tr>
<tr>
<td>Opening (mm/yr)</td>
<td>$7^{1+1}_{2-2}$</td>
<td>-</td>
<td>$11^{1+1}_{2-2}$</td>
</tr>
<tr>
<td>Deep Motion (mm/yr)</td>
<td>$20^{2+2}_{1-1}$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Azimuth (deg/North)</td>
<td>$100^{2+2}_{1-1}$</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Figure 4: Volcanic systems (orange dotted lines) and geothermal areas (red points represent boreholes). Black lines represent visible fractures mapped by Amy E. Clifton. Red squares are continuous GPS stations.
1 TECTONIC SETTING AND RECENT ACTIVITY

comes directly from powerplants situated in Hellisheidi or Nesjavellir (figure 4).

Some focal mechanisms of the background seismicity within this area presents a significative non double couple component which is related to fluid circulation in the crust (Miller et al., 1998).

Seismic tomography under south Iceland (Tryggvason et al., 2002) showed a negative anomaly of P-Waves (from 6.5 km/s to 5.5 km/s) and S-Waves (from 3.6 km/s to 3.3 km/s) velocities, as well as a decrease in the Vp/Vs ratio from 1.8 to 1.6, between 4 km and 6km depth in the Hengill area. The authors interpret this in terms of supercritical fluids within volcanic fissures. This is consistent with geothermal activity.

All these observations indicate that heat comes from a magmatic chamber situated beneath Hengill volcanic system, and fluids circulate in the various faults of the area.

The presence of geothermal power plants who pump fluids from the crust can have an impact on the deformation in the area. Geodetic survey is a way to evaluate this impact.

1.2.3 Recent activity: the 1993-1998 events

Between 1993 and 1998, several events happened in the Hengill area. These events, as well as their surface consequences have been studied in detail by Clifton et al., 2001. These events can be summarized as follows:

Background seismicity increased significantly in the Hengill area, mostly at the NE and SW parts of the area around Hromundartindur volcanic system.

This enhanced seismicity ended by two Mw ~ 5 earthquakes, in the southwest part of the area (figure 4, right). After that inflation seems to have decreased.

Geodetic measurements such as levelling and GPS (analysed by Sigmundsson et al., 1997) showed that the area was inflating during this time period. This inflation was also seen on InSAR interferograms (Feigl et al., 2000), concentric fringes on figure 4, left. Authors modelled this as a Mogi source of pressure at 7 km depth. Center of inflation is shown on figure 4, right.

Sigmundsson et al., 1997 showed that this inflation enhanced regional left lateral shear strain in the NE and South West part of the center of inflation. This explains the locations of earthquakes.

This sequence of events has also been interpreted by Clifton et al., 2001. The model of crustal deformation is described as a consequence of magma
inflow in a shallow magmatic chamber:

An inflow of magma in a preexisting magmatic chamber beneath Hro-
mundartindur volcanic system caused the inflation.

The NE part of the area presents a weak crust due to geothermal alter-
ation. Furthermore, many faults oriented NE-SW, N-S and E-W concentrate
there. As a consequence of the inflation, seismic activity begun in the NE
part of the area and migrated to the SW.

These events loaded two fault planes: first a N-S one which ruptured in
June 1998. The mechanism shows a right lateral strike slip. Then, a second
Mw $\sim 5$ event happened in November. Afterschoks of this event aligned
along an E-W zone.

The Hengill triple junction is an area tectonically active. At the same
place, active volcanic systems, strike slip faults, normal faults are present.
Geodetic measurements in this area showed during the 90s significant defor-
mation. Therefore, it is important to continue to measure the deformation
in order to better understand how the plate motion is accomodated within
the Hengill triple junction.
Other interests of surveying this area are to monitor magmatic processes, to study interaction between magmatic processes and seismicity, as well as effects of fluid extraction.


2 GPS survey between 2001 and 2005

2.1 Global Positioning System: concepts and application to crustal deformation studies

2.1.1 GPS in Earth-science

Global Positioning System (GPS) is a navigation system developed by the US army. US Congress allowed the civilian use of GPS, with some restrictions.

The basic idea of GPS is to position a point at the Earth’s surface using satellites that transmit an electromagnetic signal received by an antenna placed at the observation point. The travel time of the signal is evaluated, and the distance between satellite and receiver is obtained by multiplying this travel time by c (velocity of light). Therefore, three satellites are needed to get three measurements of distance, necessary to determine the three components of the receiver position.

The wavelengths of the signal broadcast by the GPS satellites (19cm and 24cm) allow positioning with a centimeter accuracy. In fact, it is necessary to evaluate several biases that affect the measurements, caused by atmosphere for instance, in order to improve the accuracy of the positioning.

The precision obtained allows the use of GPS in earth-science, specially in the geological deformation domain. GPS is able to detect motions of less than one cm/yr which is of the order of deformation processes. For instance, GPS is probably the easiest and most efficient way to evaluate relative motion of tectonic plates in continental domains. Furthermore, GPS is used to detect motion over several years, which is the timescale of deformation due to earthquakes or volcanic activity. Therefore, it appears to be an efficient tool to study active tectonics.

2.1.2 The GPS system

The Global Positioning System consists of a constellation of 24 satellites. Their orbit is a perturbed Keplerian and almost circular orbit around the Earth. Perturbations are due to tidal attraction, non-sphericity of the Earth, solar wind and other factors. These satellites are located on 6 orbital planes.
at 55° of the equatorial plane represented on figure 6. Each satellite is situated at 20200 km from the earth surface. Their period of rotation is 11h 58min, half a sideral day. A satellite platform is constituted by: a radio transmitter, atomic clocks, computers, solar panels for power supply and wheels and propulsioning system for orbital adjustement.

The signals transmitted by the satellite are two radiations $L_1$ and $L_2$. An oscillator generates a fundamental frequency $f_0 = 10.23MHz$. From this fundamental frequency are derived two other frequencies: $f_1 = 1575.42MHz$ and $f_2 = 1227.60MHz$. They correspond respectively to the wavelengths $\lambda_1 = 19.0cm$ and $\lambda_2 = 24.4cm$. $L_1$ and $L_2$ are then modulated by codes which consist in sequences with two states +1 and −1. There are four different codes:

- C/A-code $C(t)$ (Coarse/Acquisition).
- P-code $P(t)$ (Protected or Precise).
- D-code $D(t)$ (Navigation code).

The C/A code only modulates $L_1$. Its frequency is $f_0/10$. It is a sequence of 1023 bits, which is repeated every millisecond. The time span between two bits is $10^{-6}s$. It is satellite specific.

