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Physically-based model of downstream fining in bedrock streams with side input and verification with field data

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ABSTRACT: Bedload particles in bedrock streams receiving side input from hillslopes may or may not show a clear pattern of size reduction in the downstream direction. Both abrasion and selective sorting may play important roles in mediating downstream fining. The objective of this study is to develop a physically-based model of downstream fining in bedrock streams based on both processes. A surfaced-based gravel transport relation for size mixture due to Parker (1990) is employed here to account for the effect of selective sorting (differential transport). While the model produces silt and sand by abrasion here it is also assumed to loosely capture particle splitting (as well as crushing, cracking, and grinding) via a lumped abrasion coefficient embodied in Sternberg's law. The model has been verified against field data from Vieux-Habitants River in Guadeloupe Island, which is located in the Caribbean Sea. The river shows clear downstream fining, and this pattern is captured reasonably well by the model. The verification of the model against the field data as well as field observations, however, suggest that the uses of a full flood hydrograph and an adapted sediment transport formula that captures the effect of drag forces associated with large, immobile boulders are essential for an improved model. In addition, rock splitting may play a vital role in downstrem fining (especially for weaker rock types). The order of magnitude of size reduction obtained from the model results suggest that abrasion (including splitting) and selective sorting can be equally important for downstream fining in bedrock streams only in certain cases, such as streams with relatively smaller grain sizes and lower slopes. In some cases, the results show that selective sorting (differential transport) in one extreme flooding period may not play a dominant role in downstream fining (such as in Vieux-Habitants River). However, selective sorting due to a full flood hydrograph (or selective entrainment), such that smaller sizes predominate in the load during the lower flows but the largest sizes are only moved near the peak flow, appears to characterize Vieux-Habitants River. The results also suggest that future work may need to explicitly consider splitting mechanisms rather than lumping them in a Sternberg-type coefficient.

1 INTRODUCTION

In the past few decades, studies of changes in size distributions of bedload in gravel streams have focused on downstream fining (e.g. Knighton, 1984; Kukal, 1990; Mangelsdorf et al., 1990; Parker, 1991; Kodama, 1994a,b). Downstream decreases of median grain sizes of bedload are attributed to selective sorting, abrasion or both. Sklar et al. (2006) argued that in bedrock streams where no net deposition occurs, abrasion is responsible for downstream fining, based on the work by Kodama (1994a), while in cases of net deposition along the channel, selective sorting may often play a dominant role in downstream fining (e.g. Paola et al., 1992; Gomez et al., 2001).

Sklar et al. (2006) were probably the first to study downstream fining in bedrock streams with side input. For the case of spatially uniform supply of poorly-sorted hillslope materials, they found that abrasion of grain particles during fluvial transport in bedrock streams has a small effect on the bedload size distribution , because local re-supply offsets the size reduction from upstream. They then claimed that downstream fining must be due mainly to spatial gradients in hillslope sediment production and transport features. Their conclusions were based on numerical modeling. Here we argue otherwise. Our field observations in Vieux-Habitants River and Capesterre River, Guadeloupe Island, indicate that the material input from the hillslopes is nearly spatially invariant from upstream to downstream. This notwithstanding, both streams display downstream fining. Thus particle abrasion and/or splitting must prevail. The rock type is andesite in these field sites; rock splitting and crushing have yet been ubiquitously observed along both streams. Sklar et al. (2006) also ignored selective sorting processes in downstream fining in bedrock streams. The role of selective sorting is unclear, but the present analysis indicates that it can play a role when combined with abrasion/splitting.

The present study thus focuses on developing a physically-based model of downstream fining with side input by incorporating both abrasion and selective sorting and testing the predictions with field data. A secondary purpose is to test whether and where selective sorting plays a role in downstream fining in bedrock streams. This paper starts with the model framework. The field site in Guadeloupe Island is introduced. The model application to the field data is then outlined, and the results are evaluated and discussed.

2 MODEL FRAMEWORK

2.1 *Assumptions*

Key assumptions in the present study are as follows: 1) bedrock incision processes occur so slowly that the pattern of downstream fining can be treated as quasi-steady state, 2) the size distribution of sediment input from the hillslopes is spatially uniform along the channel, 3) all particle sizes (especially big boulders) are moved during floods, 4) size reduction due to splitting as well as silt-producing abrasion is treated implicitly within a single abrasion coefficient, 5) the denudation rate is assumed to be spatially constant, and 6) only a single channel following Hack's law is considered. The model can be extended to a distributed drainage network at a later time.

2.2 *Theoretical development*

In this section we develop the theoretical framework used in modeling downstream fining in bedrock streams with side input from hillslopes.

