Evolution of the contact network during tilting cycles of a granular pile under gravity

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We perform discrete numerical simulations of a granular pile undergoing quasi-static tilting cycles in the gravity field. The volumic deformation and the granular fabric exhibit hysteretic evolutions with the slope of the free surface. When exploring the range of metastable slopes -between the angle of repose and the angle of avalanche-, the contact network is strongly affected, as evidenced by the qualitative change of the hysteresis. A specific contribution to the observed hysteresis of the weak contacts is underlined: they carry the memory of the proximity of the slope destabilization.

1 INTRODUCTION

When inclined in the gravity field, a granular slope starts flowing when the surface slope reaches the avalanche angle θ_a, and comes to rest afterwards at the angle of repose $\theta_r < \theta_a$. In between these two angles, the slope is metastable: avalanches can be triggered by finite amplitude perturbations (Daerr & Douady 1999). Recent numerical studies investigated the mechanisms of destabilization of granular piles during continuous load in the gravity field (Staron, Vilotte, & Radjai 2002). Far before the avalanche, the authors observe the occurrence of local instabilities leading to the compaction of the pile. At the approach of the stability limit, they evidence dilatancy simultaneously with the percolation of clusters of critical contacts, that are essentially weak contacts. These results lead them to identify the major role of spatial correlations in the range of metastable slopes $[\theta_r; \theta_a]$. Similar correlations were also experimentally observed in (Deboeuf, Bertin, Lajeunesse, & Dauchot 2003; Kabla, Debregeas, di Meglio, & Senden 2004).

The metastability of the slope can be seen as a signature of a major modification of the internal state of the granular packing. In order to investigate the nature of these modifications, and to study the influence of this metastability on the following evolution, we simulate by contact dynamics the quasi-static cyclic evolution of a granular pile tilted in the gravity field towards the slopes $[\theta_r; \theta_a]$. Details of the numerical procedure are given in section 2. An analysis of the contact network in section 3 shows that the volumic strain and the granular fabric evolutions are hysteretic. The shape of the hysteresis is strongly affected when the pile slope θ explores the metastable range $[\theta_r; \theta_a]$. In section 4 we observe the contributions of the strong and weak contacts and we evidence the specific role of memory played by the latter.

2 NUMERICAL SET-UP

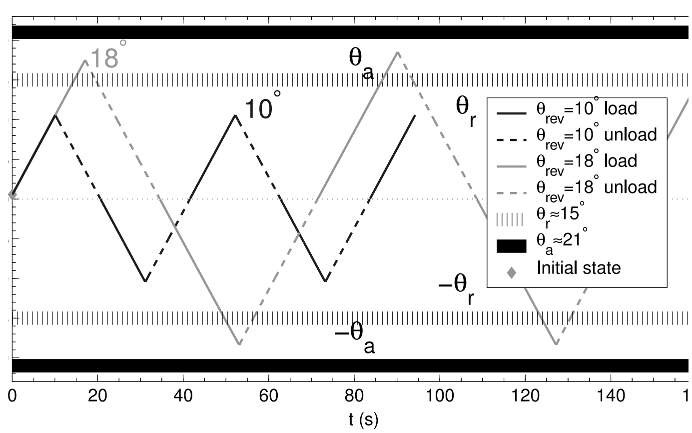

Figure 1. Measure of the surface slope of the pile during the rotation cycles of amplitude $\theta_{rev} = 10°$ and $\theta_{rev} = 18°$.

The discrete numerical simulations are performed using the contact dynamics method (Moreau 1994). The grains are perfectly rigid, and interact via frictional contacts governed by a Coulomb friction law ($\mu = 0.5$). The granular pile is made of a two dimensional rectangular box filled with 4000 circular grains of mean diameter D, with a thickness of $\approx 35D$, and a width of $\approx 120D$. The solicitation consists in quasi-static rotations in the gravity field at a rate of $0.001°$ per time step. Initially, the slope increases from $\theta = 0°$ to a maximal inclination angle θ_{rev}. At that stage, two successive cycles $[-\theta_{rev}; \theta_{rev}]$ are performed. All the simulations are averaged over 50 granular piles differing in the initial grains arrangments, and showing characteristic angles $\theta_r \simeq 15°$ and $\theta_a \simeq 21°$.

