Interface instabilities during displacements of two miscible fluids in a vertical pipe

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We study experimentally the downward vertical displacement of one miscible fluid by another in a vertical pipe at sufficiently high velocities for diffusive effects to be negligible. For certain viscosity ratios and flow rates, the interface between the two fluids can destabilize. We determine the dimensionless flow rate U_c above which the instability is triggered and its dependence on the viscous ratio M, resulting in a stability map $U_c = U_c(M)$. Two different instability modes have been observed: an asymmetric ''corkscrew'' mode and an axisymmetric one. We remark that the latter is always eventually disturbed by ''corkscrew'' type instabilities. We speculate that these instabilities are driven by the viscosity stratification and are analogous to those already observed in core annular flows of immiscible fluids. © *2001 American Institute of Physics.* [DOI: 10.1063/1.1343907]

I. INTRODUCTION

The hydrodynamic stability of two-phase flow in pipes and channels has been the subject of many experimental and theoretical investigations in the past. Yih[1] considered plane Couette–Poiseuille flow of two superposed layers of fluids of different viscosities between two horizontal plates. He showed that the flow is unstable for all Reynolds numbers, no matter how small, and that the primary cause of instability was the viscosity stratification between the fluids. The mechanism was further elucidated by Hinch,[2] who identified it as being related to the vorticity jump at the interface. Hickox[3] presented a similar linear stability analysis of steady concentric flow of two fluids in a vertical circular tube. He considered both asymmetric and axisymmetric disturbances to the primary flow using long wave expansions. No situations were encountered for which the primary flow was stable to both types of disturbances simultaneously, but the parameter range he studied was relatively limited.

In a series of companion papers,[4–8] Joseph and collaborators studied both theoretically and experimentally the stability of core-annular flow for a variety of different situations (free-fall, forced flow, with or without gravity effects) but focused on the case of lubricated pipelining, with the less viscous fluid near the wall. It was shown that core-annular flows are stable for a very small set of conditions. Chen, Bai, and Joseph[6] and Bai, Chen, and Joseph[8] performed experiments on the stability of core-annular flow of immiscible fluids (water/oil) in vertical pipes. The experiments were conducted in an inverted U-shaped pipeline, so that both upflow and downflow conditions could be studied simultaneously. They observed that the flow could be stable or unstable, contradicting Hickox's long-wave prediction, but in agreement with their prediction of a window of stability in the parameter space. The unstable flows exhibited two different patterns: a large amplitude axisymmetric wave they called ''bamboo wave'' and an asymmetric ''corkscrew'' wave.

A few papers have addressed the study of miscible fluids displacements in pipes. Petitjeans and Maxworthy[9] have performed experiments on the displacement of a viscous fluid (glycerin of viscosity η_2 and density ρ_2) by a less viscous one (a glycerin–water mixture of viscosity η_1 and density ρ_1) in a vertical tube. The authors observed the formation of a finger of injected fluid and measured both the speed of the tip of the finger V_t and the average velocity, V_m at the bottom exit of the pipe. From this measurement, they computed the average thickness t of the film of viscous fluid left behind on the tube wall. Variations of t were investigated as a function of the Péclet number $\mathrm{Pe} = 4V_mR/D$ (where D is the diffusion coefficient between the two fluids and R the pipe radius), the Atwood number $\mathrm{At} = (\eta_2 - \eta_1)/(\eta_2 + \eta_1)$, and the gravity parameter $F = 2g(\rho_2 - \rho_1)R^2/\eta_2 V_m$. For large values of Pe, $O(10^5)$, t was found to attain an asymptotic value depending only on At and tended to the asymptotic value of 0.37 when $\mathrm{At} \approx 1$, the limit of infinite viscosity ratio.

