RESEARCH ARTICLE

Near-wall velocity measurements by particle-shadow tracking

Pierre Lancien · Éric Lajeunesse · François Métivier

Abstract We report a new method to measure the velocity of a fluid in the vicinity of a wall. The method, that we call particle-shadow tracking (PST), simply consists in seeding the fluid with a small number of fine tracer particles of density close to that of the fluid. The position of each particle and of its shadow on the wall are then tracked simultaneously, allowing one to accurately determine the distance separating tracers from the wall and therefore to extract the velocity field. We present an application of the method to the determination of the velocity profile inside a laminar density current flowing along an inclined plane.

1 Introduction

Measuring the velocity of a fluid in the vicinity of a wall is relevant to a great number of fundamental and applied investigations such as understanding the structure and dynamics of boundary layers (Townsend 1976; Alfredsson et al. 1988; Buschmann and Gad-El-Hak 2003). In practice, such measurements are difficult as they require to achieve a high accuracy on both the velocity and the distance to the wall, the proximity of which makes unfortunately difficult to reach. Optical methods such as particle image velocimetry (PIV) or laser anemometry usually fail to meet these two criteria when operating too close to a

wall and are therefore inappropriate for such measurements (Adrian 1991; Somandepalli and Mungal 2004).

In this paper, we present a new and simple experimental technique developed to perform local measurement of the velocity field of a fluid near a wall. This method, that we call particle-shadow tracking (PST), consists in seeding the fluid with a very low number of fine tracer particles of density close to that of the fluid. Simultaneous tracking of each particle and its shadow on the wall allows us to accurately determine the distance separating tracers from the wall and therefore to extract the velocity field.

The paper is organized as follows. The technique is presented and discussed in Sect. 2. Section 3 describes an application of the method to the measurement of the velocity profile of a laminar density current flowing along an incline plane. We then make some brief suggestions about how the technique may be further developed and conclude.

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2 The PST method

For the sake of simplicity, we will illustrate the PST method in the case of a fluid flowing above a inclined wall as schemed on Fig. 1. The x and y axes are oriented, respectively, along and transverse to the flow direction. The z axis is normal to the wall. The fluid of density ρ and viscosity η is seeded with a small number of fine tracer particles of diameter D. Ideally the best is to use neutrally buoyant particles, that is of density $\rho_\mathrm{p} = \rho$. In practice, a perfect density matching is difficult to achieve. But, as discussed hereafter, the use of tracer particles of density slightly larger than that of the fluid can be a plus for the method.

A punctual light source enlightens the wall under a small incidence angle α. A camera oriented perpendicularly

Fig. 1 Schematic of the PST method
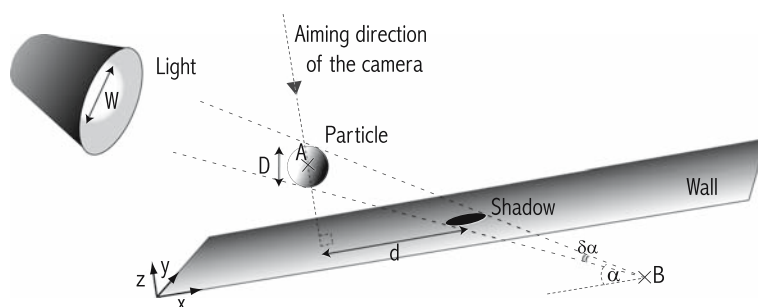
to the plane is used to acquire digitized images of the wall at regular time intervals. The images are then processed to extract the lateral coordinates (x_p, y_p) of each particle and (x_s, y_s) of the shadow it projects on the wall (see Fig. 1). The third coordinate z_p of the particle is then deduced from the distance d separating the particle from its shadow:

$$z_p = \tan\alpha \; d \;\approx \alpha d \tag{1}$$

Measurement of $x_p(t)$, $y_p(t)$ and $z_p(t)$ at regular time intervals therefore allows us to determine the velocity field along the particle trajectory:

$$u(x_p, y_p, z_p) = \frac{x_p(t + \delta t) - x_p(t)}{\delta t} \tag{2}$$

where δt is the time step separating the acquisition of two successive frames. The complete velocity profile $u(x,y,z,t)$ is extracted by repeating these measurements on a large number of particles whose trajectories cover the whole area of interest.

