

Mixing at mid-ocean ridges controlled by small-scale convection and plate motion

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Supplementary Information

S1 Numerical setup and governing equations

Mantle solid-state flow may be reasonably described by the motion of a Boussinesq, highly viscous fluid in the limit of infinite Prandtl number (*i.e.*, inertia is negligible). In this case, the dimensionless governing equations in a 2D (x, z) Cartesian domain are the conservation of mass:

$$\nabla \cdot \mathbf{u} = 0, \quad (1)$$

the conservation of momentum:

$$-\nabla P + \nabla \cdot \eta(\partial_j u_i + \partial_i u_j) - Ra T \vec{e}_z = 0, \quad (2)$$

and the conservation of energy:

$$\partial_t T + \mathbf{u} \cdot \nabla T = \nabla^2 T. \quad (3)$$

In these equations, \mathbf{u} is the velocity vector, P is the dynamic pressure, T is the temperature, t is the time, \vec{e}_z is a unit vector pointing upward, Ra is the reference Rayleigh number. The viscosity $\eta = \eta_0 \exp[-\ln(10^3) T]$. To account in a simplified way for the effect of olivine-to-perovskite phase transition occurring between the upper and the lower mantle, we impose $\eta_0 = 1$ in the top half of the domain, and $\eta_0 = 10$ in the remaining bottom half of the domain. The value for the transition depth was chosen in order to maintain the same volumetric proportions between the upper and lower mantles as in the spherical

Earth. Additional runs with shallower transition depths did not reveal significant differences.

The reference Rayleigh number that provides a measure of the convective vigor is defined using dimensional values of reference physical parameters for the Earth's mantle:

$$Ra = \frac{\rho_{\text{ref}} g_{\text{ref}} \alpha_{\text{ref}} \Delta T D^3}{\eta_{\text{ref}} \kappa_{\text{ref}}}, \quad (4)$$

where $\rho_{\text{ref}} = 4500 \text{ kg m}^{-3}$ is the reference mantle density, $g_{\text{ref}} = 9.8 \text{ m}^2/\text{s}$ is the acceleration of gravity, $\alpha_{\text{ref}} = 2.5 \times 10^{-5} \text{ K}^{-1}$ is the thermal expansion coefficient, $\Delta T = 2500 \text{ K}$ is the superadiabatic temperature difference between the surface and the core-mantle boundary, $D = 2900 \text{ km}$ is the mantle thickness. The value for the reference thermal diffusivity κ_{ref} is set to $2 \times 10^{-6} \text{ m}^2/\text{s}$ according to recent estimates¹. The most poorly constrained parameter is the reference upper-mantle viscosity η_{ref} , which can range between $\sim 10^{20} \text{ Pa s}$ and $\sim 10^{21} \text{ Pa s}$. We use $Ra = 3 \times 10^5$, which corresponds to $\eta_{\text{ref}} = 1.8 \times 10^{21} \text{ Pa s}$. In order to account for the dependence of viscosity to temperature on the convective vigor, it is more appropriate to refer to the effective Rayleigh number:

$$Ra_{\text{eff}} = Ra / \eta_{\text{eff}}, \quad (5)$$

whose definition is based on the dimensionless effective viscosity $\eta_{\text{eff}} = \exp(-\ln(10^3) T_i)$, and T_i is the average dimensionless temperature. In our numerical experiments Ra_{eff} varied between 5×10^6 and 2×10^8 .

Equations (1) and (2) are formulated in terms of a stream function. The whole system is discretized with a finite volume method, using the *StreamV* code^{2,3} that was benchmarked against various analytic and numerical solutions.