Lajeunesse *et al.*[10] studied experimentally and theoretically the axisymmetric downward vertical displacement of one miscible fluid by another lighter one in both Hele–Shaw and tube geometries. They focused on the large Peclet number regime for which a well-defined interface separates the two fluids. The shape of this interface was studied as a function of the viscosity ratio, $M = \eta_2/\eta_1$, and the normalized flow rate, $U = 8\eta_2 V_m / R^2(\rho_2 - \rho_1)g$. Three different domains were delineated in the (M,U) plane. In the first domain, the shape of the interface is a self-propagating tongue. Domains 2 and 3 are characterized by the presence of a ''shock'' along the tongue profile for which all the points on the interface travel with the same speed. In the case of Hele–Shaw flows, theoretical and experimental results were found to be in good agreement for the first two domains, but not for the third, where the thickness of the shock was observed to saturate at a value depending on the viscous ratio. Similar results were obtained for cylindrical tube flow. In experiments done in the FAST Lab, Orsay, France, one of us (E.L.), in collaboration with J. Martin and D. Salin, first

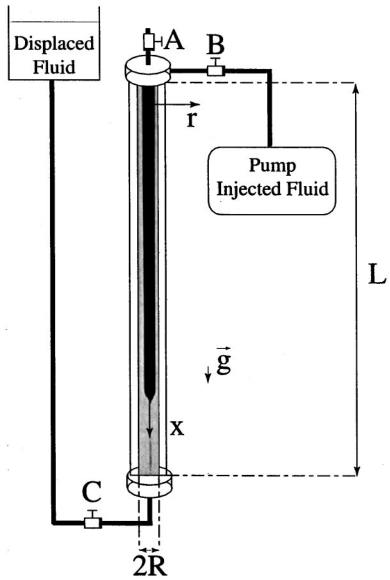

FIG. 1. Schematic of experimental apparatus.

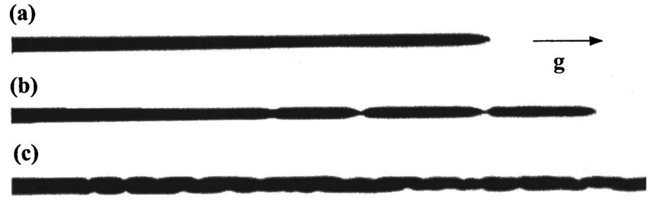

FIG. 2. Different types of flow observed: (a) Stable finger. (b) Axisymmetric mode. (c) Corkscrew mode.

discovered that such miscible flows in tubes can become unstable at sufficiently high flow rates.[11] The present work is motivated by this discovery: our main objective is to measure the stability boundary and characterize the instability modes.

II. EXPERIMENTAL APPARATUS AND PROCEDURE

The experimental setup consisted of a cylindrical tube of length $L=1$ m and radius $R=3$ mm (Fig. 1). The injected fluid was water of viscosity η_1 and density ρ_1. In order to visualize the interface, we dyed the water with Chicago Sky blue 6B until it is saturated, so that it has the same properties for all experiments: we further assume that the dye does not change the viscosity of water. The displaced fluid, of viscosity η_2 and density ρ_2, was a mixture of water and glycerin, the viscosity of which varied in the range 4–384 mPa s. Consequently, two decades in the viscosity ratio were covered in these experiments, while the density contrast $(\rho_2-\rho_1)/(\rho_2+\rho_1)$ varied from 5×10^{-4} to 0.11.

To obtain an initially flat horizontal interface, the pipe was first partially filled to within a few centimeters of the top with the displaced fluid injected from the bottom through valve C (Fig. 1) while air is removed through valve A. The pipe was then totally filled by injecting the second fluid through valve B. We made sure that this fluid flows down the walls of the pipe so that it meets the first fluid very smoothly. This procedure took a few minutes to complete so that, due to diffusion, the initial interface extended somewhat (about 0.1 mm) along the direction of the pipe axis. Also note that a stabilizing density difference was necessary to achieve the initial condition of a flat interface. Subsequently, fluid 1 was injected at a constant flow rate through valve B by means of two pumps (FMI-LAB model RP-D), connected in parallel. The injection flow rate, ranging from 0.01 to 0.25 L/min, was high enough that the two fluids do not have time to mix under the effect of molecular diffusion (Pe$\approx10^5-10^6$). In this high Péclet number regime, a well defined interface separates the two fluids.

The experimental tube was enclosed in a square box filled with glycerin, which matches the Plexiglas optical index within 1% and thus reduces lens distortion from the round walls of the pipe. Experiments were recorded using a 3 CCD XL-1 Canon camcorder and digitized at a rate of 20 frames per second through an interface video card (Scion LG3) linked to a computer (Power Macintosh G3).

