Three-dimensional seismic transmission prospecting of the Mont Dore volcano, France

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Received 1983 April 22; in original form 1983 February 21

Summary. The three-dimensional seismic structure of the Mont Dore volcano is studied by inversion of the arrival times of seismic waves. With this aim two new methods are developed. First, the arrival times are those of Moho-reflected waves at a critical distance from artificial sources in different azimuths. Secondly, the inversion uses a technique which does not require the traditional a priori partition of the space into blocks. The resulting picture reveals such features as: (1) a circular caldera within the basement, the rim of which is marked by magnetic anomalies associated with post-caldera activity; (2) a clear lower limit of the volcano-sedimentary sequence under part of the caldera, opposed to low velocity anomalies extending deeper beneath another part and which may have been the site of volcanic material transport, and (3) deeper heterogeneities possibly related to foundered basement blocks.

1 Introduction

The pioneering work on the determination of the three-dimensional (3-D) structure of the lithosphere from arrival times of seismic waves was achieved by Aki, Christofferson & Husebye (1977) using tele-seismic data and by Aki & Lee (1976) with data from local earthquakes. Numerous studies have since followed this approach successfully.

We have designed a method for the examination of the heterogeneous upper crust based on the use of artificial sources and which gives good resolution. In the inversion technique we also replaced the traditional a priori partition of the space into blocks by an approach in which the unknown is a smooth 3D function.

2 Experimental method and data set

Usual seismic prospecting methods, based on the analysis of reflected or refracted waves, have been developed to investigate the layered, quasi-horizontal sequences typical of sedimentary basins. Volcanoes and high enthalpy geothermal areas do not meet the hypotheses for the application of these methods but have to be viewed as 3-D heterogeneities
Figure 1. The seismic transmission method is defined by the use of waves sent back by a deep interface to illuminate the area under study at angles of incidence suited for 3-D resolution of the structure. In the Mont Dore we used waves generated by 1 ton borehole shots and reflected by the Moho to the critical distance of 80 km.

Figure 2. Hatched square: the 15 km area of Mont Dore within the volcanic domain of Central France. From each of the shotpoints situated in five different azimuths, waves are recorded at 100 different locations.

with complicated geometry and possibly transitional nature. The idea is to ‘undershoot’ such regions and determine their 3-D heterogeneity from the perturbation to the transmitted wavefield. We use a crustal interface at depth to send back through the studied region the wave generated by a shot at distance (Figs 1 and 2).

In Central France the crust–mantle interface (Moho) is a good reflector on a regional scale (Perrier & Ruegg 1973; Sapin & Prodehl 1973). We used $P_M P$ waves generated by shots around critical distance (80 km) in five different azimuths and a temporary receiving array providing altogether $480 P_M P$ arrival time readings on a $20 \times 20 \text{km}^2$ region on the Mont Dore (Hirn & Nercessian 1980). The frequency content of $P_M P$ signals from 1 ton borehole shots reaches at least 15 Hz and the corresponding wavelength allows a resolution of the order of a kilometre. The incidence of the waves, near to 45°, allows a comparable investigation of structure in horizontal and vertical directions. The element of Moho used to send back the waves on the 20 km aperture array is only 10 km, its deviations from a plane mirror are not very likely and would hardly affect arrival times given the wavelength. Using a distant mirror to undershoot the region means that the sampling of shallow heterogeneity is not contaminated by structural complications at greater depths.
Fig. 3, Two examples of the type of profiles used for visual correlation of the $P_{MP}$ (Moho reflected) wave. Stations are on fan-profiles at constant distance from shotpoint. These 2 to 8 Hz bandpass filtered seismograms are used for identification of the $P_{MP}$, the arrival times of which show respectively smooth or locally abrupt variations in the upper and lower example.