The P-code is encrypted with another code (W-code) by US army. This encryptage is called Anti Spoofing. P-code modulates both $L_1$ and $L_2$. Its frequency is $f_0$. It is a sequence of $2.3547 \times 10^{14}$, which is repeated every 266.4 days. The time span between two bits is $10^{-7}s$. It is therefore 10 times more accurate than the C/A code. It is satellite specific.

The navigation code is a 1500 bits message at 50Hz. It contains information about satellite orbit, satellite clock, satellite health for instance.
Before 2000, information about the satellite clock was artificially degraded (this process is called Selective Availability). Furthermore, information about orbits (broadcast ephemerides) are not precise enough (several meters) for geophysical applications, so that it is necessary to calculate precise orbits during the processing.

All the following developments about GPS theory are given with more details in the Bernese GPS software manual. Transmitted signals formulae are:

\[ L_1(t) = a_p P(t)D(t) \cos 2\pi(f_1t) + a_c C(t)D(t) \sin 2\pi(f_1t) \] (1)

\[ L_2(t) = b_p P(t)D(t) \cos 2\pi(f_2t) \] (2)

The last part of GPS consists of an antenna and a receiver. After its reception and amplification by the antenna, the signal is separated into satellite specific signals, which is achieved through C/A code. A receiver unit is constituted by: a Radio Frequency (RF) section, a control device, a storing device and a power supply.

The main part of the receiver is the RF section which processes the signal and converts it into observations. It is made of oscillators that generate a reference frequency, filters and mixers. Once separated into satellite specific signal, the different codes (P, C and Navigation) are isolated as well as fundamental frequencies \( L_1 \) and \( L_2 \). Then, oscillators generate synthetic codes \( P \) and \( C \) and synthetic reference radiations \( L_1 \) and \( L_2 \). Those synthetic signals are compared with the received signals to determine the travel time of the signal from the satellite by looking at the relative phases. In the case of \( L_1 \) and \( L_2 \) we compare the signals as follows. Assume \( y_i \) is the signal of the satellite \( i \) transmitted at time \( t - \tau \) with phase \( \phi_i \) and \( y_k \) the signal of receiver \( k \) with phase \( \phi_k \) at time \( t \).

\[ y_i = a_i \cos 2\pi \phi_i(t - \tau) \] (3)

\[ y_k = a_k \cos 2\pi \phi_k(t) \] (4)

where \( a_i \) and \( a_k \) are the amplitudes. The two signals are first multiplied:

\[ y_iy_k = \left( \cos 2\pi[\phi_k(t) - \phi_i(t - \tau)] + \cos 2\pi[\phi_k(t) + \phi_i(t - \tau)] \right) \] (5)

A low pass filter is applied to the multiplication of the two signals in order to eliminate \( \phi_i + \phi_k \) and the phase of the signal remaining is:

\[ \psi_k = \phi_k(t) - \phi_i(t - \tau) + n_k \] (6)

which is the phase observation, \( n_k \) being the number of integer wavelengths between the satellite and the receiver.
2.1.3 Observations and biases

Observation equations

The distance between receiver $i$ and satellite $k$ is called the range and is noted $\rho_k^i$. If $\tau$ is the travel time, $c$ the velocity of light, the range is:

$$\rho_k^i = c\tau$$

(7)

The range is the quantity we are looking for. Using code measurements, receivers are able to measure the code pseudo range which is not the range because of measurements biases, for instance satellite and receiver clock errors $\delta^i$ and $\delta_k$:

$$P_k^i = c((t + \delta_k) - (t - \tau + \delta^i))$$

(8)

using equation (7) the relation between code pseudo range and range is:

$$P_k^i = \rho_k^i + c\delta_k - c\delta^i$$

(9)

the subscript F refers to the frequencies $f_1$ and $f_2$.

Using phase measurements, receivers also measure the phase pseudo range given by the following formula:

$$\psi_k^i(t) = \phi_F(t) - \phi_F^i(t - \tau) + n_F^i$$

(10)

this can be rearranged using $\phi^i(t - \tau) = \phi^i(t) - f_F \tau$ and $\phi_F^i(t) = \phi_F^i(t) + (\delta_k - \delta^i)f_F$, and multiplying by $\lambda_F$ (the wavelength of the signal):

$$L_k^i = \rho_k^i + c\delta_k - c\delta^i + \lambda_F n_F^i$$

(11)

Biases

Several biases may affect the pseudorange measurements, due to the satellite, the receiver and path effects:

- Uncertainties about the satellite’s clock $\delta^i$
- Uncertainties in satellite’s orbit parameters (navigation message).
- Ionosphere refraction
- Tropospheric refraction $\Delta \rho_k^i$
- Uncertainties about the receiver’s clock $\delta_k$
- Uncertainties about the position of the phase center of the antenna.
- multipath effects

Because of the unprecise orbit information, it is necessary to calculate precise orbits during the processing, in order to get centimetric accuracy in the satellite position.

Ionosphere is the part of the upper atmosphere where the concentration of ionic particules and electrons is sufficient to affect the propagation of the radio waves. It is a dispersive medium, which means that the velocity of the signal \( v \) depends on its frequency, and is not exactly the velocity of light \( c \).

We can define the refraction index of ionosphere \( n \) which is the ratio of light velocity with the signal velocity. \( n \) can be written as the first terms of a serie:

\[
n_i = 1 \pm \frac{C}{f_i^2} \quad (12)
\]

where \( n_i \) is the refraction index of ionosphere at frequency \( f_i \), and \( C \) a constant related to the integrated concentration of electrons along the signal path in the ionosphere. The sign + is used when we consider phase velocity (for phase measurements), and the sign − when we consider group velocity (for code measurements). The codes are supposed to travel at group velocity, whereas the phase velocity describes the motion of the unmodulated radiations. Therefore, travelling through the ionosphere introduces a time delay in both the phase and code measurements. Assume that \( \rho_{im} \) is the range measured with the signal of frequency \( f_i \), and \( \rho \) the real range. The difference \( \rho_{im} - \rho \) due to ionospher refraction is given by:

\[
\rho_{im} - \rho = \int_{\text{path}} (n_i - 1)d\rho \quad (13)
\]

Then, we have, using equation (12):

\[
\rho_{im} - \rho = \pm \int_{\text{path}} \frac{C}{f_i^2}d\rho \quad (14)
\]

Therefore, if \( I_k = \rho_{1m} - \rho \) is the effect of ionosphere on the frequency \( f_1 \), the effect on frequency \( f_2 \) is \( \frac{f_2^2}{f_1^2} I_k \).

