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Body Tides of a Realistic Earth

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Abstract

A precise modelling of the Earth tides is necessary to correct the space gravimetry observations, from satellites such as GRACE and GOCE. It is also useful to correct ground measurements, and even more important for superconducting gravimeters, which have a 10 nGal precision. The Earth response (deformation and gravity) to tides or atmospheric load is generally computed assuming radial symmetry in stratified Earth models, at the hydrostatic equilibrium. Our study aims at providing a new Earth tide model, which accounts for the whole complexity of a more realistic Earth. Our model is based on a dynamically consistent equilibrium state which includes lateral variations in density and rheological parameters (shear and bulk moduli), and interface topographies. We use a finite element method and we solve numerically the gravito-elasticity equations. The deviation from the hydrostatic equilibrium has been taken into account as a first order perturbation. We investigate the impact on Earth tidal response of an equilibrium state different from hydrostatic and of the topography of the interfaces, for a simple model of lateral variation: a spherical anomaly in the mantle, which can represent plumes and superplumes. At the M2 frequency (semi-diurnal), we estimate the order of magnitude of the perturbation as a function of the radius and physical parameters of the anomaly.

Body tide Modelling

The elasto-gravitational equations are solved in two steps:
-1- the classical equations are solved for a spherical earth with hydrostatic state of pre-stresses,
-2- we solve the equations for the Earth with lateral variations using a fisrt order perturbation theory, knowing the solutions of the first step (see Dahlen & Tromp, 1998).

We use the spectral element method to solve the two systems of equation. The method is applied on the cubed sphere mesh (Ronchi et al., 1976).

Our numerical model is based on a seismological model developped by E. Chaljub (Chaljub et al., 2003), and uses the formalism and the theory proposed by Chaljub & Valette (2004).

Elasto-gravitational deformations

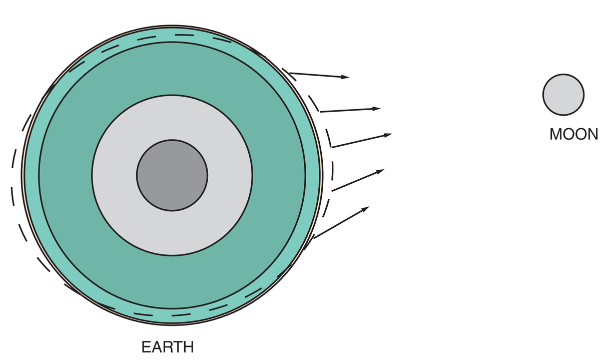

Fig. 1: The body tides of a spherical Earth with no lateral variations.

Earth parameters:

- r Density
- T_0 Pre-stress tensor
- f_0 Gravitational potential
- w Tide frequency

The unknowns:

- $\vec{u}$ Tide displacement
- f Mass redistribution potential

$$(1)\begin{cases} -r\,w^2\vec{u} + A\vec{u} + r\nabla f = \vec{f} & \text{Momentum equation} \\ \nabla\cdot(r\,\vec{u}) + \dfrac{1}{4pG}\Delta f = 0 & \text{Mass redistribution equation} \end{cases}$$

with A the elastodynamical operator defined such as

$$A\vec{u} = -\nabla.T_1^L(\vec{u}) + \vec{\nabla}(r\,\vec{u}\cdot\vec{\nabla}f_0) - \vec{\nabla}.(r\,\vec{u})\vec{\nabla}f_0$$

and T_1^L the lagrangian Cauchy stress tensor.

First order Perturbation theory

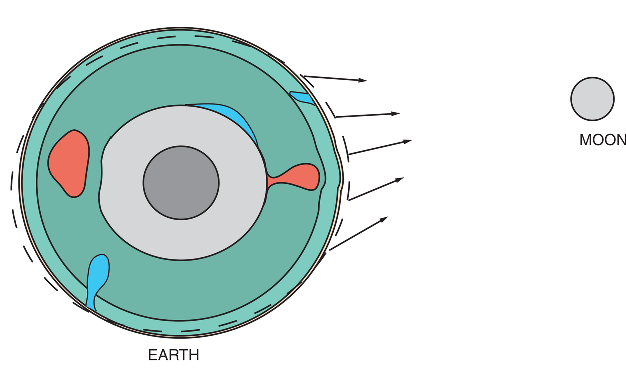

Fig. 2: The body tides of the Earth with lateral variations, interface topographies and deviatoric pre-stresses.