In terms of accuracy, it follows from Eqs. 1 and 2 that:

$$\Delta z_p = \alpha R \tag{3}$$

$$\Delta u = 2\frac{R}{\delta t} \tag{4}$$

where R is the spatial resolution of the image (i.e. the pixel size).

As for all velocimetry method based on seed particles, an important issue is to ensure that particles move at the same velocity than the fluid. In the limit of small particle Reynolds number, the difference between fluid and particle velocity is given by (Adrian 1991):

$$|v - u| = \frac{\rho_p D^2 |\dot{v}|}{36\eta} \tag{5}$$

where v is the velocity of the particle, u that of the fluid and $\dot{v}$ is the typical particle acceleration. If the flow is steady and quasi-parallel, it follows from the above equation that v is very close to u. For a non steady flow, the effect of slip

needs to be evaluated from the above equation in order to estimate the accuracy of the velocity measurement.

In practice, particle-shadow tracking forces velocity measurements to remain concentrated below a maximum distance to the wall for two reasons. First of all, both the particle and its shadow have to be present simultaneously inside the zone imaged by the camera. It follows from Eq. 1 that the largest wall-particle distance that can be detected is of the order of:

$$z_1 \approx S\alpha \tag{6}$$

where S is the typical length of the zone imaged by the camera.

The second limiting factor is illustrated on Fig. 1. In practice, a light source is not punctual. As a result, the shadow cone generated by a particle A extends up to a point B, beyond which only penumbra remains. If B is located above the wall, the shadow poorly contrasts on the wall so that it cannot be detected. Distance AB can be estimated by:

$$\|AB\| \approx \frac{D}{\delta\alpha} \tag{7}$$

where D is the particle diameter. $\delta\alpha \approx W/L$ is the source angular diameter, where W is the light source diameter and L is the distance separating the light source from the measurement point. From Fig. 1 it follows that the shadow is not detectable anymore as soon as the distance of the particle to the wall is larger than:

$$z_2 \approx D\frac{\alpha}{\delta\alpha} = \frac{D\alpha L}{W} \tag{8}$$

Although receding the light source reduces $\delta\alpha$ and increases z_2, the counterpart is a loss of luminosity degrading the signal to noise ratio during treatment. Another alternative is to use a collimated light source instead of a punctual one. In that latter case, z_2 depends on the parallelism uncertainties $\delta\alpha'$ of the collimated light: $z_2 \approx D\alpha/\delta\alpha'$ (Adrian 1991).

A key point of the method is to determine the shadow associated to a given particle. This can become difficult

when the number of particles visible on a frame is too large. A simple way to address this problem is to use tracer particles of density slightly larger than that of the fluid so that they slowly settle. When a settling particle reaches the wall, it coincides with its shadow allowing us to identify it without any ambiguity. The particle and its shadow are then tracked by playing the movie backwards. This, however, introduces a third maximum distance to the wall as the particle must remain in the field of the camera until it reaches the wall. The time necessary for the particle to settle from a height h is:

$$t = \frac{h}{V_s} \qquad (9)$$

where the settling velocity $V_s = (\rho_p - \rho)g \cdot D^2/18\eta$ is estimated from the Stokes velocity (Lamb 1945). The horizontal distance covered by the tracers during this time is of order:

$$\phi = \frac{hU}{V_s} \qquad (10)$$

where U is the typical flow velocity. The particle must remain in the field of the camera until its settling, which imposes $\varphi < S$ and determines a third boundary for the profile height of the order of:

$$z_3 \approx \frac{SV_s}{U} \qquad (11)$$

From the above discussion, it follows that the maximum wall-distance z_{max} which can be explored by PST is given by the minimum between z_1, z_2 and possibly z_3. For $z > z_{max}$, PST fails and the velocity profile has to be measured using classical techniques such as PIV.