The tropospheric refraction is the effect of the troposphere on the signal propagation. It is not frequency dependent. The tropospheric refraction depends mostly on the water vapor content of the atmosphere, which presents great spatial and temporal variations. This is the most difficult bias to estimate and most of the uncertainties come from tropospheric refraction.
The antenna phase center is the physical position of the reception in the antenna. This center doesn’t correspond to the geometrical center of the antenna, because it varies with the orientation of the vector receiver-satellite. It is possible to evaluate this effect by calibration measurements on antennas. However, such experiments are difficult, and it is possible to avoid this by aligning antennas in the same directions during measurements so that the variation of the position of the phase center is the same for each antenna during the survey; to minimize its influence in relative positioning.

The relation between the pseudorange measurements and the range becomes, taking biases into account:

\[ P_{1k}^i = \rho_k^i + c\delta_k - c\delta^i + I_k^i + \Delta\rho_k^i \]  
\[ P_{2k}^i = \rho_k^i + c\delta_k - c\delta^i + \frac{f_1^2}{f_2^2} I_k^i + \Delta\rho_k^i \]  
\[ L_{1k}^i = \rho_k^i + c\delta_k - c\delta^i - I_k^i + \Delta\rho_k^i + \lambda_1 n_{1k}^i \]  
\[ L_{2k}^i = \rho_k^i + c\delta_k - c\delta^i - \frac{f_1^2}{f_2^2} I_k^i + \Delta\rho_k^i + \lambda_2 n_{2k}^i \]

Therefore, it is necessary to evaluate how much these quantities change the pseudorange-measurements. It is possible to use for that atmospheric and clock deviation models, whose parameters are adjusted during the inversion of the position of the stations.

**Forming differences**

In order to evaluate some of these biases with a good accuracy, it can be useful to eliminate others. This can be done using linear combinations of the last equations as observations. Several linear combinations will be used in the processing:

The single difference between the signals received by two receivers k and l from the satellite i. This eliminates the satellite’s clock bias:

\[ L_{Fkl}^i = L_{Fk}^i - L_{Fl}^i \]  

The double difference, between two receivers k,l and two satellites i,j. This eliminates both the receiver and satellite clock uncertainties:

\[ L_{Fkl}^{ij} = L_{Fkl}^i - L_{Fkl}^j \]

Therefore, instead of using phase and code measurements, double difference code and phase measurements are used:

\[ P_{1kl}^{ij} = \rho_k^{ij} + I_k^{ij} + \Delta\rho_k^{ij} \]
When using double difference measurements, the positioning becomes more accurate, but we can only get relative positioning between pairs of stations. The distance between the two stations is called baseline.

To eliminate the phase ambiguity \( n_{2kl}^{ij} \) the triple difference is sometimes performed. Assuming that this ambiguity doesn’t change between times \( t_1 \) and \( t_2 \), double difference phase measurements are used at these two different epochs to define the triple difference phase measurement:

\[
L_{1kl}^{ij}(t_2) - L_{1kl}^{ij}(t_1) = \rho_{kl}^{ij}(t_2) - \rho_{kl}^{ij}(t_1) - (I_{kl}^{ij}(t_2) - I_{kl}^{ij}(t_1))
\]

\[
L_{2kl}^{ij}(t_2) - L_{2kl}^{ij}(t_1) = \rho_{kl}^{ij}(t_2) - \rho_{kl}^{ij}(t_1) - \frac{f_1^2}{f_2^2} (I_{kl}^{ij}(t_2) - I_{kl}^{ij}(t_1))
\]

Several other combinations are possible. In the following notations, \( L_1 \) and \( L_2 \) are the phase measurements (zero or double differences) and \( P_1 \) and \( P_2 \) are the code measurements. We can build (with phase and code measurements) the ionosphere free linear combination \( L_3 \), which eliminates the ionosphere path delay:

\[
L_3 = \frac{1}{f_1^2 - f_2^2} (f_1^2 L_1 - f_2^2 L_2)
\]

The geometry free linear combination \( L_4 \) is geometry independant (satellites and stations coordinates) and clock error independant. It depends only on ionospheric path delay and integer ambiguity:

\[
L_4 = L_1 - L_2
\]

The wide-lane linear combination \( L_5 \) as well as the Melbourne-Wübbena linear combination \( L_6 \) will also be used in the processing. \( L_6 \) is ionosphere, troposphere, geometry and clock error independant.

\[
L_5 = \frac{1}{f_1 - f_2} (f_1 L_1 - f_2 L_2)
\]

\[
L_6 = \frac{1}{f_1 - f_2} (f_1 L_1 - f_2 L_2) - \frac{1}{f_1 + f_2} (f_1 P_1 + f_2 P_2)
\]
2 GPS SURVEY BETWEEN 2001 AND 2005

2.2 Data sets, measurements, and processing

2.2.1 GPS Data sets used in this study

In this study, I used GPS data collected in the Hengill triple junction. The measurements were done in 2001, 2003 and 2005. The dataset consists of:

- Continuous GPS measurements at the following stations: HVER, HLID, VMEY, VOGS, REYK, OLKE.
- GPS campaign measurements from the Nordic Volcanological Center (NVC).
- GPS campaign measurements from ISOR (Icelandic Geosurvey).
- Continuous GPS measurements from reference IGS stations: MADR, ONSA, TROM and ALGO.

Some details about the GPS measurements are given in the next section. A solution for the ISOR measurements had already been calculated before by the Icelandic Geosurvey, but in a different way, without including precise orbits in the processing, and without including the observations from the continuous GPS stations and the NVC measurements.

2.2.2 GPS campaign measurements

Usually GPS campaign measurements are performed during May and June, when the snow is melted enough to see the points on the ground. I participated during May and June, in the GPS campaign in the SISZ, learning to do GPS measurements.

The different parts of the measurement system are: an antenna, a receiver, a tripod and a power supply (battery or solar panel). Different kinds of antennas were used in 2001, 2003 and 2005: Trimble Geodetic Compact L1/L2 + groundplane and Trimble L1/L2 microcentered Geodetic + groundplane.

Antennas are fixed on the tripod above a benchmark, most often a geodetic mark cemented into bedrock. (figure ??). The tripod has to be fixed on the ground and must not move during the survey. Therefore it is better to put it on a stable substrate, such as bedrock, and avoid mud or sand. The center of the antenna has to be precisely above the center of the benchmark. Furthermore, the plane of the antenna also has to be horizontal. With those types of antennas, the center of phase moves as the direction antenna-satellite...
changes. Therefore, it is necessary to align all the antennas in the north direction. Hence, relative positioning will not be affected by this uncertainty. Finally, the antenna height above the ground has to be measured. This measure can be a source of error in the determination of the vertical position of the point.