2.2.1 *Abrasion coefficient and Sternberg's law*

The size reduction of grains in the downstream direction has long been quantified in terms of Sternberg's law;

$$D = D_o e^{-\alpha x} \tag{1}$$

or its equivalent form

$$\frac{dV_p}{dx} = -\beta V_p \tag{2}$$

where $\beta = 3\alpha$. According to Eq. (1), the upstream grain size D_o abrades down to size D at distance x from the origin, at a rate given by α (1/L). Particles are presumed to abrade by shedding silt. In Eq. (2), V_p is grain volume $=(4/3)\pi(D/2)^3$ and β is an abrasion coefficient characterizing the fraction volume of a grain that is lost per unit distance traveled. By using the logarithmic grain size $D = 2^\Psi$, one finds that

$$\frac{d\psi}{dx} = -\frac{1}{3\,\ell n(2)}\beta \tag{3}$$

Before the work of Kodama (1994b), several previous researchers had found the abrasion coefficient to be around the order of 10^{-4}–10^{-3} km^{-1} for chert, quartzite, granite, and limestone (e.g. Krumbein, 1941; Kuenen, 1956; Bradley, 1970). This led to the conclusion that in many streams selective sorting may dominate abrasion in downstream fining. Kodama (1994b), however, argued that the force of impact of gravel particles in previous studies was much smaller than that in natural rivers during floods, and that tumbling mills in previous studies only shed silt, with the number of clasts in the mill remaining constant in time. In contrast, in his rotating drum the force of impact was more violent. Thus, his definition of abrasion includes all mechanism of breakdown i.e. splitting, crushing, superficial cracking, grinding, etc. Kodama found that the abrasion coefficient in his drum was around the order of 10^{-2}–10^{-1} km^{-1} for chert, quartzite, and andesite. Comparing with field data from Watarase River, Japan in which the calculated value of abrasion coefficient was found to have the same order of magnitude as the drum's values, he then concluded that abrasion (by his definition) should at least in part be responsible for downstream fining in this river.

2.2.2 *Probability density function*

When dealing with grain size distributions, it is important to use the concept of probability density function. Again, $\psi = \ell n_2$ (D) $= \ell n(D)/\ell n(2) =$ a logarithmic grain size (psi scale). The volume probability density that a bedload particle is size ψ is given by $p(\psi)$. The volume probability density that sediment in the surface layer is size ψ is given by $F(\psi)$. Both should satisfy the constraints

$$\int_1^\infty p(\psi)d\psi = 1; \quad \int_1^\infty F(\psi)d\psi = 1 \tag{4a, b}$$

In addition to volume probability density, the areal probability density that sediment in the bed surface layer is size ψ must also be used in the model; it is denoted as $F_a(\psi)$. According to Parker (1991), it should also satisfy the condition

$$F_a(\psi) = \frac{F(\psi)2^{-(1/2)\psi}}{\int^{\infty} F(\psi)2^{-(1/2)\psi}d\psi} \tag{5}$$

Likewise, $p_L(\psi) =$ volume probability density of sediment in landslide material derived from adjacent hillslopes. It satisfies the condition

$$\int^{\infty} p_L(\psi)d\psi = 1 \tag{6}$$

Given that $q_{bd}(\psi) =$ density of volume bedload transport rate per unit width of grain size ψ, the total volume bedload transport rate per unit width summed over all grain sizes, q_{bT}, is given as

$$q_{bT} = \int_{l}^{\infty} q_{bd}d\psi \tag{7}$$

2.2.3 *Abrasion terms and volume transfer rate*

Abrasion does two things: a) it produces silt as the gravel particles collide, and b) it causes the gravel particles to get finer as they do so. The volume loss density per unit bed area per unit time due to abrasion of bedload particles, $A_{bedload}$, is then given with the aid of a continuity condition as

$$A_{bedload} = \beta v_b \xi_{bd} = \beta q_{bT} p \tag{8}$$

where $v_b(\psi) =$ velocity of bedload particle of size ψ and $\xi_{bd}(\psi) =$ volume density per unit bed area of bedload particles in motion of size ψ (Parker, 1991). When bedload particles strike bedrock they do not reduce the amount of alluvium in the stream bed, but when they strike bed sediment they do reduce this amount. Assuming that as a bedload particle strikes the bed the same fraction of sediment is removed from the bed particle as the bed, the volume loss density per unit bed area per unit time due to abrasion of bed particles, A_{bed}, is given as

$$A_{bed} = \beta q_{bT} P_c F_a \tag{9}$$

where $P_c =$ fraction of bedrock surface that is covered by alluvium. In (9) the term P_c accounts for the fact that not all of the bed is covered with alluvium, and the term F_a accounts that it is areal density not volume density, that governs the exposure of bed particles to abrasion. The total abrasion rate A_{tot} is then given as

$$A_{tot} = \beta q_{bT}(p + P_c F_a) \tag{10}$$

As particles abrade they become finer, i.e. they are fluxed through ψ space from large ψ to fine ψ. The "velocity" of flux is given as $d\psi/dt$, where from (3)

$$\frac{d\psi}{dt} = \frac{d\psi}{dx}\frac{dx}{dt} = -\frac{1}{3\ell n(2)}\beta v_b \tag{11}$$