To study whether exploring the metastable regime influences the evolution of the granular pile, two values of the maximal inclination angle θ_{rev} are investigated. Figure 1 shows the time evolution of the pile slope averaged over 50 simulations. The smallest amplitude is chosen in order to keep inclination below the metastable range: $\theta_{rev} = 10° < \theta_r$. The largest amplitude is chosen such that the metastable regime is explored during each cycle, still below the minimal avalanche angle to prevent the pile from avalanching: $\theta_{rev} = 18° \in [\theta_r; \theta_a]$.

3 EVOLUTION OF THE CONTACT NETWORK

3.1 *Volumic strains of the granular pile*

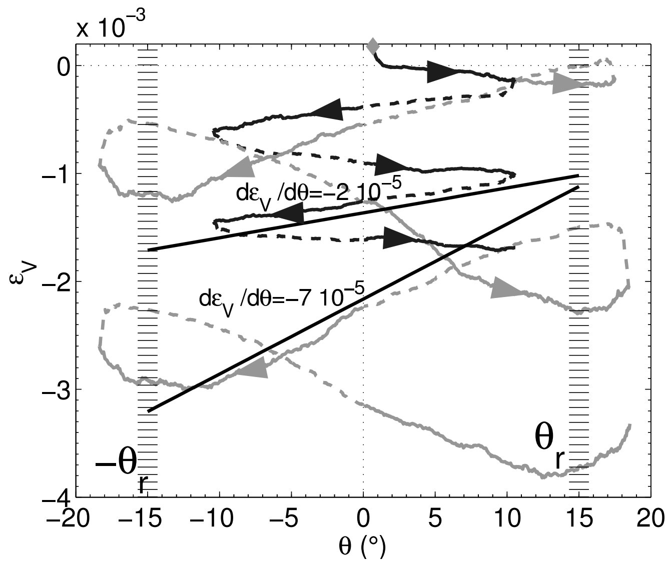

Figure 2. Estimation of the volumic strain as a function of the pile slope (same symbols as figure 1).

During each loading and unloading phase, local instabilities occur intermittently within the granular pile. The effect of these rearrangments of the grains is apparent when plotting the volumic strain $\epsilon_V = (V - V_0)/V_0$, where V is the volume of the pile and V_0 its initial volume, as a function of the slope of the pile (Fig. 2).

Irrespective of the value of θ_{rev}, cycles produce an overall densification of the pile. Note that the consolidation is of the same order (10^{-3}) as via avalanching from (Barker & Mehta 2000). For small cycles ($\theta_{rev} = 10°$), the granular pile is always contractant: ϵ_V decreases. On the contrary, for large cycles ($\theta_{rev} = 18°$), the granular pile exhibits compaction and dilatancy stages. Indeed for slopes in the metastable range $[\theta_r; \theta_a]$, the rearrangments cause the granular pile to dilate: ϵ_V increases, as already observed by Staron *et al.* . Interestingly, the dilatancy stage occurring when $\theta \in [\theta_r; \theta_a]$, changes dramatically the overall behaviour of the pile over the complete solicitation: the compaction is twice more efficient for large cycles than for small ones.