III. RESULTS

When the pump is started, an advancing finger of injected fluid is created, but the more viscous fluid is not totally displaced from the tube: a thin layer of this more viscous fluid is left on the wall of the pipe and surrounds a finger of injected fluid as it travels. Over a range of low rates, this flow is stable and steady, with an apparent sharp interface between the fluids [Fig. 2(a)].

For sufficiently high flow rates the interface destabilizes to either an axisymmetric or nonaxisymmetric shape. The axisymmetric mode consists of sausage-shaped droplets [Fig. 2(b)] that travel with the same speed as the finger tip. The nonaxisymmetric mode, which we call the ''corkscrew mode,'' is shown in Fig. 2(c). Depending on the viscosity ratio, it is either the primary mode or it evolves from the axisymmetric mode. We observed that a slight misalignment in verticality of a straight pipe can trigger the corkscrew-type instability at low flow rates: extreme care was taken to ensure that the tube was vertical.

For large values of the viscous ratios *M*, the ''usual'' evolution of an unstable flow was from the stable interface to the axisymmetric mode, eventually disturbed by a corkscrew instability. At relatively low viscous ratios *M*, the corkscrew mode sometimes formed so rapidly that it was impossible to verify if it resulted from a previous axisymmetric mode. It is therefore not clear if the corkscrew mode should be considered as a secondary instability of the axisymmetric pattern or as a possible unstable mode of the primary flow.

We constructed a map of the different flow regimes observed. Data were acquired on the thickness of the displaced fluid left on the wall and the wave length of the axisymmetric mode. These will be discussed in a later paper. Here we focus on the critical speed for onset of instability. Each experiment was characterized using the control parameters introduced by Lajeunesse *et al.*,[10] that is the viscosity ratio *M* and the normalized flow rate *U*, defined in Sec. I above. Figure 3 shows the type of flow observed as a function of *M*

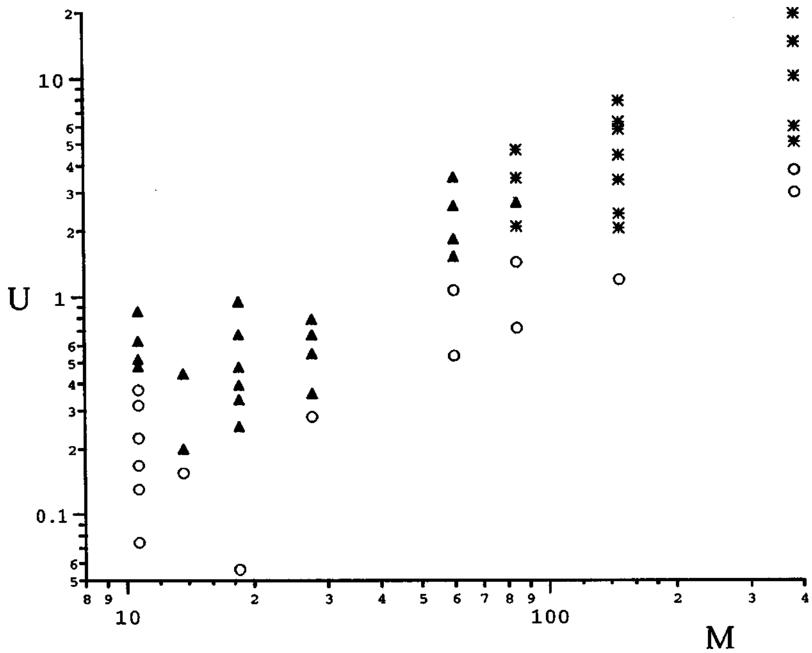

FIG. 3. Different types of flow observed as a function of M and U. See text for definition of the symbols.

and U. Circles correspond to stable flows, stars refer to flows that first destabilize in an axisymmetric pattern, eventually leading later to an asymmetric pattern, and triangles correspond to flows that first destabilize in a corkscrew pattern. Keeping in mind that the distinction we make between corkscrew and axisymmetric instability is subjective, Fig. 3 indicates that corkscrew modes seem to dominate at relatively low viscous ratios $M(M<84.4)$, whereas axisymmetric modes dominate at large $M(M>84.4)$.