Earth parameters perturbations induced by lateral variations :

- dr Density perturbation
- $d\Gamma_0$ Pre-stress tensor perturbation
- df_0 Gravitational potential perturbation
- d Interface topography

The unknowns:

- $d\vec{u}$ Displacement perturbation
- df Potential perturbation

Using the first order perturbation theory, the equations are reduced to a system similar to the system of equations (1). The second members are calculated knowing the solutions for the classical elasto-gravitational problem with no lateral variations.

$$(2)\begin{cases} -r\,w^2 d\vec{u} + A d\vec{u} + r\nabla df = \vec{g_1}\left(\vec{u}, f, dr, d\Gamma_0, \ldots\right) \\ \nabla\cdot(r\,d\vec{u}) + \dfrac{1}{4pG}\Delta df = g_2\left(\vec{u}, f, dr, \ldots\right) \end{cases}$$

Numerical solutions

The equations are expressed on the cubed sphere mesh (see fig. 3). The derivative system of equations reduces to a linear system where the vector are expressed on all points of the grid.

$$\begin{cases} \overline{\overline{A}}U + \overline{\overline{N}}^T\Phi = G_1 \\ \overline{\overline{N}}U + \overline{\overline{L}}\Phi = G_2 \end{cases}$$

The system is symmetric (the matrix A and L are symmetric), but not necessarily positive definite (due to degree one toroidal modes). We use for this reason the SYMMLQ iterative algorithm developped by Paige & Saunders (1975) to solve the linear system.

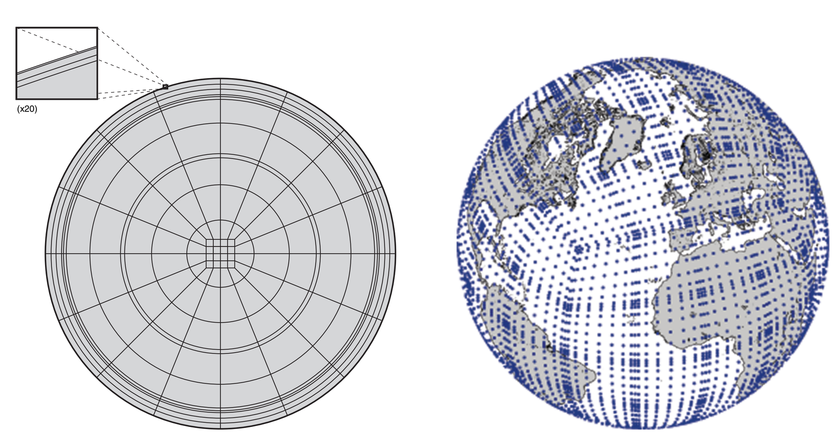

fig. 3: The grid used for the spectral element approximation. The left picture show the element distribution in a transversal section of PREM Earth Model. Each 3D element contains about 1000 points. The right picture shows the surface points of the grid.

References:

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Mantle lateral variations: example of superplume.

We modelize a referential Earth which contains a superplume in the Lower Mantle. We assume for simplicity that the plume is a large spherical heterogeneity of density rising through the Mantle. The figures 4-8 show the physical parameter perturbations induced by the thermal anomaly of the superplume. The superplume is localised in the equator plane for the present example.

The calculation has been made using a spherical harmonics approach with viscous rheological law (Greff-Lefftz, 2004).

With internal lateral variations of parameters the Earth cannot present a hydrostatic equilibrium. The pre-stress tensor is calculated considering that the plume arises in the Mantle with the appropriate Stokes velocity.

