Stress-Drop Variability of Shallow Earthquakes Extracted from a Global Database of Source Time Functions

by Françoise Courboulex, Martin Vallée, Matthieu Causse, and Agnès Chounet

ABSTRACT

We use the new global database of source time functions (STFs) and focal mechanisms proposed by Vallée (2013) using the automatic SCARDEC method (Vallée et al., 2011) to constrain earthquake rupture duration and variability. This database has the advantage of being very consistent since all the events with moment magnitudes $M_w > 5.8$ that have occurred during the last 20 years were reanalyzed with the same method and the same station configuration. We analyze 1754 shallow earthquakes (depth < 35 km) and use high-quality criteria for the STFs, which result in the selection of 660 events. Among these, 313 occurred on the subduction interface (SUB events) and 347 outside (NOT-SUB events). We obtain that for a given magnitude, STF duration is log normally distributed and that STFs are longer for SUB than NOT-SUB events. We then estimate the stress drop using a proxy for the rupture process duration obtained from the measurement of the maximum amplitude of the STF. The resulting stress drop is independent of magnitude and is about 2.5 times smaller for the subduction events compared with the other events. Assuming a constant rupture velocity and source model, the resulting standard deviation of the stress drop is 1.13 for the total dataset (natural log), and about 1 for separate datasets. These values are significantly lower than the ones generally obtained from corner-frequency analyses with global databases (∼1:5 for Allmann and Shearer, 2009) and are closer to the values inferred from strong-motion measurements (∼0.5 as reported by Cotton et al., 2013). This indicates that the epistemic variability is reduced by the use of STF properties, which allows us to better approach the natural variability of the source process, related to stress-drop variability and/or variation in the rupture velocity.

INTRODUCTION

Numerous parameters are required to estimate in advance the ground motion caused by an earthquake. The first-order parameters are the magnitude $M$ of the earthquake and its distance to the observation point. The second-order parameters are linked both to attenuation and sometimes amplification at the regional and local scale (anelastic attenuation and site effect) and to the source process itself. The source parameter generally recognized as the most important for the control of high frequencies is the stress-drop $\Delta \sigma$ (Hanks and McGuire, 1981). This parameter is, in fact, directly or indirectly an input of most of the ground-motion simulation methods (see Douglas and Aochi, 2008). Determination of stress drop is thus a major concern for the prediction of high-frequency ground motions (e.g., peak ground acceleration [PGA] and peak ground velocity levels). It is first important to mention that the term stress drop is not used unequivocally. As pointed out by Atkinson and Beresnev (1997), it can reflect various concepts that are not always associated with its true physical meaning, which is simply the difference between the stress level before and after an earthquake. We introduce hereafter the commonly used definitions of stress drop.

The original definition of stress drop is referred to as static stress drop and was introduced as a measure of the static deformation induced by an earthquake. As such, it is directly related to the strain drop, that is the ratio of seismic slip over the dimension of the rupture (Kanamori and Boschi, 1983; Vallée, 2013). The stress drop averaged over the fault plane can be simply expressed by

$$\Delta \sigma \approx \mu \frac{\bar{D}}{L},$$

in which $\mu$, $\bar{D}$, and $L$ are the earth rigidity, the average slip on the fault, and a characteristic rupture dimension. For a constant seismic moment, the stress drop is thus higher when the rupture surface is small and the average displacement is high. In a bidimensional source model, the stress drop is equal to:

$$\Delta \sigma = cM_0/L^3,$$

in which $M_0$ is the seismic moment and $c$ is a factor depending on the rupture type (Kanamori and Rivera, 2004). The rupture dimension is thus a key parameter to determine $\Delta \sigma$, but its value is inaccessible to direct observation. For large earthquakes ($M \geq 7$), the rupture dimension is often retrieved by the inversion of several datasets: teleseismic and/or local seismograms.
and/or geodetic measurements (see the database of finite-source rupture models compiled by Mai and Thingbaijam, 2014). The rupture dimension can also be deduced from the distribution of early aftershocks. For superficial events that break the surface, direct rupture length measurements can also be used (e.g., Wells and Coppersmith, 1994; Manighetti et al., 2007; Shaw, 2013). The smaller (and more numerous) earthquakes, however, are not systematically studied with such detailed analyses. It is the case only for some specific earthquakes in well-instrumented areas that had a strong impact on populated regions, for example, the recent 2015 Napa Valley earthquake or the 2009 L’Aquila event (e.g., Tinti et al., 2014; Dreger et al., 2015), among many others. Thus, even if static stress drop is directly related to the stress release on the fault, it has limited practical utility due to the difficulty in measuring slip and fault dimensions.