At the surface, horizontal velocities are prescribed in order to produce a ridge-like flow, with surface plate velocities equal to the dimensionless Péclet number, Pe , representing the dimensionless half-spreading rate. The latter can be converted into dimensional units by multiplication with the diffusive time scale: D^2/κ_{ref} . The ridge axis is initially located at the center $x_{\text{ridge}} = x_{\text{center}}$, but its position varies with time due to an imposed sinusoidal and randomly perturbed motion of small amplitude in order to avoid complete symmetry at large values of Pe :

$$x_{\text{ridge}} = x_{\text{center}} + (0.08 + p r) \sin(t f) \quad (6)$$

where, $p = 0.02$ is the amplitude of the random noise, r is a random Gaussian number ranging between 0 and 1, and $f = 5Pe$ is the frequency.

All other velocity boundaries are free-slip. Temperature sidewalls are insulating (zero flux), and top and bottom boundaries are isothermal at $T = 0$ and $T = 1$, respectively. The domain was discretized using 100 cells of equal sizes in the vertical direction and between 200 to 800 square cells in the horizontal direction depending on the domain aspect ratio.

S2 Measuring mixing efficiency

We measure the convective mixing efficiency using a Lagrangian method that determines the value of the maximum Finite Time Lyapunov Exponents (FTLE), λ^+ , as described in⁴. The latter measure the Lagrangian strain rate along the direction of maximum stretching imposed by mantle motion. Large values of FTLE ($\lambda^+ \gg 1$) indicate an efficient (also called *chaotic*) mixing, where the Lagrangian deformation is an exponential function of time. In contrast, $\lambda^+ \leq 1$ are indicative of a weak (or *regular*) mixing, where the Lagrangian deformation exhibits only a linear dependence with time⁵. To avoid the bias due to the arbitrary choice of the initial conditions, the FTLEs were calculated after each experiment had reached statistical steady state. The latter is considered to be reached whenever the average mantle temperature has converged towards a constant value (statistically speaking), which also corresponds to similar values of surface and bottom heat fluxes.

S3 Influence of the viscous rheology

We have tested the effect of a possible bias due to the temperature change with Pe between models by running cases for which viscosity does not depend on temperature, accounting only for the viscosity increase between the upper and lower mantle *i.e.*, $\eta = \eta_0$, and $H = 1$. As specified in section S1 η_0 increases from 1 to 10 the upper to the lower mantle. For these models, we set $Ra = 3.9 \cdot 10^7$, which is close to the effective Rayleigh number values for the cases shown in Fig. 1a-f. The resulting dependence of FTLE values with Pe displayed in Fig. S1 (solid line) remains very similar to cases with temperature-dependent viscosity shown in Fig. 1.

Additionally, we tested the effect of stress-dependent rheology on our results using the following viscosity law:

$$\eta = \eta_0 \left[1 + \left(\frac{\sigma_{II}}{\sigma_T} \right)^{n-1} \right]^{-1}, \quad (7)$$

where σ_{II} is the second invariant of the deviatoric stress tensor, $n = 3.5$ and $\sigma_T = 1$.

The viscous rheology may affect the value of the critical Pe at which the transition from mixed small-scale convection and large scale flow to purely plate-driven flow. However, the resulting curve for the RMS FTLEs displayed in Fig. S1 (dashed line) shows the same V-shaped form observed for other rheologies (Fig. 2c).

Overall, the additional experiments shown in Fig. S1 demonstrate that our results remain robust for various mantle rheologies.

S4 Semi-analytic mixing model

The efficiency of convective mixing is directly related to the Lagrangian deformation. An incompressible fluid parcel will, under the action of the imposed flow, deform by stretching along a principal direction and shrink along one perpendicular direction. Considering the principal direction of stretching the corresponding Lagrangian strain rate is λ . Assuming that the flow is the quasi-linear superposition of large-scale plate flow and small-scale convection, one can write:

$$\lambda \cong \lambda_{\text{SSC}} \phi + \lambda_{\text{LS}} (1 - \phi), \quad (8)$$

where ϕ expresses the relative importance of small-scale convection on deformation, and ranges between 0 (pure plate-flow) to 1 (pure small-scale convection flow). The subscripts ‘LS’ and ‘SSC’ refer to the contributions of large-scale flow and small-scale convection, respectively.