Fig. 3 shows samples of data which were arranged as a fan profiles for visual correlation of waveforms in a low frequency, 2–8 Hz band. Precise time-picking with a 0.01 s reading accuracy is done on a zero-crossing of the $P_{MP}$ signal in a 4–16 Hz playback. Preliminary maps of residuals with respect to a reflection hyperbola and topographical correction show significant spatial variations of 0.4 s within some kilometres, the structure of which differs
with azimuth. Thus the database contains information on heterogeneity on a kilometric scale, involving velocity deviations of the order of 20% if these are restricted to the upper 6 or so kilometres (Nercesselian & Hirn 1980).

3 The inverse problem

First of all we have transformed our arrival times into residuals. To do that we have assumed that, in the absence of heterogeneities, the arrival times of the reflected $P_MP$ waves should fit the hyperbola

$$t^2 = aX^2 + bX + c$$

(1)

where $X$ is the epicentral distance. The values of $a$, $b$ and $c$ are adjusted for each shotpoint by standard least-squares.

Let $\Delta t^\text{obs}_i$ be the residuals thus obtained, and let $\Delta_n(r)$ be the departure of the true slowness with respect to the model where equation (1) is assumed to hold. In the absence of experimental errors in $\Delta t^\text{obs}_i$ we should have

$$\Delta t^\text{obs}_i = \int_{\text{th ray}} ds_1 \Delta_n(r)$$

(2)

where if we assume that the slowness perturbation is small enough, the integral can simply be taken along the ray path corresponding to the starting model.

Estimated experimental errors are introduced by the covariance matrix $(C_t)_{ij}$ where, as usual, the terms $(C_t)_{ii}$ are the variances, and the terms $(C_t)_{ij}$ for $i \neq j$ are the estimated covariances. We choose the simple form

$$(C_t)_{ij} = \sigma_i^2 \delta_{ij},$$

(3)

i.e. we assume that the errors are independent and that they are of the order of $\sigma_i$.

The a priori information on the slowness model is introduced by defining the covariance function

$$C_n(r, r') = \sigma_n^2 \exp \left\{ -\frac{1}{2} \frac{||r-r'||^2}{L^2} \right\}. \tag{4}$$

Equation (4) means that we expect the values of $\Delta_n(r)$ to be of the order of $\sigma_n$ at each point $r$, and that we expect the function $\Delta_n(r)$ to be smooth with a smoothness length equal to $L$.

We can now define the least squares inverse solution as the model $\Delta_n(r)$ which minimizes the expression

$$S = \sum_i \sum_j \left( \Delta t^\text{obs}_i - \int \Delta_n(r) \, ds_1 \right) \left( C_t^{-1} \right)_{ij} \left( \Delta t^\text{obs}_j - \int \Delta_n(r) \, ds_1 \right)$$

$$+ \int ds_a \int ds_b \Delta_n(r_a) \, C_n^{-1} \left( r_a, r_b \right) \Delta_n(r_b). \tag{5}$$

As shown by Tarantola & Nercesselian (1984) the solution minimizing (5) is given by

$$\Delta_n(r) = \sum_i \beta_i \int ds_1 C_n(r, r_i)$$

(6)

where

$$\beta_i = \sum_j \left( S^{-1} \right)_{ij} \Delta t^\text{obs}_j$$

(7)
Figure 4. Horizontal cross-sections every kilometre beneath the mean surface level, of the solution of the 3D inversion without blocks of the P-wave velocity anomaly in the upper crust of the Mont Dore volcano, Central France. The scale indicates slowness deviations in s km⁻¹ from the initial layered model. The central red-orange zone with a diameter of the order of 10 km in the two upper maps is interpreted as the extension of a volcano-sedimentary sequence and other low-velocity material from the surface to 2 km depth. The girdle of green and light blue surrounding it can be associated with higher velocity Hercynian basement rocks which form the rim of a caldera and are partly outcropping and partly hidden beneath recent lava flows and pyroclastic deposits. At greater depth the yellow and green, normal to high velocity material cover the eastern part of the maps at 3 and 4 km depth indicating that the bottom of the caldera is reached. Red spots of limited extent persist in the west-central part to these depth; their location under or just to the west of the post-caldera eruptive centres of Banne d'Ordonche in the north and Puy de Sancy in the south suggests an interpretation in terms of fractured weak zones which may have served as feeding connections from depth to surface. At the greatest depth reached, 6-7 km, red, low-velocity patches persist in the north-western part whereas alternating orange and green zones suggest a segmentation of the crust beneath the bottom of the foundered caldera along NW–SE directions.