An alternative way to assess stress drop is to use seismological parameters that are easier to measure. For instance, the duration of the source time function (STF), representative of the total duration of the source process \( T \), can be inferred from distant seismograms. Introducing the rupture velocity \( V_r \), equation (2) becomes:

\[
\Delta \sigma = c M_0 / (V_r T)^3.
\]  

(3)

Thus, for a given seismic moment, a similar value of the stress drop can be obtained for a short STF duration and a high rupture velocity, or for a long STF duration and a low rupture velocity. It is well known from source studies that \( V_r \) values usually vary in the range (0.6–0.9 \( V_S \)) (e.g., Heaton, 1990) and sometimes exceed, for a portion of the rupture, the shear-wave velocity (e.g., Bouchon et al., 2001; Dunham and Archuleta, 2004; Walker and Shearer, 2009; Vallée and Dunham, 2012). Nevertheless, because \( V_r \) is difficult to obtain and is apparently not linked with \( M_0 \), most of the studies simply assume that \( V_r \) is constant and then put their efforts into determining \( T \), which is much more variable and correlated with \( M_0 \).

Practically, most of the studies using seismograms to determine stress drop are based on the corner-frequency \( f_c \), determination. \( f_c \) is generally defined on the Fourier spectrum in displacement (Thatcher and Hanks, 1973; Allmann and Shearer, 2009) as the intersection between a flat low-frequency level and an \( f^{-2} \) slope that describes the fall off of the high frequencies in the \( \omega^{-2} \) model (Brune, 1971). The relationship often used to link \( f_c \) and the stress drop assumes a circular crack model:

\[
\Delta \sigma = \frac{7}{16} M_0 f_c^2 / (k V_r)^3,
\]  

(4)

in which \( k \) depends on the assumptions of the rupture model and on the type of wave. For instance, Brune (1971) used \( k = 0.37 \) for \( S \) waves, whereas \( k = 0.21 \) for Madariaga (1976) and \( k = 0.26 \) for Kaneko and Shearer (2014). Tests of \( k \) values for different source models can be found in Dong and Papageorgiou (2003). Note that stress drop is proportional to the cube of \( f_c \) in equation (4) and the cube of \( T \) in equation (3). Its determination is then highly sensitive to these values. Determining the absolute value of an earthquake stress drop is then both sensitive to the selected parameters and to the measurements made on the data. In this article, we do not focus on the absolute values of stress drop, but rather on its relative values and variability.

A recent article by Cotton et al. (2013) examines the links between the variability of the seismic stress drop, hereafter called \( \sigma_{\text{in}}(\Delta \sigma) \), determined on seismologic data, and the variability of the PGA, \( \sigma_{\text{pga}}(\text{PGA}) \), reported in ground-motion prediction equations (GMPEs) for between-event variability (Al-Atik et al., 2010). Based on the theory of random vibrations (Hanks and McGuire, 1981), and assuming a constant rupture velocity, the relationship should be: \( \sigma_{\text{in}}(\Delta \sigma) = 1.25 \sigma_{\text{pga}}(\text{PGA}) \). Cotton et al. (2013) then compare the stress-drop variability obtained by different authors on global seismological databases and the variability obtained from GMPEs. They note that the variability obtained from seismological data is much larger than that deduced from GMPEs. They attribute this difference to a possible overestimation of \( \sigma_{\text{in}}(\Delta \sigma) \) due to the difficulty in measuring the \( f_c \) value.

We then propose to remeasure this variability from a database of STFs recently made available that analyzes all the earthquakes with \( M_w > 5.8 \) of the last 20 years. Unlike global databases that typically used \( f_c \) to calculate the stress-drop variations (Allmann and Shearer, 2009, being the most recent and complete), the SCARDEC database directly produces STFs that avoids the need of a corner-frequency estimation. In this article, we analyze the stress-drop variations directly estimated from these STFs and make subsets of earthquakes to examine their \( \sigma \) values.