Fig 4.: Density perturbation induced by a superplume

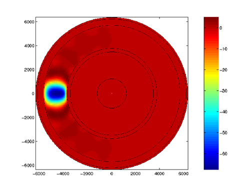


Fig 5.: Potential perturbation

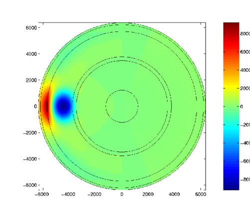


Fig 6.: Displacement

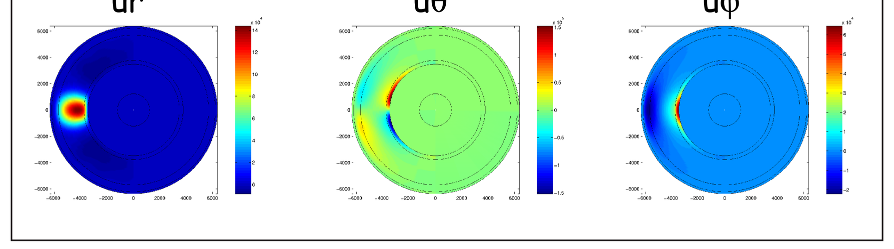


Fig 7. : Deviator pre-tress tensor

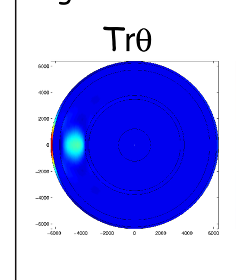
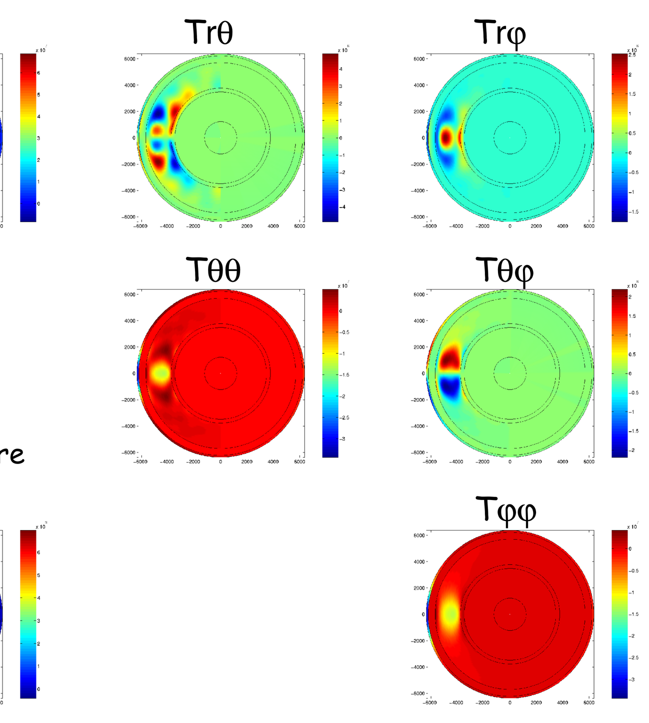
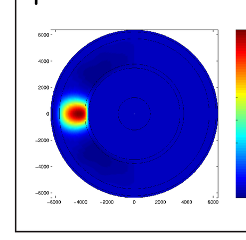



Fig 8 : Pressure perturbation



Earth body tide perturbation: Impact of the african and the pacific superplumes.

The bimodal convection

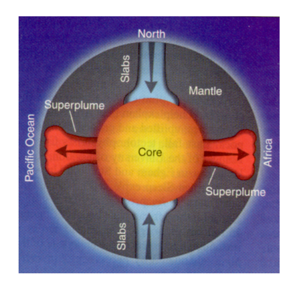
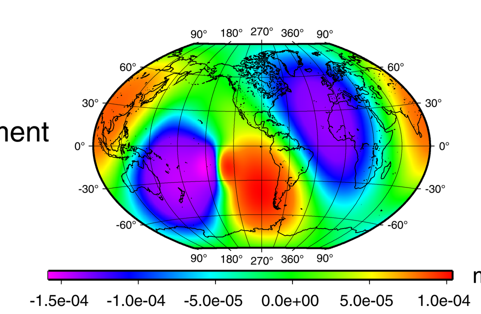


Fig 9.: The bimodal convection (figure from Kerr, 1999)

Recent models of Earth dynamics propose that the Mantle convection is mainly bimodal in the Mantle (see Gu & al., 2001, for seismic tomography; Davaille, 1999, for analogical study of thermochemical convection). These models suggest that the Lower Mantle structure is dominated by two large superplumes, one below the Pacific, the other below South Africa (Courtillot et al., 2003).

Perturbation of radial displacement
t = 0 T(M2)



Comparison with love number perturbations induced by the earth ellipticity:

x 10-3	Dk0	Dk+
Ellipticity	1.67	-0.56
Superplumes	-0.54	-0.09

We modelize the two superplumes as spherical heterogeneities, using the approach presented in the above box. The superplumes are illustrated in the figure 10, and the geoid topography of our referential Earth is presented in the figure 11.

We calculated the impact of these heterogeneities on the M2 body tides. The figures below present the perturbation of gravity on the deformed Earth surface with time (during one period T(M2)). The first row present the gravity variations of the Earth with no lateral variations (a Spherical Non Rotating Elastic and Isotropic planet), and the second row of figures present the additional perturbation of gravity induced by the superplumes (The degree 0 and the degree 1 of the solution have been substracted).

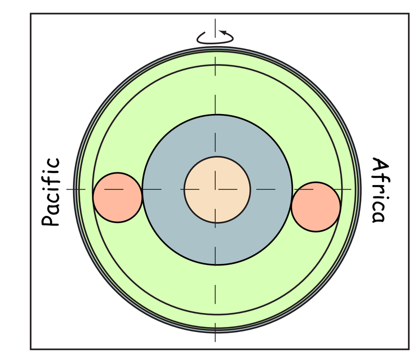


Fig 10.: The two heterogeneities.