Figure 5. Vertical cross-sections of the 3D P velocity anomaly. No vertical exaggeration. The three upper sections are in the west–east direction at locations shown by the arrows (a), (b) and (c). The red and orange low-velocity caldera filling is obvious on section (a); it is clearly limited at depth to the east whereas to the west it is connected with a red-orange low-velocity anomaly which is not of a sedimentary nature due to its depth but may be interpreted as a weak cracked zone. Similar features show up on section (b) 3 km to the south. Among the three lower sections, (d), (e) and (f), in a north–south direction, section (e) shows clearly the relation between the green-yellow caldera bottom in the north–central part, the overlying red low-velocity volcano-sedimentary filling and the deeper reaching red anomaly to its south situated just to the west of the main post-caldera volcano of Puy de Sancy. The southern rim of the caldera is sharply defined on sections (d) and its northern rim and bottom on section (f).

Figure 6 (a) (left hand side). Analysis of the spatial resolution of the velocity structure reached by our data set. The a posteriori covariance function \( C_a(r_0, r) \) for a particular point \( r_0 \) located 1 km deep at coordinates 636,63 is shown on four maps each situated 1 km deeper. The dark green dot seen only in the upper map indicates a sphere of the order of 1 km in the vicinity of \( r_0 \) with which the slowness value obtained at \( r_0 \) is highly correlated (> 50 per cent). This value is not correlated at the 5 per cent level with that at points outside the light green areas. (b) (right hand side). Reduction of the uncertainty in the velocity model. The a posteriori standard error \( C_n(r, r) \) at each point is displayed on the map at 1 km depth and on the two central west–east and north–south vertical cross-sections. The scale is in percentage of the a priori error. As expected the looser cross-sampling of the structure at the edges of the region results in a loose determination of the model at these places. Non-uniform sampling due to the lack of receiving points in a region of difficult access is measured by the brown area extending to the centre on the lower cross-section.
and

\[ S \equiv (C_i)_{ij} + \int ds_i \int ds_j C_n(r_i, r_j). \]  

(8)

Uncertainties in the solution (6) are given by the a posteriori covariance function

\[ C_n(r, r') = C_{n_0}(r, r') - \sum_i \sum_j \int ds_i \int ds_j C_{n_0}(r_i, r_j) (S^{-1})_{ij} C_{n_0}(r_j, r'). \]  

(9)

(see Tarantola & Nercissian for more details).

We see in equations (6-9) that the main computational task for calculating \( \Delta n(r) \) is the evaluation of the simple and double integration of the covariance function (4) along ray paths. As we have here simply approximated the ray paths by straight lines, the simple integrals can be performed analytically (see Appendix), while the double integrals are then computed by numerical discretization along the ray paths.

Although errors in individual arrival readings are of the order of 0.01s, the value of \( \sigma_t \) in (3) was chosen equal to 0.1s, for including the exceptional possibility of phase misidentification. The error bar in the a priori model (\( \sigma_n \) in equation 4) was taken equal to 0.1s km\(^{-1}\) and the smoothness length (\( L \) in equation 4) was chosen equal to 1 km. Different choices do not change the main trends of the solution.

The initial travel-time residuals had a standard deviation of 0.11s. Those computed after inversion and corresponding to the slowness model of our solution have a standard deviation of 0.03 s.

4 Numerical results

The solution defined by equation (5) is a function defined everywhere in space. The representation of 3-D structure is given in the form of a colour graphical display of the solution on planes at arbitrarily chosen depths (here every kilometre) which can be interpreted as geological maps at different depths, and on some vertical cross-sections.

Velocity structure is shown in Figs 4 and 5, the estimated a posteriori uncertainties, an example of resolution length (a posteriori covariance) in Fig. 6.

5 Interpretation of results

Depths described here are depths beneath the mean basement surface which is at 1000 m above sea level. Topographical relief (Fig. 7) is due to volcanic material.