Once the system has been set up and the antenna height measured, the receiver and the antenna are plugged in, and the receiver finds satellites available. The system stays at the same place for three days, measuring at a sample rate of one measurement every 15 seconds.

2.2.3 GPS data processing

I processed the data using the Bernese GPS software Version 5.0. More details are provided in the Bernese manual (edited by Rolf Dach, Urs Hugentobler, Pierre Fridez and Michael Meindl, from the Astronomical Institute, University of Bern).

The raw data downloaded from receivers are in .dat binary format, and it’s necessary to convert them in an appropriate format used for data exchange in Bernese software: the RINEX format (Receiver-INdependent EXchange format). This was done using the program teqc developed by L.H. Estey and C.M. Meertens. We have one RINEX file for one session (day) of observation and for one station, containing the unmodulated phases \( L_1 \) and \( L_2 \) measured every 15 seconds, as well as the code measurements.

The processing consists in the precise estimation of the positions of the stations in the reference frame ITRF 05 (International Terrestrial Reference Frame) defined by positions of reference stations in 2005 from all the pseudorange observations. Therefore it is necessary to estimate the uncertainties at the same time. The processing is done in two principal steps, using the Bernese 5.0 GPS software:

- PPP (Precise Point Positioning): This gives a first estimation of the coordinates from the code pseudorange measurements used in zero difference, for each session.
- RNX2SNX which uses the double difference phase observations, and the first solutions of PPP to give a more accurate relative solution

The principal steps of the two programs, and the estimated parameters are presented on table 1.
operations | observations | program
---|---|---
Preprocessing: Cycle slip detection and repair | $L_3, L_4, L_6$ (on zero difference) | RNXSMT
Instrument clock estimation | $L_3$ | CODSPP
Tropospheric delay, instrument clock error phase ambiguity estimation | $L_3$ | GPSEST
Satellite clocks synchronization | | SATMRK
Ionosphere delay estimation | $L_4$ | GPSEST

Precise orbits of the satellites are provided by the JPL (Jet Propulsion Laboratory of the NASA) as well as Earth orientation parameters and satellite clocks parameters. Parameters of ocean loading models and earth-tides models are also provided.

To estimate the atmospheric delays, atmospheric models are used. The model of ionosphere is a model of concentration of charged particles in the ionosphere (the constant $C$ in equation (12)) and the model of troposphere is a model of pressure, temperature and vapor content. The estimation of atmospheric delays corresponds to the estimation of the model’s parameters.

In general, all the parameters (station coordinates, atmospheric model parameters) are estimated through a least square inversion, using observations, equations (15) to (18) and (21) to (24).

At the end of those steps, an estimation of the station coordinates are provided for each day of observation. It is now necessary to find a solution for all the period of observation (for all the sessions in the same campaign), by combining all the session solutions. This is done with the program ADDNEQ. The reference frame (ITRF 05) is defined by fixing the positions of the well known stations of the reference network (here I used MADR, ONSA, TROM and ALGO). Then, baselines between the stations of our network and reference stations are calculated by the combination of session by session solution.

The motion is calculated relative to a fixed local station (here I used REYK). Using three different campaigns at three different years, I obtained two sets of displacements vectors, 2001-2003 and 2003-2005 (with Est, North and Vertical components) for the stations relative to REYK. I deduced the velocities from the comparison of the displacements between two different years.
2.3 Results

2.3.1 Horizontal motion

Horizontal velocities obtained for the period 2001-2005 (average velocities over four years) are represented on figure 7, top, and compared to velocities obtained by Geirsson et al., 2006, and Keiding et al., 2007. This comparison is a way to check the consistency of the solutions. My velocity estimates are consistent with previous studies (Geirsson et al., 2006, and Keiding et al., 2008). The general trend of velocities may represent plate spreading in the area: indeed, REYK is situated on the North-American plate, and stations move towards the east relative to REYK.

I also compare my velocities with those obtained by the processing of the ISOR set alone. Results are on figure 7, bottom. This could give an idea of the contribution of this set to the solution. In fact, it is difficult to conclude anything about these differences, since processing has been done in a different way, the solutions do not agree very well.

I compare the velocities for the two periods (2001-2003 and 2003-2005) in figure 8, top to look for a temporal evolution in the deformation pattern of the area, in the plate spreading signal. The velocities relative to REYK of the two time periods present the same general E-W trend. Velocities deviate from this trend in the geothermal area of Nesjavellir. This deviation appears in different ways between the two time periods. A focus on this area is provided on figure 8. For the period 2001-2003 vectors seem to converge, which indicates contraction in the area. Furthermore, the area south-west of OLKE shows a change in velocities between the two time periods. The differences between the two time periods in this area might not be caused by local noise because of spatial coherence of the difference. The pattern of deformation in the area seems to be related to both regional deformation in an E-W trend (plate spreading), and local deformation, that we need to explain. This local deformation seems to be related with geothermal exploration.

2.3.2 Vertical motion

The vertical velocities I estimated for the two time periods are presented in figure 9, relative REYK (distant station) and relative to a local station, HH38. With those vertical motion representations, it is possible to look for either regional or local deformation, as it is suggested by the analyse of horizontal velocities.

The velocities of continuous stations at OLKE and HVER are consistent with the velocities calculated by Geirsson et al., 2006 (HVER: 0.2±1.4mm/yr OLKE: 0.6 ± 1.4mm/yr). It is difficult to identify any coherent pattern
Figure 7: Top: Horizontal velocities relative to REYK (black arrows). The green arrows are velocities from Keiding et al., 2007. The red arrows are velocities from Geirsson et al., 2006. Bottom: Horizontal velocities for 2003-2005 (black arrows) compared with ISOR’s previous results (green arrows, from ISOR’s reports, 2003 and 2005), relative to HH04. The red dots represent boreholes, red squares continuous GPS stations. The blue points correspond to Nesjavellir and Hellisheidi power-plants. The blue triangle is the top of mount Hengill.
Figure 8: Top: 2001-2003 horizontal velocities (green arrows), 2003-2005 horizontal velocities (black arrows) relative to REYK. Bottom: Zoom in on Nesjavellir geothermal area.
Figure 9: 2001-2003 vertical velocities (green arrows), 2003-2005 vertical velocities (black arrows). Top: relative to REYK. Bottom: relative to HH38.
of long-term regional deformation in vertical velocities relative to REYK. Furthermore, there is no coherence between the two time periods: from 2001 to 2003 uplift seems to occur, whereas most stations are subsiding in the following years.