The volume transfer rate per unit time per unit bed area, or flux, through ψ space of bedload sediment $T_{bedload}$ is thus given as (volume bedload sediment per unit area * "velocity")

$$T_{bedload} = \xi_d \frac{d\psi}{dt} = -\frac{1}{3\ell n(2)}\beta \xi_{bd} v_b = -q_{bT}\frac{1}{3\ell n(2)}\beta p \tag{12}$$

The minus sign ensures that the transfer by abrasion is from coarser to finer sizes. If a bedload particle strikes bedrock, it abrades bedrock, which does not change the amount of alluvium in the bed, because the products of abrasion are assumed to be transported as wash load. If a bedload particle strikes a bed particle, however, it reduces the amount of alluvium in the bed. The volume transfer rate per unit time per unit bed area of bed particles through ψ space T_{bed} is then given as

$$T_{bed} = -q_T \frac{1}{3\ell n(2)}\beta P_c F_a \tag{13}$$

The total transfer rate per unit bed area per unit time T_{tot} through ψ space at size ψ is thus given as

$$T_{tot} = T_{bedload} + T_{bed} = -q_T \frac{1}{3\ell n(2)}\beta(p + P_c F_a) \tag{14}$$

2.2.4 *Sediment mass conservation*

Consider a reach of river extending from x to $x + \Delta x$, and a grain size range extending from ψ to $\psi + \Delta\psi$. A 1D conservation of bed sediment for gravel size mixtures moving as bedload can be formulated using the active layer concept (e.g. Hirano, 1972) in a way analogous to Parker (1991), but using densities instead of fractions. The reduced form of the sediment conservation relation is found to be

$$\begin{aligned} &B(1-\lambda_p)\left[\frac{\partial}{\partial t}(L_a F) + f_I \frac{\partial}{\partial t}(\eta - L_a)\right]\Delta x \Delta\psi = \\ &\left[(Bq_{bT}p)\big|_x - (Bq_{bT}p)\big|_{x+\Delta x}\right]\Delta\psi + I_{LT}p_L \Delta x \Delta\psi \\ &-BA_{tot}\Delta x \Delta\psi + (T_{tot}\big|_{\psi} - T_{tot}\big|_{\psi+\Delta\psi})B\Delta x \end{aligned} \tag{15}$$

where B is channel width, which can vary with downstream distance, λ_p is bed porosity, L_a is thickness of the active layer (=effective thickness of alluvium averaged over bedrock surface), $f_I(\psi)$ is volume probability density of size ψ of sediment exchanged at the interface between the surface and any substrate as the bed aggrades or degrades, t is time, η is bed elevation and I_{LT} is volume input of sediment of all sizes per unit distance downstream per unit time input by landslides. The above form can further be reduced to

$$\begin{aligned} &B(1-\lambda_p)\left[\frac{\partial}{\partial t}(L_a F) + f_I \frac{\partial}{\partial t}(\eta - L_a)\right] = \\ &\quad -\frac{\partial Bq_{bT}p}{\partial x} + I_{LT}p_L - BA_{tot} - B\frac{\partial T_{tot}}{\partial \psi} \end{aligned} \tag{16}$$

Substituting (10) and (14) into (16), one finds

$$B(1-\lambda_p)\left[\frac{\partial}{\partial t}(L_aF)+f_I\frac{\partial}{\partial t}(\eta-L_a)\right]=-\frac{\partial Bq_{bT}p}{\partial x}+I_{LT}p_L \quad (17)$$
$$-B\beta q_{bT}(p+P_cF_a)+\frac{B\beta q_{bT}}{3\ell n(2)}\frac{\partial}{\partial\psi}(p+P_cF_a)$$

Integrating the above equation from $\psi=1$ to $\psi=\infty$ and invoking (4a, b), (5), and (6), (17) reduces to

$$B(1-\lambda_p)\frac{\partial\eta}{\partial t}=-\frac{\partial Bq_{bT}}{\partial x}+I_{Lt}-B\beta q_{bT}(1+P_c)$$
$$-\frac{B\beta q_{bT}}{3\ell n(2)}(p|_{\psi=1}+P_cF_a|_{\psi=1}) \quad (18)$$

The second-to-last term on the right-hand side of (18) quantifies the rate of loss of gravel volume to silt by grinding, and the last term quantifies the rate of loss to sand (as gravel particles are ground so fine that they become sand).

A flood intermittency I (fraction of time the river is in the considering flood) is used to characterize the fact that the river is morphodynamically active only for a small fraction of real time. The Eqs. (17) and (18) become, respectively

$$B(1-\lambda_p)\left[\frac{\partial}{\partial t}(L_aF)+f_I\frac{\partial}{\partial t}(\eta-L_a)\right]=-I\frac{\partial Bq_{bT}p}{\partial x}+I_{LT}p_L \quad (19)$$
$$-IB\beta q_{bT}(p+P_cF_a)+I\frac{B\beta q_{bT}}{3\ell n(2)}\frac{\partial}{\partial\psi}(p+P_cF_a)$$

$$B(1-\lambda_p)\frac{\partial\eta}{\partial t}=-I\frac{\partial Bq_{bT}}{\partial x}+I_{Lt}-IB\beta q_{bT}(1+P_c)$$
$$-I\frac{B\beta q_{bT}}{3\ell n(2)}(p|_{\psi=1}+P_cF_a|_{\psi=1}) \quad (20)$$

