

$$\begin{aligned}\vec{\nabla}_n (\vec{\nabla} \varphi) &= \epsilon_{ijk} d_j d_k \varphi \\ &= \frac{1}{2} \epsilon_{ijk} (d_j d_k \varphi - d_k d_j \varphi) = 0\end{aligned}$$

$$\begin{aligned}(2) \quad \vec{\nabla} \cdot (\vec{\nabla}_n \vec{u}) &= d_i \epsilon_{ijk} d_j u_k \\ &= \frac{1}{2} (\epsilon_{ijk} d_i d_j u_k + \epsilon_{jik} d_j d_i u_k) \\ &= \frac{1}{2} u_{k,ij} (\epsilon_{ijk} + \epsilon_{jik}) = 0.\end{aligned}$$

$$(3) \quad \vec{\nabla} \cdot (\varphi \vec{\psi}) = d_i (\varphi \psi) = \varphi d_i \psi + \psi d_i \varphi$$

$$\begin{aligned}(4) \quad \vec{\nabla} \cdot (\varphi \vec{A}) &= d_i (\varphi A_i) = \varphi d_i A_i + (d_i \varphi) \cdot A_i \\ &= \varphi \vec{\nabla} \cdot \vec{A} + \vec{\nabla} \varphi \cdot \vec{A}\end{aligned}$$

$$\begin{aligned}(5) \quad \vec{\nabla}_n (\varphi \vec{A}) &= \epsilon_{ijk} d_j (\varphi A_k) \\ &= \epsilon_{ijk} [\varphi d_j A_k + d_j \varphi A_k] \\ &= \varphi \vec{\nabla}_n \vec{A} + \vec{\nabla} \varphi \wedge \vec{A}\end{aligned}$$

$$\begin{aligned}(6) \quad \vec{\nabla} \cdot (\vec{A} \wedge \vec{B}) &= d_i \epsilon_{ijk} A_j B_k \\ &= A_j d_i \epsilon_{ijk} B_k + B_k \epsilon_{ijk} d_i A_j \\ -\vec{A} \cdot \text{rot } \vec{B} + \vec{B} \cdot \text{rot } \vec{A} &= A_j (-\epsilon_{jik} d_i B_k) + B_k \epsilon_{kij} d_i A_j\end{aligned}$$

$$\begin{aligned}
\textcircled{7} \quad \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{A}) &= \epsilon_{ijk} \partial_j \epsilon_{klm} \partial_l A_m \\
&= \epsilon_{ijk} \epsilon_{klm} \partial_j \partial_l A_m \\
&= \epsilon_{kij} \epsilon_{klm} \partial_j \partial_l A_m \\
&= (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \partial_j \partial_l A_m \\
&= \partial_i \partial_m A_m - \partial_j \partial_j A_i \\
&= \vec{\nabla} \cdot (\vec{\nabla} \cdot \vec{A}) - \Delta \vec{A}
\end{aligned}$$

$$\begin{aligned}
\textcircled{8} \quad \vec{\nabla} \cdot (\psi \vec{\nabla} \psi) &= \epsilon_{ijk} \partial_j (\psi \partial_k \psi) \\
&= \epsilon_{ijk} (\partial_j \psi) (\partial_k \psi) + \cancel{\epsilon_{ijk} \psi \partial_i \partial_k \psi} \\
&= \vec{\nabla} \psi \cdot \vec{\nabla} \psi
\end{aligned}$$

$$\begin{aligned}
\textcircled{9} \quad \vec{\nabla} \cdot (\vec{A} \cdot \vec{B}) &= \epsilon_{ijk} \partial_j \epsilon_{klm} A_l B_m \\
&= \epsilon_{ijk} \epsilon_{klm} (\partial_j A_l B_m) \\
&= (\delta_{il} \delta_{jm} - \delta_{im} \delta_{jl}) \partial_j (A_l B_m) \\
&= \partial_j (A_i B_j) - \partial_l (A_l B_i) \\
&= A_i \partial_j B_j + B_j \partial_j A_i - \partial_l A_l \cdot B_i - A_l \partial_l B_i \\
&= \vec{A} \cdot (\vec{\nabla} \cdot \vec{B}) + (\vec{B} \cdot \vec{\nabla}) A_i - (\vec{\nabla} \cdot \vec{A}) B_i - (\vec{A} \cdot \vec{\nabla}) B_i
\end{aligned}$$