Local deformation is more visible when represented relative to HH38 on figure 9. Some important motion occurs within the Nesjavellir area, with a large difference between the two time periods. Subsidence seems to appear in Olkelduhals area (south-west of OLKE) after 2003.

Vertical motion seems to confirm local deformation in Nesjavellir and Olkelduhals (OLKE) geothermal areas. However, vertical motion is less well constrained than horizontal motion because of uncertainties of measurements. The vertical motion can be compared to synthetic aperture radar interferograms discussed in the next section.

3 InSAR observation of deformation: 2005-2006

3.1 InSAR concepts

Interferometric Synthetic Aperture Radar (InSAR) is a RADAR method that can be applied to measure Earth’s surface displacements and topography from space, with an accuracy of few ten meters. A SAR satellite transmits an electromagnetic pulse and detects the echo from the target on the Earth’s surface. The system’s geometry is represented in figure 10. The direction of the satellite’s motion is called azimuth, and the perpendicular distance is the range. The line between the sensor on the satellite and the point at the Earth’s surface is called the Line of Sight (LOS).

Two acquisitions are necessary to detect topography or deformation of the Earth’s surface. The first radar image is called master and the second slave. For the same point on the ground, the difference in the phase of the master echo and the slave echo $\Delta\phi$ is proportional to the difference in range $\Delta r$ between the master position and the slave position relative to the point:

$$\Delta\phi = -\frac{4\pi}{\lambda} \Delta r$$

where $\lambda$ is the wavelength of the signal ($\lambda = 5.6cm$). $\Delta r$ is first due to the difference in the position of sensor between master and slave acquisition, with respect to a point situated on a reference ellipsoid (here WGS84 is used), the phase introduced is called flat earth phase. $\Delta r$ is also due the presence of
Figure 10: Geometry of SAR system. The sensor is moving in the plane perpendicular to the paper, and looking in a perpendicular direction, at an angle $\theta$ from the vertical. $\theta$ is called look angle and is usually $23^\circ$. The position of the satellite during the master acquisition is $m$, and $s$ for the slave acquisition. $B$ is the length of the baseline between the two positions, and $B_\perp$ its perpendicular component. $r$ is the range between the sensor and the earth’s surface (from Hooper et al., 2007).
topography on the area studied, and to possible ground displacements occurring between the two acquisitions. Therefore, those phenomenons introduce a phase difference between the master and slave images, that is approximately, according to Bürgmann & al (2000) [?]:

\[
\Delta \phi = \frac{4\pi}{\lambda} (-\vec{B} \cdot \vec{l} + \vec{D} \cdot \vec{l})
\]

where \( \vec{l} \) is a unit vector in the line of sight direction, \( \vec{D} \) is the ground displacement vector. Therefore, InSAR can be used to estimate topography, if the flat earth phase is substracted from the observation, assuming that no ground deformation occured between the two acquisitions. If deformation occurred, after removing the flat earth phase and the one due to topography from equation (32), it is possible to measure the displacement for each point on the ground in the line of sight. Phase due to the flat Earth is deduced from the positions of the satellite and the reference ellipsoid. The phase due to topography is calculated using a Digital Elevation Model (DEM), obtained from other observations. In order to convert the phase observations into displacements observations, the phase differences have to be integrated, from a reference point over the area to get the displacements. This last operation is called unwrapping.

InSAR is able to detect motion in the LOS at a centimeter level over an area with a ten meter resolution. The motion detected is the motion in the line of sight, which is close to the vertical. Therefore, the combination of GPS (accurate horizontal displacements) and InSAR (accurate vertical motion) gives complementary results about on-going deformation of an area. Also, the spatial cover of InSAR is nearly continuous, contrary to GPS that gives point displacements.

However it is sometimes difficult to estimate deformation over an area because of several sources of noise: the presence of unstable reflectors such as vegetation produces a temporal decorrelation in the phases of the echo. In this case the deformation signal is hidden by noise. Moreover, reflectors might be removed between the two acquisitions (cultivated field). This kind of noise is refered as temporal decorrelation. It is therefore necessary to identify stable reflectors called persistent scatterers. Several other biases affect the measurements such as geometric decorrelation (caused by the difference in the orientation of the sensor relative to the reflector between the master and slave acquisition), atmospheric delays (comparable to the GPS case), and instrumental noise.
3.2 Data sets used

The radar data used for this study are ENVISAT images from track 367 acquired between June 2005 and September 2006. Unfortunately, data from the period 2001-2005 were not available for this area, or the images did not extend over whole Hengill area. However, this data set provides an idea about what kind of vertical local deformation occurred after the period studied by GPS, which is close enough to check the consistency of GPS results. I used seven pictures acquired in July, August and September 2005, and in June, July, August and September 2006.

3.3 Processing: the StaMPS method

SAR images were processed using the Stanford Method for Persistent Scatterer (StaMPS) developed by Andy Hooper (Hooper et al., 2007). This method addresses the problem of temporal decorrelation of the phase by identifying a set of persistent scatterers in the interferograms, using a phase stability criterion. The main parts of processing are described in the following paragraphs. More details can be found in the StaMPS/MTI manual written by Andy Hooper.

Formation of interferograms

Interferograms are formed using the DORIS software developed by the Delft Institute for Earth-Oriented Space Research, Delft University of Technology.

Each image contains a value of the amplitude and the phase of the signal reflected by each pixel. An interferogram is formed by multiplying the signal of an image with the complex conjugate of the second image. The phase of the interferogram is the difference of the phases of each image. In general all images are compared to a master that is chosen by minimization of the sum of decorrelation of all interferograms.

Before differentiating the phases, each image is resampled to the master one. This is called coregistration and it is a way to eliminate differences in geometry. More details about coregistration algorithm are provided by Hooper & al (2007) [10].

Then, interferograms are formed and phase is corrected from flat earth phase using the reference ellipsoid WGS84 and from the topography using a DEM. Errors introduced by this last step are due to the uncertainties about the DEM, and will be referred to as look angle error.
Phase stability estimation and PS selection

PS pixels are pixels whose phase is stable enough over all the interferograms. A first set of pixels is selected assuming a statistical relation between phase stability and amplitude stability. For that, the amplitude dispersion index $D_A$ is used:

$$D_A = \frac{\sigma_A}{\mu_A} \quad (33)$$

where $\sigma_A$ and $\mu_A$ are respectively the standard deviation and the mean of the amplitude of a pixel over the interferograms. Pixels with $D_A < 0.4$ are selected to be potential PS pixels.