A consideration of the scaling of terms (19) and (20) indicates that the time evolution terms are no larger than the order of 1/100 of the other terms. Thus we can say that bedrock incision occurs so slowly that the pattern of downstream fining can be taken as quasi-steady state. Thus, (19) and (20), respectively, become

$$\frac{dBq_{bT}p}{dx}=\frac{I_{LT}p_L}{I}-B\beta q_{bT}(p+P_cF_a)+\frac{B\beta q_{bT}}{3\ell n(2)}\frac{\partial}{\partial\psi}(p+P_cF_a) \quad (21)$$

$$\frac{dBq_{bT}}{dx}=\frac{I_{LT}}{I}-B\beta q_{bT}(1+P_c)-\frac{B\beta q_{bT}}{3\ell n(2)}(p|_{\psi=1}+P_cF_a|_{\psi=1}) \quad (22)$$

From (21) and (22), we obtain the final forms of the governing equations for downstream fining in bedrock rivers with side (hillslope) input as

$$\frac{dq_{bT}}{dx}=-\frac{q_{bT}}{B}\frac{dB}{dx}+\frac{I_{LT}}{IB}-\beta q_{bT}(1+P_c)$$
$$-\frac{\beta q_{bT}}{3\ell n(2)}(p|_{\psi=1}+P_cF_a|_{\psi=1}) \quad (23a)$$

$$Bq_{bT}\frac{dp}{dx}=\frac{I_{LT}(p_L-p)}{I}-B\beta q_{bT}P_c(F_a-p)$$
$$+\frac{B\beta q_{bT}}{3\ell n(2)}\left[\frac{\partial}{\partial\psi}(p+P_cF_a)+p(p|_{\psi=1}+P_cF_a|_{\psi=1})\right] \quad (23b)$$

Discretized form of (23 a, b) for use in numerical computations can be obtained as follows. The grain size distributions are discretized into N grain size ranges bounded by $N+1$ sizes $D_{b,1}..D_{b,N+1}$ from finest to coarsest. The characteristic size of the ith grain size range is D_i, where $D_i=\sqrt{D_{b,i}D_{b,i+1}}$ or on the logarithmic psi scale $\psi_i=(\psi_{b,i}+\psi_{b,i+1})/2$. Thus

$$q_{bT,k+1}=q_{bT,k}-\frac{q_{bT,k}}{B_k}(B_{k+1}-B_k)+$$
$$\left[\frac{I_{LT,k}}{B_k}-\beta q_{bT,k}(1+P_{c,k})-\frac{\beta q_{bT,k}}{3\ell n(2)}\frac{(p_{1,k}+P_{c,k}F_{a1,k})}{\Delta\psi_1}\right]\Delta x \quad (24a)$$

$$p_{i,k+1}=p_{i,k}+\frac{I_{LT,k}}{B_kq_{bT,k}}(p_{Li,k}-p_{i,k})-\beta P_{c,k}(F_{ai,k}-p_{i,k})\Delta x$$
$$+\frac{\beta}{3\ell n(2)}\left[\frac{p_{i+1,k}+P_{c,k}F_{a,i+1,k}}{\Delta\psi_{i+1}}-\frac{p_{i,k}+P_{c,k}F_{ai,k}}{\Delta\psi_i}+\frac{p_{i,k}}{\Delta\psi_1}(p_{1,k}+P_{c,k}F_{a1,k})\right]\Delta x \quad (24b)$$

2.2.5 *Other pertinent relations used in the model*

The downstream variation of channel width B can be computed from empirical relations that can be calibrated to be site-specific. For example, let x denote a down-channel distance (with an appropriately defined origin near the headwaters of the basin) and $A(x)$ denote the drainage area upstream of point x. It is commonly assumed that B depends on A (e.g. Montgomery & Gran, 2001) as

$$B=\alpha_bA^{n_b} \quad , \quad A=K_hx^{n_h} \quad (25\text{ a, b})$$

Eq. (25b) is known as Hack's law. Reasonable values for α_b, n_b, K_h and n_h are 0.02, 0.4, 6.7 and 1.7, respectively.

A gross estimate of volume input of sediment per unit distance downstream per unit time by landslides I_{LT} is readily written as

$$I_{LT}=v_d\frac{dA}{dx} \quad (26)$$

where v_d is an estimate of the denudation rate (speed of erosion of the surface) of the hillslope area ΔA adjacent to the stream between x and $x+\Delta x$.

The simplest possible form for the fraction of bedrock surface that is covered by alluvium has been found to be

$$P_c=\begin{cases}\dfrac{q_{bT}}{q_{bTc}} & , \quad q_{bT}<q_{bTc}\\ 1 & , \quad q_{bT}=q_{bTc}\end{cases} \quad (27)$$

where q_{bTc} denotes the capacity transport rate of gravel (e.g. Sklar & Dietrich, 2004 and Chatanantavet & Parker, in preparation). By definition, if bedrock is exposed, then the supply of gravel is below capacity; otherwise the bedrock would be covered by alluvium.