The end-member Lagrangian strain rates λ_{SSC} and λ_{LS} are estimated to be directly proportional to the corresponding characteristic velocity of each flow field. This leads to $\lambda_{\text{LS}} = a Pe$ and $\lambda_{\text{SSC}} = b Ra_{\text{eff}}^{2/3}$ (*e.g.*,⁶), where Ra_{eff} is the effective Rayleigh number defined in (5).

The dimensionless constants $a = 0.15$ and $b = 3.5 \times 10^{-2}$ were determined by fitting the results of the numerical experiments carried out at various Pe and Ra_{eff} values. Note that upon determining a , FTLE values corresponding to the weakly-mixed isolated areas observed in the large-scale flow regime (*e.g.*, Fig. 1i-j) were excluded, because these regions are not sampled by ridges.

The function ϕ depends on the relative importance of the Péclet number Pe compared to the critical Péclet number Pe_c above which small-scale convection is inhibited. The latter is estimated from our numerical experiments (see Fig. 3) and can be expressed as:

$$Pe_c \cong 500 \exp[\ln(2) H]. \quad (9)$$

The best fit to our experiments for ϕ in the range $H = 0.5 - 2$ is the following exponential decrease with increasing Pe :

$$\phi(Pe, H) \cong \exp(-3.8 Pe/Pe_c). \quad (10)$$

The corresponding curve along with the associated fit to the numerical experiments are displayed in Fig. S2a and Fig. S2b, respectively.

Making the reasonable assumption that mixing is chaotic (as systematically observed in the vicinity of the ridge axis, see Fig. 1g-l), the evolution of passive heterogeneities is governed by the presence of *hyperbolic* points where the deformation occurs essentially via pure shear⁶. In the 2D (x, z) plane, assuming that compression and extension occur along the z -direction and the x -direction respectively, the velocity components (u, v) in the vicinity of an *hyperbolic* point located at $x = z = 0$ are: $(u = \lambda x, v = -\lambda z)$. Consider a circular passive heterogeneity of radius δ , centered on the *hyperbolic* point. Along the extensional

direction the rate of change of the heterogeneity at $x = \delta$ is then:

$$\frac{d\delta}{dt} = u(x = \delta) = \delta\lambda. \quad (11)$$

Similarly, along the compressional direction, the rate of change of the heterogeneity at $z = \delta$ is $-\delta\lambda$. Assuming a constant Lagrangian strain rate λ , Equation (11) can be integrated with respect to time from $t = 0$ to t :

$$\ln\left(\frac{\delta(t=0)}{\delta(t)}\right) = \ln\left(\frac{\delta_0}{\delta}\right) = \lambda t. \quad (12)$$

Equation (12), can be used to express the time $t = \tau$ necessary to stretch or to reduce a heterogeneity of initial size δ_0 to a given final size δ_f , *i.e.*, the ‘mixing time’:

$$\tau = \frac{1}{\lambda} \ln\left(\frac{\delta_0}{\delta_f}\right). \quad (13)$$

Equation (13) can be combined with Equations (8),(9) and (10) to estimate a mixing time along various mid-ocean ridges. For models with pure large-scale flow we use $\phi(Pe, H) = 0$, while models that account for both large-scale flow and small-scale convection we use Equation (10) to estimate the value of $\phi(Pe, H)$. For the calculations shown in Fig. 3c, we use $\delta_0/\delta_f = 8$, *i.e.*, we calculate the time necessary to stretch or to reduce the size of mantle heterogeneities by about one order of magnitude.