Then, the phase stability is examined. According to Hooper & al (2007), the wrapped phase is written as:

$$\psi = W [\phi_{def} + \phi_{atm} + \Delta \phi_{orbit} + \Delta \phi_{theta} + \phi_{noise}] \quad (34)$$

where $\phi_{def}$ is the phase due to displacement in the line of sight, $\phi_{atm}$ is the phase due to atmospheric delay, $\Delta \phi_{orbit}$ is the phase introduced by uncertainty about the satellite’s position, $\Delta \phi_{theta}$ is the phase due to look angle error, $\phi_{noise}$ is the phase introduced by noise, and $W$ is the wrapping operator. PS pixels are pixels whose $\phi_{noise}$ is minimum. To estimate $\phi_{noise}$, it is necessary to estimate the first four terms. Assuming that the first three terms and part of the fourth are spatially correlated, the phase is low pass filtered in space, and the result gives an estimation of these terms. Then, an estimation of noise, master image’s contribution to noise and spatially uncorrelated look angle error are done by a least square inversion. After several iterations of this process, an estimation of $\phi_{noise}$ is provided and PS pixels are selected.

Displacement estimation

In order to estimate the absolute phase due to deformation, the phase has to be unwrapped: the phase difference between each adjacent pixels are integrated over all the interferogram, from a reference pixel, whose absolute displacement is unknown. This can be achieved with a good accuracy only if the phase difference between each neighboring pixels is less than $\pi$. The unwrapped phase of a pixel is:

$$\phi = \phi_{def} + \phi_{atm} + \Delta \phi_{orbit} + \Delta \phi_{theta} + \phi_{noise} + 2\pi k \quad (35)$$

which is also corrected from master’s contribution to noise and spatially uncorrelated look angle error found in the phase stability estimation step. $\Delta \phi_{theta}$ is the remaining spatially correlated look angle error and $k$ is an
integer constant for each interferogram if the unwrapping has been done correctly.

In order to get $\phi_{def}$ from $\phi$, the other terms are estimated. For that, they are separated in two parts:

- The master’s contribution to $\phi_{atm} + \Delta\phi_{orbit}$ which is supposed to be correlated in time (it is the same over each interferogram). This part is noted $\phi_m$

- The slave’s contribution to $\phi_{atm} + \Delta\phi_{orbit}$ as well as $\Delta\phi_{\theta_{\text{c}}}$ which are supposed to be uncorrelated in time. This term is noted $\phi_s$

In order to eliminate the $2\pi k$ term, phase differences between adjacent pixels whose $k$ is supposed to be the same, are made. The resulting difference is low pass filtered in time to get $\phi_m$. Then, it is high pass filtered in time to get $\phi_s + \phi_{\text{noise}}$ and low pass filtered in space to get only $\phi_s$.

After removing $\phi_m$, $\phi_s$ and $\phi_{\text{noise}}$ from $\phi$, $\phi_{def}$ is obtained.

### 3.4 Results

Interferograms are presented on figure 11. Results of the PS processing are presented on figure 13, as well as spatially correlated look angle error and the master’s contribution to atmosphere and orbit error phase.

Interferograms present a good coherence in the summer months, whereas the two interferograms from September have less coherence. This could be a consequence of the presence of snow in September. Furthermore, the south-east part (south-east of HVER) is a cultivated area, so temporal decorrelation is important due to human activity. This explains the lack of PS pixels in this area on figure 13.

The mean LOS velocity represented on figure 13, has two important characteristics: a subsidence located within Nesjavellir, Olkelduhals and Hellisheidi geothermal areas, and uplift located south of the area between Olkelduhals and Hellisheidi. Those signals remain over all interferograms with enough coherence. The uplift is persistent over the interferograms even after correction of the master and orbit error phase, so it might be a real signal.

Deformation during the period 2005-2006, following the GPS measurements shows the same characteristics in terms of local deformation as what was inferred by GPS for 2003-2005: subsidence in areas where fluids are drilled. This kind of deformation is therefore a persistent feature of the area. Furthermore, an area south of the GPS measurements presents uplift after 2005. Unfortunately, the closest continuous station (HLID) is not located...
Figure 11: Interferograms of the Hengill area. One cycle of colors represents an evolution of phase from $-\pi$ to $\pi$.

Figure 12: Spatially correlated error terms, evaluated from PS processing. Black dots represent GPS measurements between 2001 and 2005. Black squares are the continuous GPS stations of HVER and OLKE.
Figure 13: Mean LOS velocity, corrected for spatially correlated look angle error. Black dots represent GPS stations measured from 2001 to 2005. Black squares are the continuous GPS stations of HVER and OLKE. For the mean velocity: blue colours represent an increase of the LOS distance, which corresponds to subsidence, and red indicates uplift. Velocities are given relative to all the pixels of the 22th July 2006.
4 Modelling and interpretation of the deformation

4.1 The strain rate field

4.1.1 The calculation of strain rate

In order to calculate the strain rate in the Hengill area, I used the method described by Beaven et al., 2001. The authors applied this to New Zealand deformation. Keiding et al., 2008 used this method in the Reykjanes Peninsula. I only used horizontal velocities. The calculation of the strain rate is another way to see crustal deformation in the area, independent of a reference frame.

The method estimates both a continuous velocity field and a continuous strain rate field that satisfy the two following conditions:

- the velocity field has to minimize the misfit to observed velocities
- the strain rate field has to minimize its variance

The velocity is decomposed as follows:

\[ \vec{v}(\vec{r}) = \vec{W}(\vec{r}) \times \vec{r} \quad (36) \]

Where \( \vec{r} \) is the velocity field at a position \( \vec{r} \) in a geocentric reference frame, and \( \vec{W} \) is a continuous function that describes the velocity at each position on the earth’s surface. Then, it is possible to express the strain rate in terms of \( \vec{W} \):

\[ \dot{\epsilon}_{\phi\phi} = \frac{\vec{\Theta}}{\cos \theta} \cdot \frac{\partial \vec{W}}{\partial \phi} \quad (37) \]

\[ \dot{\epsilon}_{\phi\theta} = \frac{1}{2} \left( \vec{\Theta} \cdot \frac{\partial \vec{W}}{\partial \theta} - \frac{\vec{\Phi}}{\cos \theta} \cdot \frac{\partial \vec{W}}{\partial \phi} \right) \quad (38) \]

\[ \dot{\epsilon}_{\theta\theta} = -\vec{\Phi} \cdot \frac{\partial \vec{W}}{\partial \theta} \quad (39) \]
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Where $\dot{\varepsilon}$ are the rate of strain, $\theta$ is the latitude of the considered point, $\phi$ is its longitude. $\hat{\mathbf{\Theta}}$ is a unit vector in the north direction and $\hat{\mathbf{\Phi}}$ in the east direction like on figure 14.