Here the capacity transport rate q_{bTc} is computed using the gravel transport relation of Parker (1990). This relation is convenient because it tracks only gravel transport. The tracking of sand transport in a bedrock river would be difficult. A full description of the relation can be found in Parker (1990). Here, the relation can be written in shorthand form as

$$q_{bTc} p_i = \alpha_p u_*^3 F_i G_i(\overline{\psi}_s, \sigma_s, u_*) \tag{28}$$

where α_p is a constant($=0.00218/(Rg)$), $R=1.65$ denotes gravel submerged specific gravity, $g=9.81\,\mathrm{m}^2/\mathrm{s}$ denotes gravitational acceleration, and G_i is a messy grain size-specific function that ultimately involves only shear velocity u_*, the surface arithmetic mean size $\bar{\psi}_s$, and the surface arithmetic standard deviation σ_s. The surface geometric mean size D_{sg} and surface geometric standard deviation σ_{sg} are given as

$$D_{sg} = 2^{\overline{\psi}_s} \quad , \quad \sigma_{sg} = 2^{\sigma_s} \tag{29 a, b}$$

where

$$\overline{\psi}_s = \sum_{i=1}^{N} \psi_i F_i \quad , \quad \sigma_s^2 = \sum_{i=1}^{N} (\psi_i - \overline{\psi}_s)^2 F_i \tag{30 a, b}$$

The shear velocity u_* can be estimated from

$$u_* = 2^{[(\overline{\psi}_s + 1.27\sigma_s)/20]} \hat{u}$$
$$\hat{u} = \alpha_r^{-3/10} n_k^{1/20} \left(\frac{iA}{B} \right)^{3/10} g^{7/20} S^{7/20} \tag{31 a, b}$$

where α_r is a coefficient in the Manning-Strickler resistance relation that can be set equal to 8.1, $n_k \sim 1.5$ to 3, i is effective rainfall rate, and S is channel slope (e.g. Parker, 1991).

It is assumed here that appropriate forms for A, B, S and i are specified. Then (31a, b) can be used to reduce (28) to a simpler functional form in which the only unknowns are $\bar{\psi}_s$ and σ_s;

$$q_{bTc} p_i = \alpha_p \Omega(\overline{\psi}_s, \sigma_s) \hat{u}^3 F_i \widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u}) \tag{32}$$

where

$$\Omega(\overline{\psi}_s, \sigma_s) \equiv 2^{[3(\overline{\psi}_s + 1.27\sigma_s)/20]}$$
$$\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u}) \equiv G_i\left(\overline{\psi}_s, \sigma_s, 2^{[(\overline{\psi}_s + 1.27\sigma_s)/20]} \hat{u}\right) \tag{33}$$

2.2.6 *Completion: solution for surface size fractions*

The goal of this model is to compute the downstream variation of $\bar{\psi}_s$ and σ_s. In order to calculate these values, associated equations are developed as follows. The capacity transport rate of the channel, q_{bTc} can be computed by solving (32) for F_i and summing, so that

$$q_{bTc} = \frac{\alpha_p \Omega(\overline{\psi}_s, \sigma_s) \hat{u}^3}{\displaystyle\sum_{i=1}^{N} \frac{p_i}{\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u})}} \tag{34}$$

F_i is then computed from (32) and (33) as

$$F_i = \frac{\dfrac{p_i}{\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u})}}{\displaystyle\sum_{i=1}^{N} \frac{p_i}{\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u})}} \tag{35}$$

F_{ai} is then computed from F_i and the discretized form of (5), which is

$$F_{ai} = \frac{F_i 2^{-(1/2)\psi_i}}{\displaystyle\sum_{i=1}^{N} F_i 2^{-(1/2)\psi_i}} \tag{36}$$

The basis for computing $\bar{\psi}_s$ and σ_s is (35) and the definitions (30a, b). Applying (30a, b) to (35),

$$\Phi_1(\overline{\psi}_s, \sigma_s) = 0 \quad , \quad \Phi_2(\overline{\psi}_s, \sigma_s) = 0 \tag{37 a, b}$$

where

$$\Phi_1 = \overline{\psi}_s - \frac{\displaystyle\sum_{i=1}^{N} \frac{\psi_i p_i}{\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u})}}{\displaystyle\sum_{i=1}^{N} \frac{p_i}{\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u})}} \quad , \quad \Phi_2 = \sigma_s^2 - \frac{\displaystyle\sum_{i=1}^{N} \frac{(\psi_i - \overline{\psi}_s)^2 p_i}{\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u})}}{\displaystyle\sum_{i=1}^{N} \frac{p_i}{\widetilde{G}_i(\overline{\psi}_s, \sigma_s, \hat{u})}} \tag{38 a, b}$$

The above equations can be solved iteratively using e.g. a Newton-Raphson technique. The formulation leads to a matrix equation which can be solved using Doolittle's method.