Figure 3 also shows that the $(U-M)$ plane can be divided into two areas: a stable one and an unstable including both axisymmetric and asymmetric instability modes. We therefore determined M-dependent thresholds in U above which the interface destabilizes. Figure 4 plots this critical dimensionless speed U_c as a function of the viscosity ratio with error bars extending from the highest value of U reached for stable flows to the lowest value reached for unstable flows. For $M>18$, U_c increases with M. On the contrary, for $M<18$, U_c decreases with M. This is not surprising as we expect U_c to diverge for $M=1$ where both fluids have the same viscosity. Because of the limited capacity of our pumps, we were not able to reach the unstable area below $M=10.7$.

IV. SUMMARY AND CONCLUSIONS

We report the observation of instabilities during the downward vertical displacement of one miscible fluid by another lighter and less viscous one in a vertical pipe. The high Péclet numbers regime was considered so that diffusive ef-

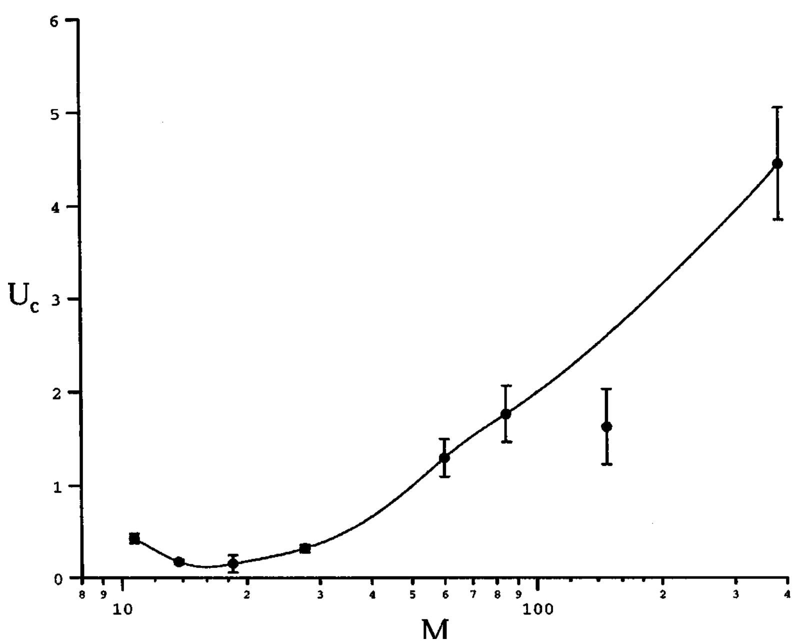

FIG. 4. Dimensionless critical speed vs the viscosity ratio. The points correspond to the experimental measurements: the smooth curve is to guide the eye and has no theoretical significance.

fects were negligible and an apparently sharp interface could be defined between the two fluids. For sufficiently high flow rates, the finger interface destabilizes into either an axisymmetric pattern or an asymmetric corkscrew mode. After this work was complete, we became aware of some similar observations by Balasubramaniam *et al.*[12] In a very preliminary study, they observe the corkscrew mode for displacement of the less viscous fluid by a more viscous one. Formation of a corkscrew mode was also observed by Gabard *et al.*[13] in the case of miscible displacement of viscoelastic fluids in a vertical pipe. Further work must be done to establish the connection between these recent observations and the study reported here.

The curve of critical speed vs mobility ratio shows a minimum around $M=18$, with stabilization for larger M and a suggestion of stabilization at lower M as well. The latter could not be confirmed due to limitations of the pumps. The apparent increase of U_c with M should not be misinterpreted: it must be remembered that all other parameters, including the density difference and the thickness ratio, t, simultaneously vary as M is varied.

These two instability patterns present striking similarities with the ones observed by Chen *et al.*[6] and Bai *et al.*[8] in core annular flows of immiscible fluids. We speculate that the instabilities observed here are driven by the viscosity stratification between the fluids, similar to what occurs in the more well-studied case of immiscible fluids. The justifications for our speculation are that (i) our experiments were done at very high Pe, for which the region between the fluids is sharp and can be considered sharp, and (ii) miscible fluids under similar conditions have been observed to mimic immiscible fluids in other respects.[9] This speculation, together with a more extensive set of data on the instability wavelength and the depth ratio, t, and a discussion of the predictions of linear theory, will be pursued in a longer paper.

ACKNOWLEDGMENTS

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