5.1 THE CALDERA AND ITS RIMS TO 2 OR 3 KM DEPTH (FIG. 4)

The obvious feature of the 1 km deep map of velocity anomalies is a ring of high velocity material surrounding a strong low velocity patch of about 7 km in diameter. At places west of km 631 and around points 642.63 and 632.60 surface geology shows the outcrops of hercynian basement not covered by volcanics which allows us to attribute these part of the high velocity ring to basement rocks. The central low velocity zone of our seismic section can then be viewed as a basement depression filled by volcano-sedimentary material. In the caldera thus defined two smaller zones of extreme low velocity show up near the towns of La Bourboule and Le Mont Dore; they may correspond to places where the most low-velocity sediments have the largest thickness. Magnetic anomalies reduced to the pole (Bayer & Cuer 1981) exhibit a series of strong peaks of small dimensions on a half circle.
Figure 7. Topography of the study area: contours at 1200 and 1500 m elevation above sea-level.

Figure 8. Magnetic anomalies from aeromagnetic survey, reduced to the pole, adapted from Bayer & Cuer (1981). Contours of 5 nT.

Figure 9. Local Bouger anomaly map (density 1.8 g/cm³ for corrections), adapted from Varet et al. (1980). Scale in mgal.

from north to east and south (Fig. 8). These anomalies fall exactly along the inner limit of the high velocity basement ring which obviously controlled volcanic extrusion and effusion during the latest volcanic cycle dated 2.2 to 1.95 Myr and even 0.83–0.25 Myr in the south-eastern part by Varet et al. (1980). This last eruptive stage which furnishes part of the volcanic pile in the caldera postdates the emptying of a large magma chamber leading to caldera foundering.

The locally low velocities in the north-western corner and at the eastern edge are probably not significant. In the south the local low velocity at Puy de Sancy volcano has better data coverage.

The velocity anomaly map at 2 km depth shows essentially the same features as above, the central low velocity region indicating the persistence of the caldera trough to this depth. The eastern velocity minimum near Le Mont Dore is still expressed. To the north and north-west the low velocity zone has a lesser extent. In contrast near 635.62 the low velocity seems to extend further towards south beneath the higher velocity material on top. This
could be caused by the huge cover of lava flows superposed on caldera sediments by later eruptions. This slight southern displacement of the low velocities associated to the deeper part of the caldera could account partly for the fact that the Bouguer gravity low (Varet et al. 1980) is centred more to the south than the low velocity anomaly at only 1 km depth (Fig. 9).

5.2 Caldera Bottom and North-South Low-Velocity Trends from 2-5 km Depth

A major change of the shape of anomalies occurs from 2 to 3 km depth. The eastern high velocity rim, confined east of km 639 in the upper part reaches now west to km 636 at depths of 4-5 km. As a high velocity underlies here the zone of lowest velocity at the surface, the caldera bottom is reached.

Beneath the western part of the caldera low velocities are still present but their north-south elongated shape no longer follows the circular patch attributed to the filling of the basement trough. Within this north-south trending zone around km 635 extreme low velocity is distinguished at km 69, 63 and possibly 59. Such structures identify perturbations in the deeper basement beneath and on the western side of the founndered block. They may mark remainders of the precaldera magma chamber and, for instance for the northern one, fractured zones having served as feeding connections for the important later volcanic transport. The southern zone where such feeding must have occurred for the most recent activity at Puy de Sancy could possibly correspond to the anomaly at km 59, just to the west of that volcano but its sampling is of lesser quality. On the east-west sections at km 64 and 67, these deep low velocity regions and their relation to the western edge of the surface caldera are well pictured. The high velocity eastern rim turning to bottom the caldera is also obvious.

5.3 Deeper Heterogeneity

At greater depths, down to 7 km a north-south trend of the low velocity anomalies seems to persist in the west but the one described above at km 635 looses its southern part and possibly merges with one further west which also extends to the south-western corner of the study area, where, however, data coverage degrades.