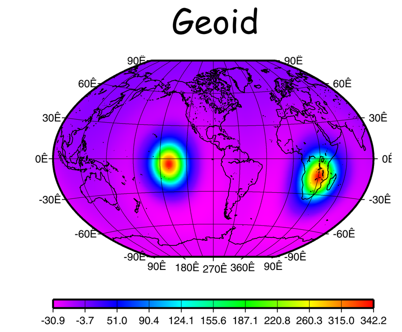
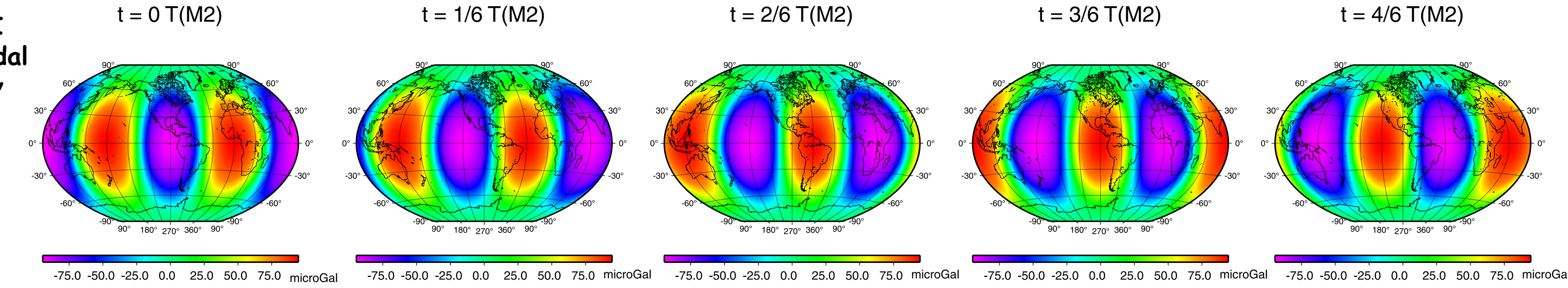


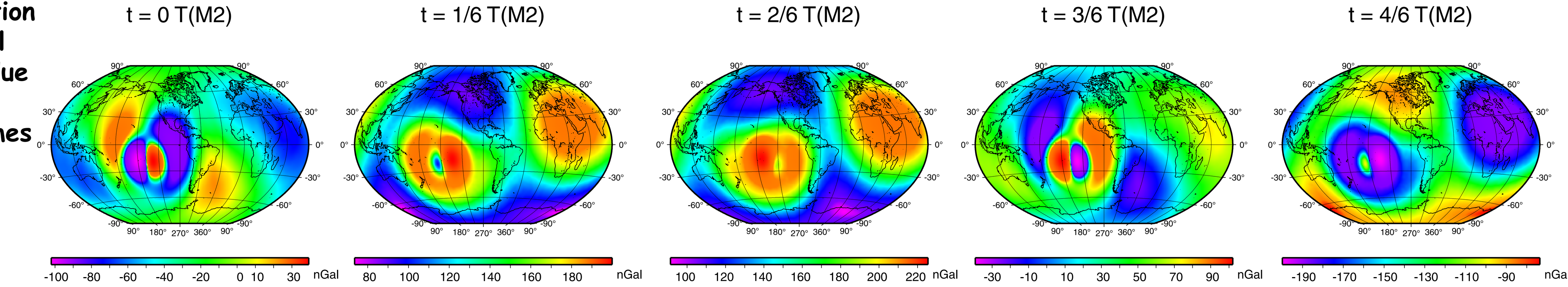
Fig 11.: The geoid of our referential Earth.

Surface tide gravity response during 1 M2 period:

SNREI Earth tidal gravity



Perturbation of tidal gravity due to superplumes



Discussion and conclusion

In this study, we have considered superplumes as very large heterogeneities. So we have computed what is expected to be the maximum lateral variations possible in the Mantle. The maximum perturbation of the displacement due to the M2 tide is about 0.1 mm, and the maximum perturbation of gravity on the deformed Earth surface is about 110 nGal. The displacement is too small to be detected in the tidal signal. The perturbed gravity is 10 times higher than the present instrumental precision of superconducting gravimeters, but cannot be presently detected because body tide models depend on oceanic tide loading models which present a lack of precision (about 0.4 microGal presently, see).
We show here that lateral variations have a weak impact on body tides but should be detectable in a next future with the improvement of oceanic tidal models. With a seismic tomographic model, our model shall be able to provide a very precise body tide model that should be of interest for many geophysical and geodetic domains, including spatial gravimetry. Moreover this type of study in the future should be useful to understand and validate the Mantle convection models and the inner structure of the Earth.