2.2.7 *Flow of the calculation*

The model calculation flows as follows.

i) The input variables *S, B, i*, p_{Li}, I_{LT} and û need to be specified in advance, either as prescribed constants or known functions of x.
ii) Upstream values of q_{bT} and p_i are imposed as boundary conditions.
iii) From the given upstream values of p_i and û, F_i and q_{bTc} are computed from (35) and (34), and then P_c and F_{ai} are computed from (27) and (36).
iv) Having done this, (24a, b) can be used to find p_i and q_{bT} one step downstream.
v) Once p_i is known one step downstream, a repeat of step iii) determines F_i, q_{bTc}, P_c and F_{ai} at that node.
vi) Repeat the above steps downstream.

3 FIELD SITE: A GUADELOUPEAN BEDROCK RIVER

A bedrock river of the Vieux-Habitants watershed on the Basse-Terre Island of Guadeloupe in the French West Indies has been used as a basis for a preliminary test of the numerical model in this study against field data. The Vieux-Habitants watershed has a drainage area of 19.7 km^2 and a relief of about 955 m. Since the area is within a national park, anthropogenic forcing is expected to be weak. The lithologic condition is found to be relatively uniform andesite. Vegetation is very dense and precipitation is high in this area. The channel long profile is concave upward. The decreasing bed slope from upstream to downstream can be readily approximated by an exponential law.

4 MODEL APPLICATIONS TO THE FIELD

The results below (Figs 2 to 5) are based on numerical modeling using the following input parameters from the field measurements in the stream and watershed of Vieux-Habitants except $q_{bT,u}$ (by model calibration): reach length $(L) = 9.53$ km, effective precipitation rate $(i) = 10$ mm/hr, channel slope at the upstream end $(S_u) = 0.056$, channel slope at the downstream end $(S_d) = 0.027$ (with exponential in between), value of x at the upstream end of the reach $= 8.56$ km (where $x = 0$ at the drainage divide upstream), volume bedload transport rate per unit width at the upstream end $(q_{bT,u}) = 0.04\ m^2/s$ and intermittency of rainfall $(I) = 0.01$. The abrasion coefficient (β) $(= 3\alpha)$ is estimated from the value $\alpha = 0.00015\ m^{-1}$ for andesite from Kodama (1994b). The denudation rate $(v_d) = 0.1$ mm/yr based on field estimation. The measured inputs for the grain size distributions of bedload at the upstream end and of material supplied from the hillslopes are taken to be the same, and are shown in Fig. 1 below.

The model output for the downchannel variation in surface geometric mean grain size is plotted in Fig. 2 together with the field data collected from Vieux-Habitants River. Within considerable scatter, the model results show reasonable agreement with the field data. The grain size distributions of bedload material from the numerical model output are also shown in Fig. 3.

Because the model uses a Sternberg formulation that grinds gravel and boulder to sand and silt with no explicit splitting or shattering, the model produces a large, and perhaps unreasonable amount of silt and sand (Fig. 4). Therefore, in order to obtain more realistic values of sediment transport along the stream, splitting processes will need to be described explicitly. Also, only one characteristic large flood is considered here (via the intermittency). As a result, selective sorting due to a full hydrograph (or selective entrainment), such that smaller sizes predominate in the load at lower flood flows but very large material is moved only at peak flow, are not generated by this model. As a result, the areal fraction of alluvial coverage decreases downstream (Fig. 5), making it difficult to describe the bedrock-alluvial transition (if any) without a slope

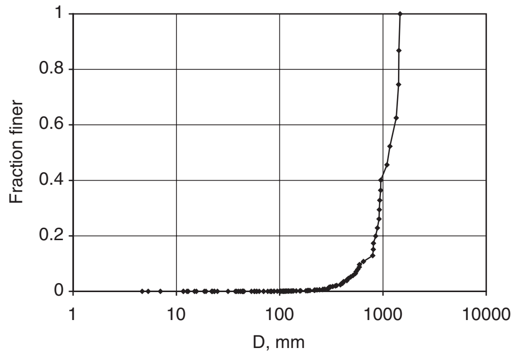

Figure 1. Grain size distributions of upstream boundary and hillslope materials in Vieux-Habitants River, which are assumed to be identical.

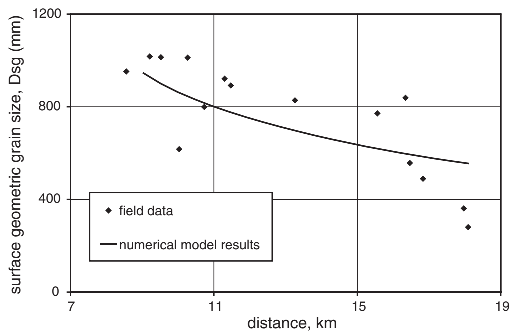

Figure 2. Comparison of downstream variation of surface geometric mean grain size predicted by the model against field data from Vieux-Habitants River.