The high velocity medium which was present in the whole eastern half of the region east of km 637, at 3-4 km depth and was interpreted as the basement outside and beneath the eastern caldera filling is broken at larger depth by the intercalation of two low-velocity segments oriented roughly north-west-south-east and possibly coalescing at their south-eastern end. Geographically they are situated beneath the eastern and southern limit of the low velocity patch at surface. The south-western one at 5 km depth, around 637.62 may prolongate to depth and towards east the central low velocity extremum which was part of the north-south trending anomaly present from 3 to 5 km depth. The high-velocity blocks centred on 635.61 and 638.61 beneath 4 km could be viewed as the blocks of founndered basement going over to the deeper undisturbed basement whereas the low velocity segments which are respectively to their south-west, between them and which separate them from the undisturbed region to the north-east would then represent lateral discontinuities along their edges.

Acknowledgments

Yan Bottinga and Georges Jobert encouraged Al. H. to present proposals for such investigations in different heterogeneous regions since 1976. To pour money into the finally quite shallow caldera was risking by the R & D Program Geothermal Energy of the
References


Appendix: evaluation of the integrals along straight ray paths

As we assimilate here the ray paths to straight lines, part of the integrations can be made analytically. We recall that we have used the Gaussian choice for the covariance function

\[ C_{n_0}(r, r') = \sigma^2 \exp \left( -\frac{1}{2} \frac{(r-r')^2}{L^2} \right). \]  
(A1)

For the simple integral along a straight line (Fig. A1) it can easily be found that

\[ \int_{t_1}^{t_2} ds \ C_{n_0}(r_0, r) = \sigma^2 \sqrt{(\pi/2)} L \exp \left( -\frac{1}{2} \frac{D^2}{L^2} \right) \left( \Psi \left( \frac{s_2}{L} \right) - \Psi \left( \frac{s_1}{L} \right) \right) \]  
(A2)

where \( \Psi \) represents the error function:

\[ \Psi(t) = \sqrt{(\pi/2)} \int_0^t dx \exp(-x^2/2). \]  
(A3)

For the double integral along two straight lines (Fig. A1) we obtain

\[ \int_{t_1}^{t_2} ds \int_{t_1}^{t_2} dt \ C_{n_0}(r_s, r_t) = \sigma^2 \sqrt{(\pi/2)} L \exp \left( -\frac{1}{2} \frac{K^2}{L^2} \right) \int_{t_1}^{t_2} dt \exp \left( -\frac{1}{2} \frac{D^2(t)}{L^2} \right) \times \left( \Psi \left( \frac{s_2-s(t)}{L} \right) - \Psi \left( \frac{s_1-s(t)}{L} \right) \right) \]  
(A4)
and the last sum remains for numerical integration. In the particular case where the straight lines are parallel we obtain

$$\int_{s_1}^{s_2} ds \int_{t_1}^{t_2} dt C_n(r_s, r_t) = a^2 L^2 \exp \left( -\frac{1}{2} \frac{K^2}{L^2} \right)$$

$$\times \left\{ \sqrt{\pi/2} \left[ \left( \frac{s_2 - t_1}{L} \right) \Psi \left( \frac{s_2 - t_1}{L} \right) + \left( \frac{s_1 - t_2}{L} \right) \Psi \left( \frac{s_1 - t_2}{L} \right) \right. \right.$$

$$\left. \left. - \left( \frac{s_2 - t_2}{L} \right) \Psi \left( \frac{s_2 - t_2}{L} \right) - \left( \frac{s_1 - t_1}{L} \right) \Psi \left( \frac{s_1 - t_1}{L} \right) \right] \right.$$

$$+ \exp \left( -\frac{1}{2} \frac{(s_2 - t_1)^2}{L^2} \right) + \exp \left( -\frac{1}{2} \frac{(s_1 - t_2)^2}{L^2} \right)$$

$$- \exp \left( -\frac{1}{2} \frac{(s_2 - t_2)^2}{L^2} \right) - \exp \left( -\frac{1}{2} \frac{(s_1 - t_1)^2}{L^2} \right) \right\}.$$