The influence of the predicted mixing efficiencies on the isotopic variability at ridges can be tested by relating the standard deviation to the mixing time. Under the assumption of chaotic mixing, the normalized variance that expresses the spatial dispersion of passive mantle heterogeneities decreases exponentially with time from one towards a statistical limit close to zero^{7,8} is:

$$\text{var} = \exp(-t/\tau). \quad (14)$$

We assume that at some ‘initial’ stage from present, the mantle is uniformly heterogeneous such that the eight regions considered share the same degree of heterogeneity (*i.e.*, initially, $\text{var}(t_0 = 0) = 1$ everywhere). During the subsequent period of time, t_e , differences in mixing efficiencies may locally impact the chemical variability. The latter can also be expressed by the standard deviation:

$$\sigma = \sqrt{\text{var}} = \exp\left(-\frac{1}{2} \frac{t_e}{\tau}\right). \quad (15)$$

Fig. S3a-c illustrates the influence of the elapsed time t_e on the chemical variability, for different values of Pe and H . The rate at which the standard deviation decreases with the elapsed time increases with decreasing Pe (Fig. S3a). However, the transition from the small-scale convection to the plate-driven flow regime corresponds to the largest values of the standard deviation (Fig. S3b-c).

This inverse V-shaped curve results directly from the observed dependence of the mixing efficiency with Pe and H (e.g., Equation (8), Fig. S2 and Fig. 2). While the values of these parameters are constrained for the calculations displayed in Fig. 3c, a meaningful value for t_e must be specified. A reasonable choice is $t_e = 200$ Myrs which corresponds roughly to half the duration of a Wilson cycle. During this period of time one can reasonably assume that the tectonic plate configuration considered in Fig. 3a and Table S1 (i.e., the values of Pe and H for each of the eight mid-ocean ridge regions) has remained approximately steady. However, this elapsed time needs to be compared with an estimate for the age of mantle material extracted at mid-ocean ridges.

For this purpose, we evaluate the sampling time, τ_{sampling} : the age of recycled lithospheric material being processed again at the ridge. In practice, using Lagrangian tracers we track for tens of overturn times the location and the corresponding age of lithospheric material that leaves a processing zone bounded by a dimensionless distance to the ridge axis equal to 0.04 (~ 100 km). When material re-enters the ridge processing zone, its age is recorded before being reset to zero.

The resulting sampling times distribution is Gaussian, with an average value $\bar{\tau}_{\text{sampling}}$ and a standard deviation σ_{sampling} :

$$\tau_{\text{sampling}} \cong \frac{1}{\sigma_{\text{sampling}} \sqrt{2\pi}} \exp \left[-\frac{(t - \bar{\tau}_{\text{sampling}})^2}{2\sigma_{\text{sampling}}^2} \right]. \quad (16)$$

In the large-scale, plate-driven regime the sampling time is bounded by the imposed geometry and half-spreading rate: $\bar{\tau}_{\text{sampling,LS}} = 2(1 + H)/Pe$. In the small-scale convection regime the average sampling time was found to be inversely proportional to the convective velocity: $\bar{\tau}_{\text{sampling,SSC}} = c Ra_{\text{eff}}^{-2/3}$, where $c = 1.78 \times 10^2$ was determined by fit to the numerical experiments. The variability of sampling times expressed by σ_{sampling} decreases continuously as small-scale convection progressively vanishes (i.e., with increasing Pe and decreasing H). In the plate-driven regime σ_{sampling} corresponds to the imposed random perturbation of the ridge position (see Equation (6)), whose amplitude $p = 0.02$ is small). Therefore, we assume the following relationship for σ_{sampling} :

$$\sigma_{\text{sampling}} \cong \left[\phi (\bar{\tau}_{\text{sampling,SSC}})^{-1} + (1 - \phi) (p \bar{\tau}_{\text{sampling,LS}})^{-1} \right]^{-1}, \quad (17)$$

where ϕ is the function defined earlier that expresses the relative importance of small-scale convection (Equation(10)). Harmonic averaging was preferred to a weighted arithmetic mean in order to avoid singularities in the vicinity of $Pe = 0$. Similarly, the average dimensionless sampling time between the two end-members is described by:

$$\bar{\tau}_{\text{sampling}} \cong \left[\phi (\bar{\tau}_{\text{sampling,SSC}})^{-1} + (1 - \phi) (\bar{\tau}_{\text{sampling,LS}})^{-1} \right]^{-1}. \quad (18)$$