3.2 *Coordinancy of the grains*

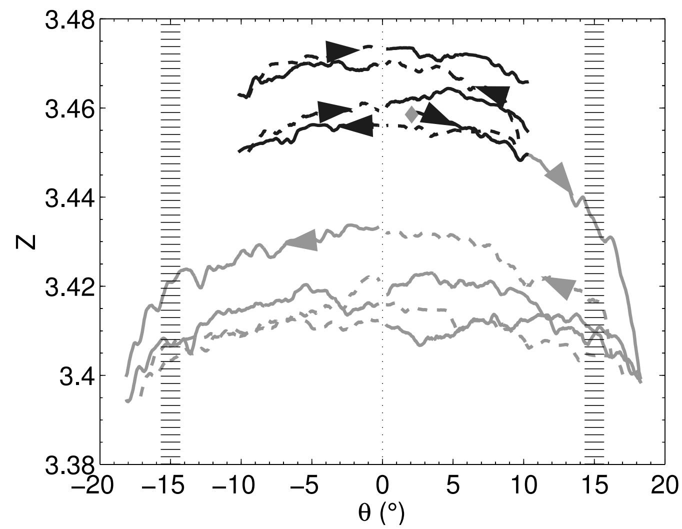

Figure 3. Coordinancy of the grains as a function of the pile slope (same symbols as figure 1).

For both values of the amplitude θ_{rev}, the coordination number averaged over all the grains Z, *i.e.* the mean number of contacts per grains, varies less than 1% (Fig. 3). Nevertheless, it exhibits a weak hysteresis according to the sense of rotation: Z tends to decrease during load, but to increase during unload.

Small and large cycles are identified by two different states: the value of the coordination number depends on θ_{rev}. During the small cycles, Z remains approximately at its initial value, in contrast with the large cycles, for which a variation of the coordination number occurs when exploring the metastable regime during the first load phase. Exploring the metastable range of slopes $[\theta_r; \theta_a]$ enhances the susceptibility for grains to rearrange and deeply modifies the state of the granular packing according to its initial one.

3.3 *Granular fabric of the pile*

To analyse further the micro-mechanical state of the granular pile, we investigate the statistics of the orientation of contacts. To do so, we compute the fabric tensor $\boldsymbol{t}$ using the following definition:

$$t_{ij} = \frac{1}{n^c} \sum_{\alpha=1}^{n^c} n_i^\alpha n_j^\alpha, \qquad (1)$$

where i and j denote the space dimension, $\boldsymbol{n}$ is the unity vector normal to the contact surface, and n^c is the total number of contacts α. The anisotropy of the granular pile is analysed with respect to the slope in terms of the anisotropic intensity Λ ($= 2\times$ the deviatoric component of $\boldsymbol{t}$) and the direction of anisotropy Φ with respect to the normal direction to the free surface (Fig. 4 and 5).

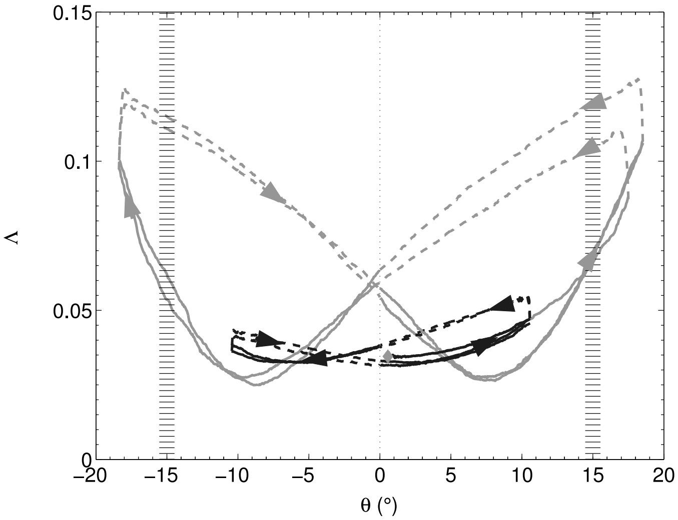

Figure 4. Anisotropic intensity of the contact network as a function of the pile slope (same symbols as figure 1).