3 Application

We first developed the PST method in order to measure the velocity profile of a dense current running out on a slope (Lancien et al. 2005a, b). We were particularly interested in the profile in the immediate vicinity of the wall as our goal was to measure the shear stress applied on the latter by the current. We will therefore illustrate PST for this particular application. The corresponding experimental setup is sketched on Fig. 2. It consists of a 100 cm $\times$ 50 cm incline, immersed in a 200 cm $\times$ 50 cm $\times$ 50 cm flume filled with fresh water. The flow is generated by injecting a brine of density ρ larger than that of fresh water at a constant flow rate Q from the top of the ramp. The resulting gravity current is laminar, steady and varies slowly along the x axis (Lancien et al. 2005a, b).

Our camera definition is 700×570 pixels, the width of the imaged zone is $S = 7$ cm and the spatial resolution of the camera is $R = 100$ μm/pixel. The acquisition rate is 25 frames s^{-1} corresponding to $\delta t = 40$ ms. The experimental plane is enlightened with a projector under an incidence angle $\alpha = 20°$. The angular diameter of the lamp is $\delta\alpha \approx 0.4°$. The tracers are fine plastic particles (Rilsan) with a characteristic size of 30 μm. Their density $\rho_p = 1,080$ kg m^{-3} is slightly larger than that of the brine. As a result, they slowly settle so that one particle allows us to explore a wide range of z. Note that, as our flow is steady and varies slowly along the x axis, tracking only one particle is enough to establish the velocity profile $u(z)$.

Equations 6, 8 and 11 lead to: $z_1 \approx 10$ mm, $z_2 \approx 1.5$ mm and $z_3 \approx 5$ mm. In this experimental configuration, the maximum wall-distance z_{max} which can be explored by PST is set by the size of the light source which forbids to measure velocity above $z_{max} = z_2 \approx 1.5$ mm. Figure 3 shows typical velocity measurements obtained by applying PST to five different particles located in the same area. All data collapse on the same velocity profile $u(z)$. The flow that we consider in this section is laminar and stationary. By performing several repeated measurements and performing slide averages of the data, we were able to establish the velocity profile with an accuracy of 0.3 mm s^{-1} and a vertical resolution of 30 μm (that is inferior to the pixel size) up to a maximum distance $z_{max} \approx 1.5$ mm.

Above $z \approx 1.5$ mm, the velocity profile was measured with a more classical particle tracking technique using side-views of the flow acquired by a camera placed on the side of the tank. The lower and upper part of the resulting velocity profile are perfectly consistent and overlap in the vicinity of $z \approx 1.5$ mm, as visible on Fig. 4. To our knowledge, this is the first measurement of a complete velocity profile obtained for laminar density current experiments.
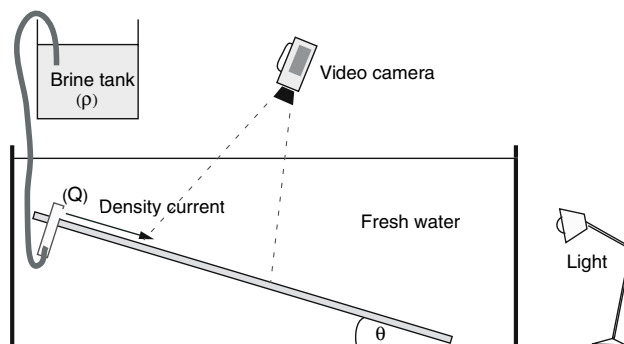

