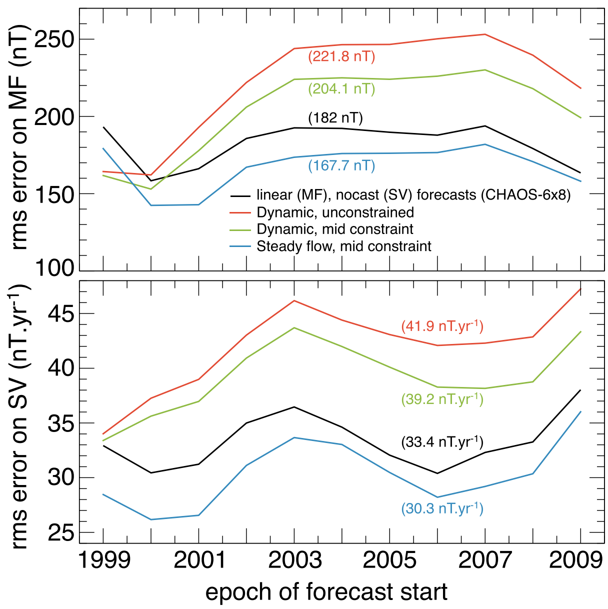

Figure 11. Same as Fig. 10, for forecasts at 10-yr range, with numbers between parentheses indicating the level of error averaged over the epoch range 1999–2009.

Introducing a steady flow forecast that is representative of the results that would be obtained if the QG-MAC balance was perfectly enforced throughout the core, we notice that the resulting predictions now slightly improve over the quality of the mathematical extrapolations. This shows that the estimation of the magnetic acceleration parts captured by a steady flow approach [i.e. those resulting from the interaction of $\mathbf{u}$ with $\partial_t \mathbf{B}$ and the diffusive acceleration $\nabla^2 \partial_t \mathbf{B}$ in eq. (47)] is adding relevant information into the forecast. A closer inspection of the contributions to error from individual Gauss coefficients reveals that much of the improvement over mathematical extrapolations is carried by the large-scale coefficients represented in Figs 8 and 9, where the steady flow forecasts frequently capture the correct future trend, though with a lower acceleration than that required by the data. The missing part underlines the importance of the geomagnetic acceleration resulting from the interaction of $\partial_t \mathbf{u}$ with $\mathbf{B}$ in eq. (47), which can only be correctly rendered if levels of residual inertia are reduced enough relative to the MAC forces in order to enable the correct magnetohydrodynamic wave dynamics at interannual timescales (Aubert 2018; Aubert & Finlay 2019). This goal is outside the scope of the dynamics rendered by the coupled earth model because of its position on the parameter space path.

4 DISCUSSION

4.1 Force balance in Earth's core

In this study, an inverse geodynamo framework has been extended in order to quantitatively assess the compatibility of recent geomagnetic main field and secular variation data with the prior hypothesis of a QG-MAC force balance in Earth's core. This hypothesis is to some extent already embedded into the statistics used to constrain the inversion, since these are obtained from a numerical dynamo model that already respects this balance. We have seen indeed (Fig. 3) that the sole specification of statistics already leads to states that respect the linear part of the QG-MAC balance, that is the thermal wind equilibrium between the pressure gradient, buoyancy and Coriolis forces. However, the Bayesian inference that is used in the framework is not sufficient to account for the nonlinear effects of the Lorentz force. Enforcing the full QG-MAC balance through a direct implementation in the hydrodynamic inverse problem, with nonlinear Lorentz terms being explicitly treated (eq. 26), we have seen that it is possible to obtain core states that adequately fit the geomagnetic data (Figs 1 and 2, Table 2) while producing residuals of the QG-MAC balance smaller than each term in this balance (Fig. 3). As these residuals induce a spurious inertial response, decreasing their amplitude is also systematically beneficial when performing forecasts of the future geomagnetic evolution by using the inverted states as startup points (Figs 8–11). Up to spherical harmonic degree $\ell_{\mathrm{asm}} = 30$, and at the core surface, the Lorentz force is found to be subdominant relative to the pressure, Coriolis and buoyancy forces (Fig. 3), thereby confirming the absence of system-scale magnetostrophy (equilibrium between the magnetic and pressure force). All these elements complement the bulk of recent direct modelling results (Aubert *et al.* 2017; Schwaiger *et al.* 2019) and provide further data-driven support to the proposal of a QG-MAC force balance controlling the geodynamo.