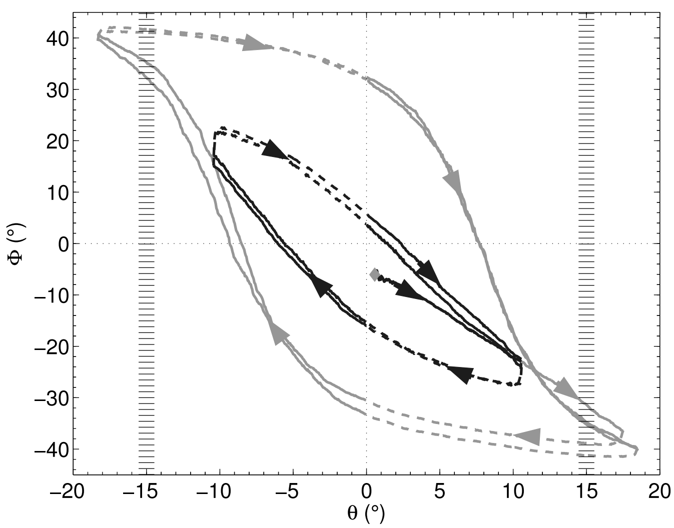

Figure 5. Anisotropic direction of the contact network as a function of the pile slope (same symbols as figure 1).

During the small cycles, a rather isotropic state is observed, and only small variations with θ are observed ($\Lambda \simeq 0.05$): the granular fabric is only weakly affected by the solicitation and remains close to the initial one. On the contrary, for large cycles, the granular fabric evolves significantly up to $\Lambda \simeq 0.12$. Note that during large cycles, the asymmetry of the initial state is removed on the granular fabric, by contrast with small ones.

Irrespective of the value of θ_{rev}, the direction of anisotropy tends to approach that of the free surface. For large inclinations, Φ tends towards $\pm 45°$. This behaviour corresponds with the mechanisms of creation of contacts in the direction of compressive stress, and the loss of contacts in the direction of extension. Indeed in the particular case of a hydrostatic infinite slope inclined in the gravity field, continuous media mechanics predicts that the principal direction of stress is $\pm 45°$ with respect to the free surface.

Besides these general observations, both $10°$ and $18°$-cycles exhibit hysteresis in the evolution of the granular fabric. Furthermore when the granular pile explores the metastable regime, the shape and the amplitude of the hysteresis are dramatically changed, pointing out the peculiar effect of this regime on the granular packing evolution. In particular, the fabric evolves less rapidly during unloading than during loading.

Altogether, this makes the state of the pile at an angle θ depend strongly on the history: the previous proximity of the destabilization is apparent on the observed characteristics. Knowing that dense granular media exhibit two contact subnetworks, strong and weak, depending on the intensity of the force transmitted at the contacts (Radjai, Wolf, Jean, & Moreau 1998), the role of the respective contributions of the two subnetworks is now investigated.

4 BI-MODAL RESPONSE OF THE FABRIC

A vertical gradient of the contact forces is related to the gravity field, such that the force intensity averaged over h-altitude contacts obeys: $\langle f\rangle(h) \propto h$. Therefore a h-altitude contact is defined as strong (resp. weak) if it transmits a normal force larger (resp. smaller) than the normal force averaged over all h-altitude contacts. The statistical analysis of the fabric is now restricted to strong and to weak contacts, from which the anisotropic direction of the strong (Φ^s) and the weak subnetwork (Φ^w) are calculated.

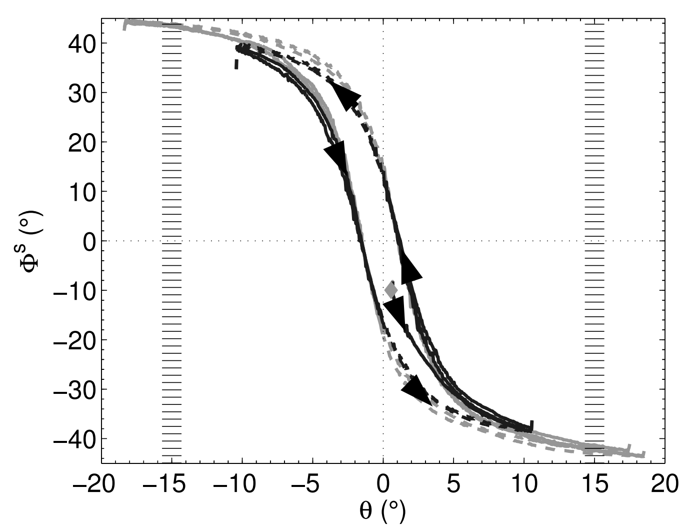

Figure 6. Fabric anisotropic direction of strong contacts as a function of the pile slope (same symbols as figure 1).