**STF DURATION DETERMINATION FROM THE SCARDEC DATABASE**

A recently developed method, called SCARDEC (Vallée et al., 2011), provides simultaneous access to the focal mechanism, seismic moment, depth, and STFs of most earthquakes with moment magnitude \( M_w > 5.8 \). As SCARDEC is fully automated, the STFs can be obtained for an unprecedented number of earthquakes (2892 events analyzed from 1992 to 2014). The STF determination is obtained by deconvolution of teleseismic waveforms by a Green’s function computed in the global IASP91 model (Kennett and Engdahl, 1991). The STF obtained must be causal and positive, and its integral (which corresponds to the seismic moment) must be constant at each station.

Because our aim is to reduce the epistemic uncertainty and to have better access to the natural variability of the source process, we work on a restricted database. We first exclude the strike-slip events whose STF determination is generally more complex and often poorly constrained by \( P \)-wave analysis, and also all the events which do not provide a fully consistent STF determination (based on the measurement of the teleseismic interstation STF coherence). This can occur because of focal...
mechanism complexities, large rupture depth extent, and strong directivity effects (e.g., Ben-Menahem, 1962; Ammon et al., 2006; Vallée, 2007; Courboulex et al., 2013). Finally, we restrict our analysis to shallow events (depth < 35 km), whose influence on ground motions is stronger than deeper ones. In the database of 1754 shallow events, 660 are then selected based on the previous criteria. Among them, 313 occurred on the subduction interface and 347 outside (Fig. 1a and 1b).

The total duration \( T \) of the STF obtained is not simple to determine. Indeed, the practical determination of \( T \) may suffer from subjective criteria to determine when the STFs actually begin and end. Moreover, the relation between the total duration and the source process characteristics (stress drop in particular) is biased when the STF displays two or more slip patches separated in time. Another approach is to measure the peak value of the STF (maximum moment rate) \( F_m \), and to compute a characteristic duration from \( F_m \) and \( M_0 \) (using, for example, a triangular shape for the STF). In this case, the presence of a late complexity of the STF only has a minor effect on the characteristic duration. This method has been preferred by Vallée (2013) for a more robust and consistent determination of the duration.

In our study, we test the two approaches. We first measure a STF-duration-based \( T \) as the duration of the STF between the first amplitude above 0.1\( F_m \) with increasing trend, and the last amplitude above 0.1\( F_m \) with decreasing trend. We then measure an \( F_m \)-based \( T \) as the width of the isosceles triangular-shaped STF with same maximum \( F_m \) and area \( M_0 \): \( T = 2M_0/F_m \). We find overall similar results for both approaches, but the variability of \( T \) (standard deviation of \( \ln[T] \)), hereafter referred as \( \sigma_{\ln[T]}(T) \) is always reduced when using \( F_m \) instead of the STF duration. The mean duration obtained being almost the same, we chose to determine \( T \) from \( F_m \) for the following analysis. We found that \( \sigma_{\ln[T]}(T) \) has values between 0.3 and 0.4 (natural log) without any clear dependence on the magnitude.

### STF DURATION IN DIFFERENT CONTEXTS

We separate the SCARDEC dataset into two main subsets: subduction events, that is, thrust events occurring on the subduction interface (SUB), and all the other event types (NOT-SUB). It is clear from Figure 2a that the STF duration \( T \) is longer for subduction events than for the others (an illustration of this behavior is shown in Fig. 1c and 1d for two earthquakes belonging to each of the contexts). This has been pointed out by Chounet and Vallée (2014) with the SCARDEC database and already observed in other global databases (Kanamori and Anderson, 1975; Bilek and Lay, 1999; Houston, 2001; Allmann and Shearer, 2009). Active and well-developed subduction plate boundaries can lead to smoother ruptures; also, the hydration of the contact can weaken the frictional properties. Those two features may induce lower rupture velocity and/or lower stress-drop earthquakes. Simple regressions can be obtained for both regions (Fig. 2a).