4.2 General circulation in Earth's core and its main driving source

The core flow solutions constrained with the QG-MAC balance confirm the general circulation features found in previous studies (Pais & Jault 2008; Gillet *et al.* 2013, 2015, 2019; Aubert 2013, 2014, 2015; Pais *et al.* 2015; Barrois *et al.* 2017; Livermore *et al.* 2017; Bärenzung *et al.* 2018), and again highlight a planetary-scale, eccentric columnar gyre in Earth's core, the surface signature of which consists of a strong equatorial westward drift in the Atlantic hemisphere and an intense, roughly equator-symmetric jet at high latitudes in the Pacific hemisphere (Fig. 4). By directly imposing dynamical consistency into the inverse problem, the present results confirm that this circulation is driven by buoyancy, rather than magnetic forces (Fig. 3). At the core surface, the longitudinal hemispheric flow circulation is therefore in thermal wind balance with a similarly hemispherical density anomaly distribution (Fig. 6). Similarly to Gillet *et al.* (2019), our results also incite to revise the amount of acceleration seen in the high-latitude Pacific jet during the past two decades. During this period indeed (Fig. 5), we find significant acceleration only in the Northern Hemisphere, with an average value of 0.4 km yr^{-2} that is more than four times smaller than that reported by Livermore *et al.* (2017). This is probably the consequence of our inversions accounting for truncation errors and magnetic diffusion effects. While the strong accelerations found by Livermore *et al.* (2017) were supportive of a magnetic driving of the flow, we note that with a typical jet velocity of 30 km yr^{-1}, the acceleration timescale of the gyre is at least $30/0.4 = 75$ yr, a value compatible with convective advection of density anomalies.

The geophysical origin of the striking hemispherical buoyancy pattern (Fig. 6) is not yet entirely clear. As in our earlier work (Aubert 2013; 2014; Aubert *et al.* 2013), the present results confirm that the hypothesis of generating this pattern through faster inner core freezing in the Eastern Hemisphere (0°E–180°E) is reasonably compatible with the geomagnetic data. The geodynamic justification for differential inner core freezing initially came from the idea that the inner core surface should be considered as an open surface permeable to transfers of matter (Monnereau *et al.* 2010), and that the solid inner core could then be subject to translational convective instabilities. This however requires the background density pattern of the inner core to be convectively unstable, a possibility which has been questioned owing to the ongoing debate on the value of thermal conductivity in the inner and outer core (e.g. Pozzo *et al.* 2012; Konôpková *et al.* 2016). In the event that the inner core is thermally stratified, it has been recently underlined that it can still be chemically unstably stratified because of a time-dependent light element partitioning between the solid and the liquid as the inner core grows (Gubbins *et al.* 2013). In this case, the net stratification can be unstable and the resulting translational hemispherical anomalies in the inner core freezing rate may be of the same order as the homogeneous freezing rate (Deguen *et al.* 2018). This is precisely the situation explored in the coupled Earth model, and the present inverse geodynamo modelling results then provide a consistent buoyancy-driven explanation of the eccentric gyre and high-latitude jet. As we have seen in the cases when we strongly enforce the QG-MAC dynamic constraint in our inversions, this explanation however still leaves room for improvement. In this situation indeed, the coupled earth model typically favours strong upwellings in the Eastern Hemisphere, which are not entirely consistent with the recent secular variation data (Fig. 2). Furthermore, the possibility still remains that the hemispherical circulation spontaneously arises in a homogeneously forced system, as shown by the recent simulations of Schaeffer *et al.* (2017) at extreme conditions. The tool presented here opens interesting prospects towards further investigation of this issue, that could for instance be explored by changing the prior model underlying the inversions in order to assess the compatibility of each scenario with the available data. It is also important to promote the acquisition of additional archeomagnetic information on the historical geomagnetic field in order to assess the persistence of the eccentric gyre over several centuries.

4.3 Towards predictions of geomagnetic jerks

We have seen that enforcing the QG-MAC balance is an essential prerequisite to accurately render the geomagnetic field evolution at subdecadal to decadal timescales. Once the inertial residuals of this balance have been sufficiently decreased indeed, the information brought by the numerical dynamo prior leads to relevant partial estimates of the geomagnetic acceleration and to forecasts that can slightly improve over the mathematical linear extrapolations (Figs 8–11). While the present approach therefore represents a step towards better geomagnetic forecasts, it remains limited as we have only achieved a moderate reduction of the inertial residuals compared to the unconstrained situation (Fig. 3). In the present scheme, the QG-MAC balance is also enforced only at the core surface. Another problem is that the estimation of the internal magnetic field and the associated Lorentz force remains of statistical nature, and its compatibility with the other force components (buoyancy, Coriolis) is not guaranteed, leading to an increase of the misfits in the constrained situation (Table 2). Co-estimation of the magnetic field, flow and buoyancy anomaly, possibly in an iterative scheme, represents a viable avenue for further developing the inverse geodynamo modelling framework.

With inertial forces at least five orders of magnitude below the MAC forces (Aubert *et al.* 2017; Aubert 2019) in Earth's core, achieving a realistic level of the QG-MAC balance and relevant geomagnetic forecasts including Alfvén-wave driven dynamics is a very delicate task. It is however essential to reach this level of accuracy if one wishes to be able to predict geomagnetic jerks, which may result from tiny magneto-inertial deviations to the QG-MAC balance that propagate as quasi-geostrophic Alfvén waves and undergo an amplification effect as they reach the core surface (Aubert & Finlay 2019). In that sense, future progress towards this fundamental goal of geomagnetism should come from parallel improvements in the three pillars of geomagnetic prediction: additional satellite geomagnetic data providing insight into a larger number of rapid geomagnetic acceleration events, better direct numerical models that operate at the correct level of inertial forces, and refinements in data assimilation frameworks able to infer core states with a finer level of dynamic balance.

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