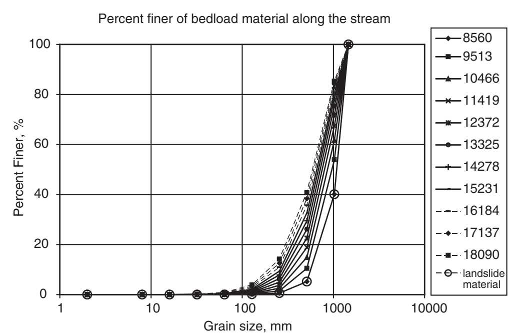

Figure 3. Numerical model results showing the downstream variation of the bedload size distribution.

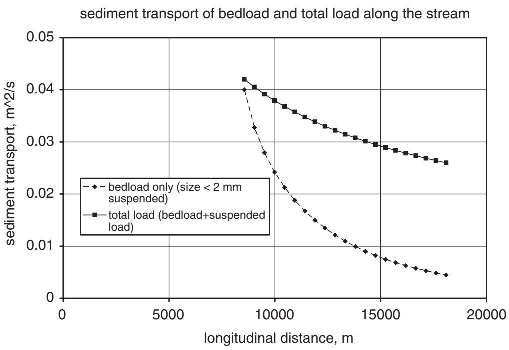

Figure 4. Numerical model results for the downstream variation of sediment transport. Note that since the conceptual model lacks an explicit grain splitting mechanism, the model produces a large amount of sand and silt (<2 mm).

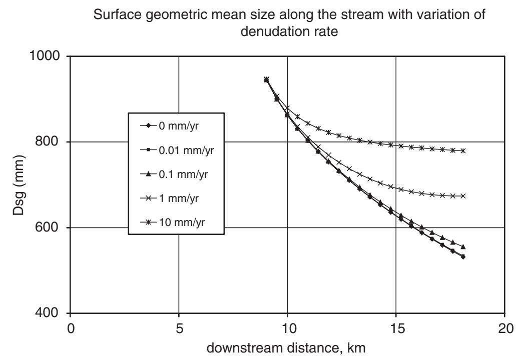

Figure 6. Numerical model results for the downstream variation of surface geometric mean size, with variation of denudation rates varying from 0 to 10 mm/year.

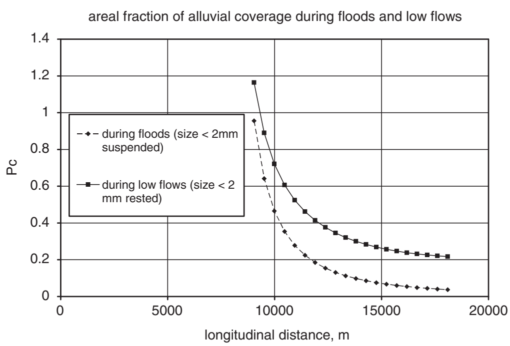

Figure 5. Numerical model results for the downstream variation of the areal fraction of alluvial coverage.

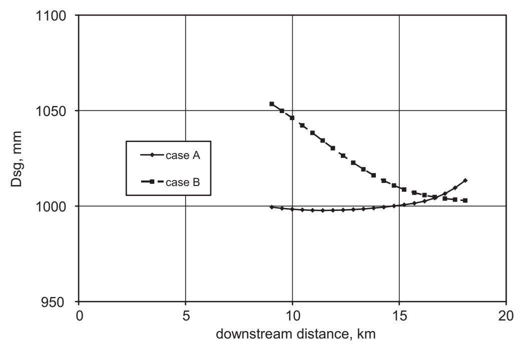

Figure 7. A test of the effect of selective sorting for Vieux-Habitants River. In both cases, the abrasion coefficient $\beta = 0$.

discontinuity. The model can be improved by incorporating a) grain splitting, b) a full hydrograph and c) a bedload transport formulation that adjusts for the fact that much of the drag of the flow is expended on large boulders that may never move.

Fig. 6 shows the numerical model results for the downstream variation of mean grain size keeping all input parameters the same except for the denudation rate, which is varied from 0 to 10 mm/year. Note that the side input from the hillslopes suppresses downstream fining.

5 ADDITIONAL MODEL RESULTS

To test the effect of selective sorting (differential transport) in downstream fining process, the investigation can be done by setting the abrasion coefficient equal to zero as shown in Fig. 7 below. In case A of that figure, the input parameters are the same as the original values for Vieux-Habitants River except the abrasion coefficient. In case B, the input parameters are the same except rainfall= 15 mm/hr, $S_u = 0.03$, $S_d = 0.02$, and $q_{bT,u} = 0.003\ m^2/s$. Note that in case A, there is no size reduction as the stream is overwhelmed with hillslope input, and in case B, the size reduction due to selective sorting is relatively small (5% reduction).