Our next goal is to estimate the duration distribution for earthquakes of a given magnitude \( M_w \). To have a sufficiently large amount of data, we compute this distribution using earthquakes with moment magnitude of \( M_w \pm 0.3 \). Variations of \( T \) due to variation of magnitude inside this range are scaled to be comparable. The STF duration \( T(s) \) roughly follows a lognormal distribution (see Fig. 2b, bottom, an example for \( M_w 6.4 \)) with \( \sigma_{\ln[T]}(T) = 0.37 \) for the whole dataset, 0.32 for subduction events and 0.34 for the others.

### IMPLICATION FOR STRESS DROP

If, like many authors, we use equation (4) for a circular crack to determine stress drop, we have to determine \( k \) and \( V_S \) and we...
also have to convert the values of $T$ obtained from $F_m$ measurements versus magnitude ($M_w$) for earthquakes that occur on the subduction interface (SUB) and away from it (NOT-SUB). Linear regressions are represented for both subsets. (b) Histogram of $T(s)$ values for events with $M_w = 6.4 \pm 0.3$ (a correction is applied to account for the differences of magnitude). Lines correspond to the lognormal function that best fits the total distribution (bold line), the NOT-SUB subset (gray dotted line) and the SUB subset (black dotted line).

To quantify the influence of the input parameters of equation (4) on the mean stress drop, we test $V_S$ values from 3300 to 3900 m/s, $k$ values for Madariaga (1976) and Kaneko to $k = 0.32$ for $P$ waves and $S$ Madariaga to $k = 0.21$ for $S$ waves (Madariaga, 1976). $P$ Kaneko corresponds to $k = 0.26$ (Kaneko and Shearer, 2014) for $P$ and $S$ waves. The color version of this figure is available only in the electronic edition.

**Figure 3.** Variation of the mean stress-drop values obtained from the SCARDEC database using two relationships between the total duration $T$ and $f_c$, three values of $V_S$ from 3300 to 3900 m/s and four values of $k$: $P$ Madariaga corresponds to $k = 0.32$ for $P$ waves and $S$ Madariaga to $k = 0.21$ for $S$ waves (Madariaga, 1976). $P$ Kaneko corresponds to $k = 0.38$ and $S$ Kaneko to $k = 0.26$ (Kaneko and Shearer, 2014) for $P$ and $S$ waves. The color version of this figure is available only in the electronic edition.

**DISCUSSION AND CONCLUSION**

Stress-drop variability is the subject of many studies that aim to better understand and constrain both source processes on faults and ground-motion predictions. Many different datasets have been used by several authors to try to constrain this value. Using surface-slip observations, Manighetti et al. (2007) found $\sigma_{\text{rms}}(\Delta\sigma) = 0.9$ from 250 continental earthquakes and Shaw
(2013) found 0.7 from 37 strike-slip events. Similar values were obtained by Mai and Beroza (2000) ($\sigma_{\text{mag}}(\Delta\sigma) = 0.8$), using rupture surface and average slip derived from 31 slip inversion models of 18 earthquakes ($5.5 < M < 8$), and by Causse et al. (2014) ($\sigma_{\text{mag}}(\Delta\sigma) = 0.7$) from finite-rupture models of 21 crustal events.

Using local or regional seismological data, Cotton et al. (2013) reported values from 0.57 (earthquakes in Greece, Margaris and Hatzidimitriou, 2002) to 1.83 (earthquakes in Switzerland, Edwards and Fäh, 2013), whereas Baltay et al. (2013) found a value of 0.9 for earthquakes in Japan. Nevertheless, our study is based on a global database, including hundreds of events with $M > 5.8$ recorded from 1992 to 2014. In this respect, the largest and most recent worldwide database has been built by Allmann and Shearer (2009). As reported by Cotton et al. (2013), Allmann and Shearer (2009) found $\sigma_{\text{mag}}(\Delta\sigma) = 1.67$ for interplate events and $\sigma_{\text{mag}}(\Delta\sigma) = 1.46$ for intraplate ones. For the same range of magnitudes, we obtained much lower values (around 1), which indicate that the use of the SCARDEC database significantly reduces epistemic variability. One of the reasons is likely related to the use of $F_m$, which is expected to be more meaningful than $f_c$-derived measurements, in particular when the source is complex: even in the case of an STF with several peaks, $F_m$ can always be nonequivocally determined (while the concept of a single corner frequency becomes unclear), and the values derived from this peak moment rate will at least approximate the behavior of the dominant patch of the rupture.