Then, the idea is to find $\vec{W}$ under the two conditions evocated. Once $\vec{W}$ is found, both rate of strain and velocity field are found. We have to introduce for that the curvilinear coordinates $(\xi,\eta)$. The correspondence to latitude and longitude is:

$$\begin{align*}
\xi &= R\phi \cos \theta \\
\eta &= R\theta
\end{align*}$$

(40) (41)

The area is divided in a grid whose nodes are $(\xi_i,\eta_j)$ i and j being integers. Within each cell $\vec{W}$ is expressed as a bicubic interpolation function:

$$\begin{align*}
\vec{W}(\xi,\eta) &= \sum_{p=0}^{1} \sum_{q=0}^{1} \left[ \vec{W}(\xi_{i+p},\eta_{j+q}) h_p \hat{\xi} h_q \hat{\eta} + \frac{\partial \vec{W}}{\partial \xi}(\xi_{i+p},\eta_{j+q}) \Delta \xi k_p \hat{\xi} h_q \hat{\eta} \\
&\quad + \frac{\partial \vec{W}}{\partial \eta}(\xi_{i+p},\eta_{j+q}) h_p \hat{\xi} \Delta \eta k_q \hat{\eta} + \frac{\partial^2 \vec{W}}{\partial \xi \partial \eta}(\xi_{i+p},\eta_{j+q}) \Delta \xi k_p \hat{\xi} \Delta \eta k_q \hat{\eta} \right] (42)
\end{align*}$$

Where we have: $\Delta \xi = \xi_{i+1} - \xi_i$, $\Delta \eta = \eta_{j+1} - \eta_j$, $\hat{\xi} = \frac{(\xi - \xi_i)}{\Delta \xi}$, $\hat{\eta} = \frac{(\eta - \eta_j)}{\Delta \eta}$ and $h_0, h_1, k_0, k_1$ being degree 3 polynomials.

Thanks to this interpolation, we just need to find the values of $\vec{W}$ and its derivatives at the nodes of the grid to get a continuous field for $\vec{W}$. For the derivatives of $\vec{W}$, finite difference formulae are used, for instance, for the middle of the grid:
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\[ \frac{\partial \vec{W}}{\partial \xi}(\xi_i, \eta_j) = \frac{\vec{W}(\xi_{i+1}, \eta_j) - \vec{W}(\xi_{i-1}, \eta_j)}{2\Delta \xi} \] (43)

Thus, \( \vec{v} \) is a linear combination of the values of \( \vec{W} \) at the nodes of the grid.

To express the strain rates in terms of \( \vec{W} \), we need the values of \( \frac{\partial \vec{W}}{\partial \theta} \) and \( \frac{\partial \vec{W}}{\partial \phi} \). Following expressions are used:

\[ \frac{\partial \vec{W}}{\partial \phi} = \begin{bmatrix} \frac{\partial \vec{W}}{\partial \xi} & \frac{\partial \vec{W}}{\partial \eta} \\ \frac{\partial \vec{W}}{\partial \eta} & -\frac{\partial \vec{W}}{\partial \xi} \end{bmatrix} \] (44)

\[ \frac{\partial \vec{W}}{\partial \theta} = \begin{bmatrix} -\frac{\partial \vec{W}}{\partial \xi} & \frac{\partial \vec{W}}{\partial \eta} \\ \frac{\partial \vec{W}}{\partial \eta} & \frac{\partial \vec{W}}{\partial \xi} \end{bmatrix} \] (45)

Strain rates are, like velocities, linear combinaison of the values of \( \vec{W} \) at the nodes of the grid. Instead of the strain rates, average strain rates over cells are calculated. The cells are defined by dividing each square \([\eta_j; \eta_{j+1}; \xi_i; \xi_{i+1}]\) in four quadrants. These average strain rates \( \vec{\varepsilon} \) are also linear combinaisons of the values of \( \vec{W} \) at the nodes of the grid.

The last thing is to find those values of \( \vec{W} \) at the nodes of the grid. This is done by an inversion in which the following function is minimized:

\[
\sum_{\text{cells}} \nu S \left( \varepsilon_{\phi\phi}^2 + \varepsilon_{\theta\phi}^2 + \varepsilon_{\theta\theta}^2 \right) + \sum_{\text{points}} \sum_{\alpha,\beta} \left( v^\text{fit}_\alpha - v^{\text{obs}}_\alpha \right) V^{-1}_{\alpha,\beta} \left( v^\text{fit}_\beta - v^{\text{obs}}_\beta \right) \] (46)

where \( \text{obs} \) stands for observations, fit for calculated values. \( V_{\alpha,\beta} \) is the covarience for the observations, with \( \alpha \) and \( \beta \) corresponding to latitude and longitude \( \theta \) and \( \phi \). Subscripts points refer to points where GPS measurements have been done. \( S \) is the area of a cell over which the strain rates are averaged. Finally, \( \nu \) is a weighting factor we have to choose for each cell. It is used to allow or not large values of the strain rate (or deformation) over each cell. Here I used a constant value of \( \nu \) since I don’t expect large gradients of deformation within the area. \( 1/\nu \) is called "strain rate variance parameter".

In practice the size of the grid and the strain rate parameters are to be fixed before the calculation.

4.1.2 Results

I tested different values of the grid size and rate of strain parameters. The best fit to the observations, that also limits the artefacts due to interpolation,
are obtained by dividing the area in 20×20 squares, each of them being divided in four cells, and with \( 1/\nu = 1e - 5s^{-1} \).

On figure 15 are represented the following quantities:

- the areal strain rate \((\ddot{\varepsilon}_{\phi\phi} + \ddot{\varepsilon}_{\theta\theta})\). Positive values represent expansion whereas negative values represent contraction.

- the shear strain \((\ddot{\varepsilon}_{\phi\theta})\).

- the maximum shear strain rate \((\sqrt{\frac{(\ddot{\varepsilon}_{\phi\phi} - \ddot{\varepsilon}_{\theta\theta})^2}{2}} + \ddot{\varepsilon}_{\phi\theta}^2)\) which corresponds to the shear strain when oriented 45° from the principal axes.

In figure 15, I just show the area covered by measured GPS sites (black triangles) that constrain the strain rate calculation. Signal observed in areas not measured are more likely to be artefacts due to interpolation in the calculation. I also took into account some parts of the signal that appeared consistent during variations of the grid size and the rate of strain, since artefacts are not stable if we change parameters of the calculation.