Using Equations (13) and (14), the average normalized variance at a given

elapsed time t_e is:

$$\overline{\text{var}} = \frac{\int_{t=0}^{t_e} \tau_{\text{sampling}} \exp \left[t_e \lambda \ln \left(\frac{\delta_f}{\delta_0} \right) \right] dt}{\int_{t=0}^{t_e} \tau_{\text{sampling}} dt}. \quad (19)$$

Using Equations (15) and (19) yields the corresponding average standard deviation:

$$\bar{\sigma} = \sqrt{\overline{\text{var}}} \quad (20)$$

We use Eq. (20) to calculate the predicted isotopic variability in the eight mid-oceanic ridge regions. Fig. 3d shows the corresponding predicted standard deviations.

S5 Standard deviation of helium ratios along ridges

We selected eight regions along the main mid-oceanic ridges (Table 1). These regions were chosen in order to avoid complexities such as contamination by plume material, which is why the Icelandic ridge or triple junctions were not considered. Along the selected ridge portions we gathered R/R_{atm} data measured in basalt glasses and peridotite using the *PetDB* tool⁹. To avoid statistical bias, multiple measurements on the same samples were not considered and only the most recent value was retained. We used the data presented in¹⁰ to determine for each region the average half-spreading rate, and the oldest oceanic age within each region, averaged along the edge of oceanic plates moving away from the ridge axis. The ratio of these two parameters yields the effective horizontal distance from the ridge axis, H , over which small-scale convection can develop under oceanic plates. (see Table S1) used in Fig. 3c.

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Figure S1: Numerical experiments results: effect of rheology on mixing efficiency | RMS values of Finite-Time Lyapunov Exponents for a case with constant viscosity (plain curve) and stress-dependent viscosity.

Figure S2: Results of the semi-analytical mixing model for a half-domain aspect ratio $H = 2$ | (a) The function ϕ vs. Pe . ϕ expresses the relative importance of small-scale convection on deformation, and ranges between 0 (pure plate-flow) to 1 (pure small-scale convection flow). (b) Predicted Finite-Time Lyapunov Exponent value as a function of the Péclet number, Pe (curve), and average values measured from the numerical experiments (circles).

Figure S3: Effect of Pe , H and the elapsed time on the chemical variability | Standard deviation σ of chemical heterogeneities as a function of (a) the elapsed time t_e , (b-c) the dimensionless spreading rate Pe for $H = 1$ and various t_e (b), or for $t_e = 200$ Myrs and for various values of H (c).

Table S1: Characteristics of the Mid-Ocean Ridge regions considered and displayed in Fig. 3a. H represents the dimensionless horizontal distance where small-scale convection can develop under the oceanic lithosphere. The Average value for the normalized $^3\text{He}/^4\text{He}$ ratios along each region and the corresponding standard deviation, σ , and number of samples, n , are indicated. See section S5 for further details.

Region acronym	Ridge name	Half-Spreading rate (cm/yr)	H	R/R _{atm}	σ	n
SWIR	Southwest Indian	0.8	0.5	7.0	0.4	85
MAR	Mid Atlantic	1.5	1.0	8.2	0.9	102
SAR	South Atlantic	2.0	0.8	8.1	1.7	75
CIR	Central Indian	2.5	1.3	7.9	0.9	14
PAR	Pacific-Antarctic	3.0	0.9	7.1	0.7	25
CR	Chile ridge	3.0	1.0	7.2	1.4	23
SEIR	Southeast Indian	4.0	1.4	8.1	1.4	84
EPR	East Pacific Rise	7.0	2.8	8.6	0.9	131

Dimensionless half-spreading rate, Pe