Fig. 8 below shows a case where selective sorting (differential transport) and abrasion processes play equally important roles in downstream fining. The following input parameters have been used in this case: reach length $(L) = 30$ km, effective precipitation rate $(i) = 20$ mm/hr, channel slope at the upstream end $(S_u) = 0.02$, channel slope at the downstream end $(S_d) = 0.01$, value of x at the upstream end ($x = 0$ at the drainage divide upstream) $= 1.5$ km, volume bedload transport rate per unit width at the upstream end $(q_{bT,u}) = 0.0005\ m^2/s$, intermittency of rainfall $(I) = 0.01$, abrasion coefficient $= 0.0003\ m^{-1}$, and denudation rate $(v_d) = 2$ mm/yr. The inputs for the grain size distributions of bedload at the upstream end and of landslide material are shown in Fig. 9 below.

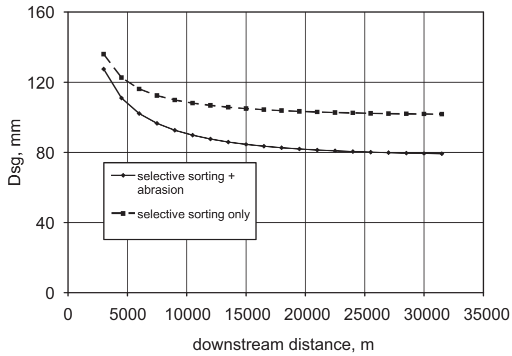

Figure 8. A test of the effect of selective sorting in a stream with smaller grain sizes than those in Vieux-Habitants River. In this case, both selective sorting and abrasion play equally important roles in downstream fining (∼25% reduction).

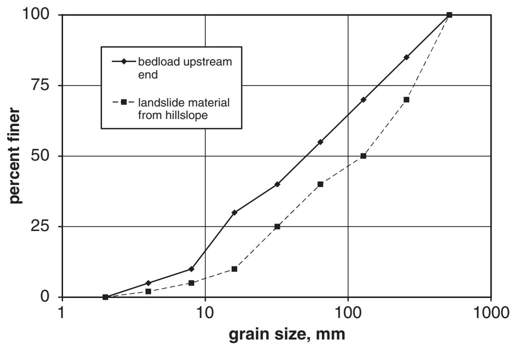

Figure 9. Grain size distributions of bedload at upstream end and of landslide material from hillslopes for results in Fig. 8.

6 DISCUSSION

This study shows that with appropriate calibration, the model provides reasonable agreement with the field data on downstream fining for Vieux-Habitants River in the Guadeloupe Island. The results suggest, however, that the model can be improved by incorporating a) grain splitting, b) a full hydrograph and c) a bedload transport formulation that adjusts for the fact that much of the drag of the flow is expended on large boulders that may never move.

The large amount of silt and sand produced by the model (Fig. 4) may simply be transported to the sea, so that this material is abundant only in the vicinity of the estuary. It may also be the case that the sand/silt production is artificially forced by using a high value of the abrasion coefficient that in fact includes splitting as well as wear. This issue cannot be resolved in the absence of a model that includes shattering as well as wear.

Model results suggest that both abrasion (including splitting) and selective sorting (differential transport) can be equally important for downstream fining in bedrock streams only in certain cases, such as streams with relatively smaller grain sizes and lower slopes (Fig. 8). In some cases, the results show that selective sorting in a single extreme flood may not play a role in downstream fining (such as in Vieux-Habitants River for certain input parameters). However, selective sorting due to a full flood hydrograph (selective entrainment), so that smaller sizes are moved during relatively lower floods flows but large sizes are only moved near peak flow or hardly moved, must prevail in Vieux-Habitants River if areal fraction of alluvial coverage is to increase downstream until it reaches the bedrock-alluvial transition, rather than decreasing downstream as predicted by the model results (Fig. 5). The results also suggest that future work may need to consider splitting or shattering mechanisms separately from the Sternberg-type coefficient characterizing abrasion.

In a bedrock stream with a high denudation rate, hillslope materials with large clasts may suppress downstream fining, as shown in Fig. 6.

Sklar et al. (2006) suggest that abrasion of grain particles during fluvial transport in bedrock streams has a small effect on the bedload size distribution for the case of spatially uniform supply of hillslope material, so that downstream fining must be due primarily to spatial gradients in hillslope sediment production. We, on the other hand, have found that abrasion (including splitting), should at least in part play a role in downstream fining in bedrock streams, possibly together with the selective sorting process due to a full flood hydrograph.

7 CONCLUSION

Model verification against field data in Vieux-Habitants River shows reasonable agreement in regard to downstream fining. The model results indicate that in some cases, both abrasion (including splitting) and selective sorting processes can be equally important for downstream fining in bedrock streams. Moreover, contrary to the common belief, the results here suggest that where no net deposition occurs such as in rivers actively cutting through bedrock, abrasion process alone cannot explain the behavior of the bedrock-alluvial transition. Here it is suggested that selective sorting due to a full flood hydrograph or selective entrainment may be the missing piece. Future work should therefore include a) the use of a full flood hydrograph, b) an adapted sediment transport formula with a reduction in effective transport capacity due to drag forces generated by large boulders and c) an explicit rock-splitting mechanism.

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