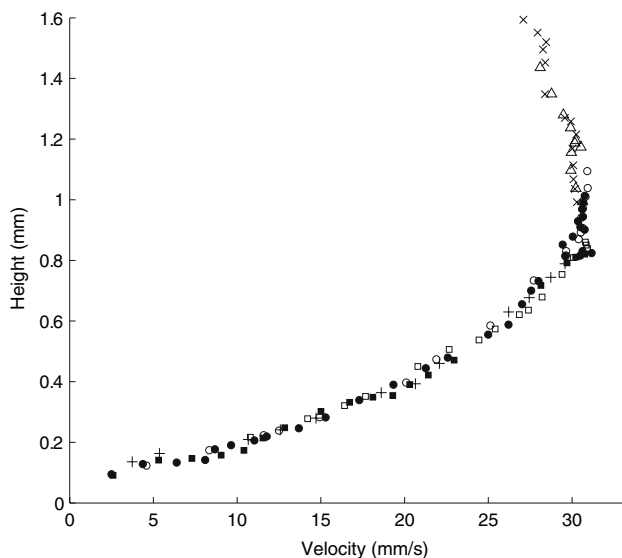Fig. 2 Setup for the density current experiment

Fig. 3 Near-wall velocity profile of the density current, measured 0.3 m downslope of the entrance with PST. Each *symbol* correspond to a different particle. $\rho = 1{,}025$ kg m^{-3}, $Q = 0.95$ ml s^{-1}, $\theta = 16°$
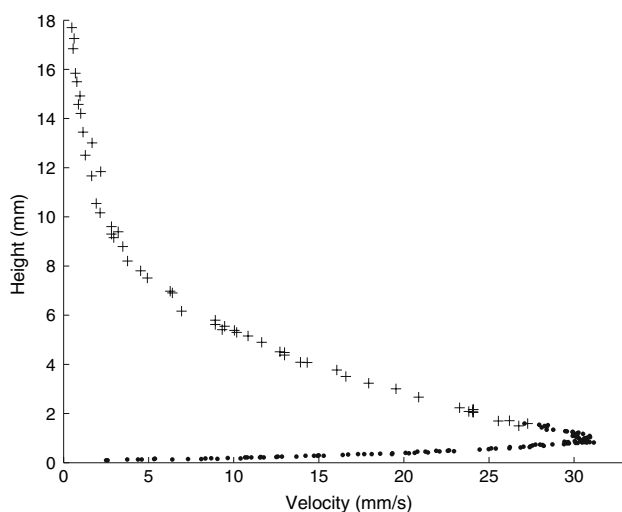

Fig. 4 Complete velocity profile. PST data from Fig. 3 (*dots*) are completed using classical side-view particle tracking (*crosses*)

4 Conclusion

We have described a new method to measure near-wall velocity profiles. This so-called particle-shadow tracking method (PST) consists in seeding the fluid with a very low number of fine tracer particles. Tracking simultaneously both the particles and their shadow permits to measure the velocity profile in the direction transverse to the wall. As illustrated through an example, this new method is cheap,

simple and accurate. Measurements are, however, restricted in a layer at the wall of thickness which mainly depends on light conditions, flow velocity and particles size and density. Determination of the velocity profile out of this layer have to be performed using more classical techniques such as PIV. PST should be therefore considered as a complementary method particularly adapted to the investigation of boundary layers.

Using two light sources with two different incidence angles, a small one and a large one, might improve PST method by extending the thickness of the measurement layer. In fact, each particle would therefore project two different shadows on the wall. When possible, the particle-wall distance would be calculated from the most distant shadow. When the latter is not in the field of view, particle-wall distance would be calculated from the nearest shadow. It would thus be possible to increase the thickness of the measurement layer without degrading the vertical resolution.

Our motivation to develop the PST method was indeed to measure the velocity profile inside a laminar density current. Therefore, we did not apply the method to a turbulent flow. In principles, PST (that is simultaneous tracking of the position of a particle and its shadow) should work in a turbulent flow. Of course, in practice, applying the PST technique to a turbulent flow would be more difficult: a higher frame rate is needed and the size and density of the particles need to be adapted to prevent slip effects.

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