The evolution of Φ^s as a function of the pile slope exhibits also hysteresis (Fig. 6). However, by contrast with all previous observations, the shape of the hysteresis is the same for both small and large cycles. Exploring the metastable regime does not affect the strong contact subnetwork.

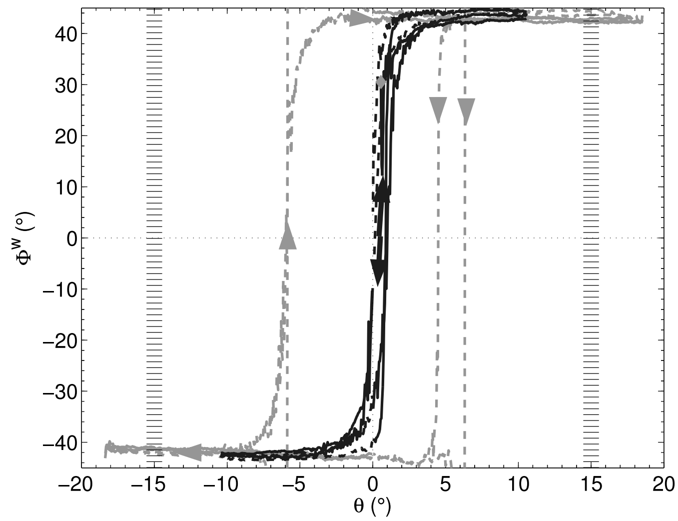

Figure 7. Fabric anisotropic direction of weak contacts as a function of the pile slope (same symbols as figure 1).

On the contrary, the evolution of Φ^w as a function of the pile slope (Fig. 7) is very different according to the cycle amplitude θ_{rev}. For small cycles ($\theta_{rev} = 10°$), Φ^w exhibits almost no hysteresis, and the hysteretic behaviour of the pile (see Fig. 5) is therefore governed by the strong contacts only. During small cycles, Φ^w remains approximately always normal to Φ^s, as observed by Staron *et al.* in the case of continuous load. On the contrary, for large cycles, a significant hysteretic behaviour is observed in the evolution of Φ^w. It consists in an early rotation of Φ^w, which leads to a remarkable orientation of the fabric of the two contact subnetworks: Φ^s and Φ^w are now aligned during a large part of the cycle, when exploring the metastable regime.

Two conclusions can be drawn from these observations:

- For small cycles, the hysteretic behaviour of the pile can be seen as a result of the strong contact subnetwork contribution;

- For large cycles, the weak contact subnetwork becomes strongly affected, and the hysteretic behaviour of the whole pile is strongly modified despite the identical response of the strong subnetwork.

5 CONCLUSIONS

When performing quasi-static tilting cycles in the gravity field, the overall behaviour of a granular pile is towards densification at constant coordination number. However differences of consolidation are observed between small and large cycles, due to the dilatancy stage occurring in the metastable range of slopes, that tends to enhance rearrangments of grains. Accordingly the evolution of volumic strains on geomaterial samples could be measured for the detection of previous proximities of the destabilization. Further analyses confirm the peculiar effect of the metastable regime on the grain packing organization: the hysteretic evolution of the fabric is deeply modified when exploring the metastable regime. Finally, this hysteretic behaviour is the result of a complex interplay between the strong and weak contact subnetworks. Such a specific interaction has to be considered when investigating the mechanical properties. For instance, it suggests to distinguish strong and weak contacts when appreciating stability limit analyses.

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