The most important characteristic of the area is the variation in areal strain. Between 2001 and 2003 a signal of contraction is visible in Nesjavellir geothermal area, that corresponds to the deformation due to fluid extraction. This signal is also visible in the second time period but less strong and shifted towards the south-west. This signal in 2003–2005 might also be an artefact but it was one of the persistent signal.

Between 2003 and 2005 some contraction is also present near the GPS station of OLKE. This corresponds to the area where velocities where slightly different between the two time periods. Since the area has also been drilled in that period of time, it is possible that this signal is caused by geothermal activity. However, the impact of an eventually inflating area located to the south is also possible, from the following InSAR observations (2005-2006)
Figure 15: Strain rates obtained from horizontal velocity field. Black triangles represent GPS measurements, red squares are continuous stations. Red points are boreholes.
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4.2 Nesjavellir deformation

Horizontal GPS velocities obtained in Nesjavellir for the period 2001-2003 show a convergence of the vectors. One possible source of this contraction is the geothermal exploration in Nesjavellir, where hot water and hot steam is extracted from the ground. According to Björnsson et al., 2006, one well has been extracting around 40 kg/s of hot water during the period between 2000 and 2005. Wells are drilled down to one or two kilometers depth in the area.

I attempt to model the deformation at Nesjavellir using a Mogi spherical source of pressure in an elastic halfspace. The approximation of an elastic halfspace is appropriate since we are looking at a local source in the upper 2 kilometers of the crust. This area could be considered as elastic, although the high temperature gradients. According to Sigmundsson & al (1997) the equations for the displacements caused by a spherical source of radius $R$ at depth $d$ are, in cylindrical coordinates ($r$ and $\phi$ in the horizontal plane are respectively the horizontal distance to the source and the azimuth, and $z$ along a vertical axis):

\begin{align}
  u_r &= C \frac{r}{(d^2 + r^2)^{3/2}} \\
  u_z &= C \frac{d}{(d^2 + r^2)^{3/2}}
\end{align}

(47)  (48)

where $u_r$ is the radial displacement and $u_z$ the vertical displacement. The tangential displacement $u_\phi$ is null. $C$ is the source strength parameter and is defined by:

$$C = \frac{3R^3 \Delta P}{4\mu}$$

(49)

with $\Delta P$ the variation of pressure and $\mu$ the shear modulus.

In order to calculate the displacements from the source parameters, I used program written by Peter Cervelli (1998). There are four model parameters: depth, XY location of the source, and change in volume of the pressure source (instead of change in pressure).

The GPS data used for the inversion are shown in figure 4.2. I only considered horizontal velocities. The plate spreading was considered uniform over the area, so a constant vector was added to the displacements found for the Mogi source, in order to approximate this motion. The velocity of HH41, since the station is far enough from Nesjavellir to be free from any geothermal signal, was considered to be the plate spreading velocity and added to the calculated displacements.
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Figure 16: Observed (black arrows) and calculated (green arrows) velocities relative to REYK for 2001 2003. The velocity of HH41 represent plate spreading over the area. The cross represents the center of contraction.

Using a grid search, I estimated the model parameters that give the best fit to the data in a least square sense. The best fitting model corresponds to a source at $1.5^{+0.1}_{-0.2} km$ depth, whose volume decreases by $0.1(\pm 0.05) \times 10^6 m^3$ (uncertainties are estimated by a bootstrap method of resampling the data set). It’s location is represented on the map on figure 4.2 as well as the predicted velocities.

The volume of steam and water extracted from wells in the area is much larger than the volume decrease I estimated. The difference can be explained by the presence of reinjection wells in Nesjavellir. Furthermore, the reservoir is filled by natural processes: rain and lake water circulate in the crust through the faults to the reservoirs.
4.3 Seismicity at Hengill

Seismicity in the Hengill area between 2001 and 2005 is shown in figure 17. Relocation of the events has been done at the Meteorological Office. In general, magnitudes are between 0 and 4, and most of the seismicity is located at 3 to 9 km depth, up to 12 km depth. This is similar to what was observed before 1993. This background seismicity concentrates near the areas of Nesjavellir and Olkelduhals. This corresponds to the areas where GPS velocities where deviating from the general East-West trend.

Temporal evolution of seismicity in Nesjavellir and Olkelduhals is represented in figure 18. This is a way to check if the change in displacements observed between 2001-2003 and 2003-2005 near Olkelduhals is correlated with an increase in seismicity. A little increase is visible after 2003 in Olkelduhals.

Seismicity evolution seems to be correlated with variation of the surface displacement field. However, most of the events are quite deep and not strong enough to generate significant surface displacements. Increase in seismicity and surface displacement variations may be consequences of another source of deformation in the crust.
Figure 17: Seismicity represented by depth and magnitude. Black triangles represent GPS stations. Blue squares are the area where the evolution of seismicity was observed.

Figure 18: Evolution of seismicity in Nesjavellir and Olkelduhals between 2001 and 2005.
Conclusion

According to GPS horizontal measurements, the main regional characteristic of the deformation in the Hengill area is an eastward motion of the area relative to the station REYK situated in Reykjavik. This motion represents the accumulation of strain at the plate boundary (Keiding et al., 2008). It is difficult to define the position of the plate boundary. It appears that all the area is moving relative to Reykjavik, but it is not possible to define a rigid block motion, so that all the area is considered as plate boundary. Furthermore, the regional motion pattern is affected by local deformation due to fluid pumping from the upper two km of the crust: the most important signal is present in Nesjavellir where ground contraction occurred in the drilling area between 2001 and 2003. This was interpreted as an extraction of $0.11 \times 10^6 m^3$ of fluids from 1.5 km depth. Subsidence observed with InSAR between 2005-2006 within all the area is certainly also due to pumping. Furthermore, the velocity field changes after 2003 in Nesjavellir and Olkelduhals areas: those changes are local and could be due to variations of the amount of water pumped from the boreholes. Fluids might also be reinjected in reservoirs, that might explain why contraction stops after 2003 in Nesjavellir. InSAR shows uplift in an area south of Hengill after 2005. It might be interesting to check if this inflation is also present before 2005 and before 2003, so that it could explain the deviation of the motion near Olkelduhals. For that, and also in order to better constrain vertical motion in zones of contraction, it is necessary to compare GPS and InSAR data for the period 2001-2005. Finally, the local motions due to fluid extraction in the area shows that the continuous stations of OLKE and HVER might record local noise, that is necessary to take into account when studying long term plate motion in Iceland.

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