To check the effects of the initial data selection procedure (removal of strike-slip earthquakes and events without a fully consistent determination of the STF), we also computed the sigma value considering all of the 1750 events with depths shallower than 35 km. We found a variability of the STF duration $\sigma_{\text{mag}}(\Delta T)$ equal to 0.38. The resulting $\sigma_{\text{mag}}(\Delta\sigma)$ equal to 1.14 is then only marginally larger than for the database of selected events (see the ALL dataset represented by a bold line

Figure 4. (a) Stress-drop values (MPa) obtained from the SCARDEC catalog of STF for shallow earthquakes (depth < 35 km) using equation (4) for $k = 0.32$ (value for $P$ waves in the Madariaga, 1976, model), $V_S = 3900$ m/s and $f_c = 0.6/T$. Mean values are computed for bins of $M_w \pm 0.1$. (b) Distribution of $\log_{10}(\Delta\sigma)$. The color version of this figure is available only in the electronic edition.

Figure 5. (a) Mean stress-drop values obtained for the whole database, the selected database, SUB and NOT-SUB datasets. Stress-drop values are mean values computed using equation (4), with $V_S = 3900$ m/s, $k = 0.32$ and $f_c = 0.6/T$. (b) $\sigma_{\text{mag}}(\Delta\sigma)$ with magnitude for all events (for $\pm 0.1$ bins), the selected events, and the SUB and NOT-SUB subsets. The values obtained by Cotton et al. (2013) for source studies on global databases and derived from ground-motion prediction equations (GMPEs) are indicated. The color version of this figure is available only in the electronic edition.

(2013) found 0.7 from 37 strike-slip events. Similar values were obtained by Mai and Beroza (2000) ($\sigma_{\text{mag}}(\Delta\sigma) = 0.8$), using rupture surface and average slip derived from 31 slip inversion models of 18 earthquakes ($5.5 < M < 8$), and by Causse et al.
in Fig. 5b). We can then suppose that a large part of the epistemic variability has been removed and that we are closer to the natural variability of the source process.

Nevertheless, the question raised by Cotton et al. (2013) remains open. Why is the variability obtained (around 1) still two times larger than the one derived from PGA between-event variability observed in ground-motion databases (e.g., Al-Atik et al., 2010), which is generally around 0.5 (Cotton et al., 2013). This result is surprising and counterintuitive. One would expect that the details of the high-frequency rupture process and the natural heterogeneity of the distribution of stress on the fault (e.g., Noda et al., 2013) affect more the observed PGA than the full duration of the STF. The source duration, which is a global source property, should intuitively vary less than the corresponding PGA.

The large values of the stress-drop variability obtained from global seismological databases can be partially explained by the fact that the variability of the STF duration (or $f_c$) is always converted into stress drop using a single source model and fixed input parameters. It is probable that if the ad hoc parameters were specifically chosen for each earthquake, the stress-drop variability would be lower. It is also clear that the rupture velocity plays a major role in the variability of the corner frequency (Kaneko and Shearer, 2015) and of the STF duration (e.g., Kanamori and Rivera, 2004), and that a possible correlation or anticorrelation (as proposed by Causse and Song, 2015) between stress drop and rupture velocity would modify the stress-drop variability. The resolution of this problem must wait for more reliable determination of $V_r$ and $\Delta \tau$.

Finally, we cannot exclude that the low values of the stress-drop variability deduced from the PGA variability (Cotton et al., 2013) arise from an underestimation of the observed between-event variability of PGA. In addition, the simple relationship used to relate PGA and static stress drop may be more complex in real cases.

**DATA AND RESOURCES**

Source time functions (STFs) can be obtained from SCAR-DEC database at http://scardec.projects.sismo.ipgp.fr (last accessed April 2016).

**ACKNOWLEDGMENTS**

We thank Annemarie Baltay, Adrian Ort, and Zhigang Peng for their constructive remarks that helped us to improve the article. This study has benefited from the comments of Raul Madariaga. It has been partially funded by Institut National de Sciences de l’Univers (CNRS) through CT3 project.

**REFERENCES**


