

## Experimental investigation of the response of an alluvial river to a vertical offset of its bed

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**ABSTRACT:** We study the question of the lifetime of a vertical offset generated in the bed of an alluvial gravel-bed river by faulting. This problem is addressed experimentally by investigating the response of a micro-scale river to a vertical offset of its bed. Calibration of the experimental sediment transport law and of the velocity – water depth relationship allow us to write the equation describing the evolution of the bed elevation. Scaling considerations further reveal that, in their dimensionless form, the equations governing the evolution of a natural or an experimental river are identical. The differences of time and length scales at work in experimental and natural rivers are encoded in the expression of only two parameters. This result justifies the use of a microscale river and provides the key to (and the extent up to which) extrapolate our experimental results to the field scale.

### 1 INTRODUCTION

In 1999 the Gölçuk ( $M = 7.4$ ) and Duzce ( $M = 7.2$ ) earthquakes in Turkey induced disruption and offset of all the streams crossing the north anatolian fault along a 50 km path, some of these offsets being more than 4 meters in amplitude (Armijo (2000)). Adding to the desolation induced by each one of these deadly events, more than 17000 lives lost, 60000 homeless and a cost of more than 25 billions of euros (European Environmental Agency 2003), farmers and people leaving along the stream banks faced an unusual threat in the following months : the reaction of the streams to the strong perturbation of their bed. Entrenchment due to vertical offset induced bank collapse, endangering neighboring constructions. It also changed the hydraulics of floods. Furthermore bank erosion due to lateral offset along the fault resulted in rapid lateral shift of the river that threatened both neighboring constructions and agricultural crops. Understanding the dynamics and estimating the timescale governing the response of a river to an external perturbation such as the ones produced by earthquakes is therefore an important issue for risk assessment and civil protection in seismic tectonically active countries.

At a larger time scale it is also an important issue in earth sciences as such perturbations often lead to the abandonment of morphologic markers such as alluvial terraces. These features are then used as passive markers of the deformation that can be used to constrain uplift rates or to date earthquakes and climatic changes. Indeed active faults generate offsets

of features such as rivers, ridges, terraces levels and terrace risers. Such morphological markers combined with dating are widely used in earth sciences to constrain the kinematics of active faults (Burbank (2001), Dade (1998), Van-der-Woerd (2002)).

This use of morphological markers raises the question of their characteristic time scales of both formation and resilience (e.g. Bull (1991) and Meunier (submitted)). This question has been addressed quantitatively in the case of scarp degradation by Wallace (1977) and later Avouac (1993) who established a method of dating based on the estimate of the degree of degradation of a scarp (vertical offset) through diffusive-like slope processes. Armstrong (2004) later investigated experimentally the evolution of a micro-scale river to a lateral offset of its bed. He showed that bank erosion develops a kinematic wave and derived an equation describing the evolution of the bank. These studies all share in common the conclusion that depending on the timescale at which they are looked upon morphologic markers may or may not just be considered as passive. The formation of such markers depends on the hydrodynamics of flow and sediment transport at the time of abandonment and results from the progressive propagation (up or downstream) of erosion waves with characteristic velocities and timescales.

Our objective in this paper is to extend the work of Armstrong (2004) by investigating the response of a river to a sudden vertical uplift of its bed. As an example the Chi-Chi earthquake which stroke Taiwan in 1999 produced a vertical offset of about 2 meters along

the bed of the Da-Jia river. However this offset could not be detected anymore six months later: the river had completely erased the marker in such a short time. This observation motivated the work reported in this paper. The response of a river to a sudden vertical uplift of its bed is investigated. We focus on the transient regime following the creation of the scarp. How does the scarp erode? On what timescale? What parameters govern the dynamics of return to equilibrium?

Because of the lack of reliable natural data sets, these questions are addressed experimentally by investigating the response of a micro-scale river to a vertical offset of its bed.

The paper is organized as follows. The experiments are described in section 2.

The equations governing the response of our micro-scale river to an offset of its bed are developed and solved in section 3. The results of the model are compared to experimental results. Section 4 is devoted to the problem of the upscaling of our experimental results to natural rivers

## 2 EXPERIMENTS

### 2.1 Setup and procedure

We conducted experiments in a small inclinable flume of cross section  $5 \times 5 \text{ cm}^2$  and length  $L = 1 \text{ m}$  schemed on figure 1. The flume was filled with a 4 cm thick sediment bed of glass beads of size between 50 and  $100 \mu\text{m}$  and density  $\rho = 2500 \text{ kg.m}^{-3}$ . A pump was used to inject water at the flume inlet. The flow discharge remained constant during an experiment. It was measured with a flow-meter (accuracy 0.01 L/minute) and was varied between 0.1 and 2 L/minute. The walls of the flume were rigid so that our micro-scale river kept a constant width of 5 cm. The flume reposed on an inclinable plane allowing us to vary the slope of the river bed. This latter measured with a digital inclinometer (accuracy  $0.1^\circ$ ) was varied between  $0.3$  and  $4^\circ$ . At the flume outlet, sediment particles settled in a constant water level overflowing tank. The tank was positioned on a high precision scale (accuracy 0.1 g) connected to a computer collecting the weight of the tank at regular time intervals (10 s). This allowed us to precisely measure the sediment cumulated mass at the outlet of the river from which the sediment discharge was derived (figure 1b).

The experimental procedure was the following. An initial river with a flat bed of slope  $S_i$  and elevation  $h_i(x)$  was prepared. About ten minutes after the flow had been initiated, sediment discharge reached a stationary state as illustrated on figure 1b. Once this stationary state was reached, we generated a vertical offset of the river bed using a gate located at the downstream end of the flume which can be suddenly dropped down (see figure 1). This offset was applied

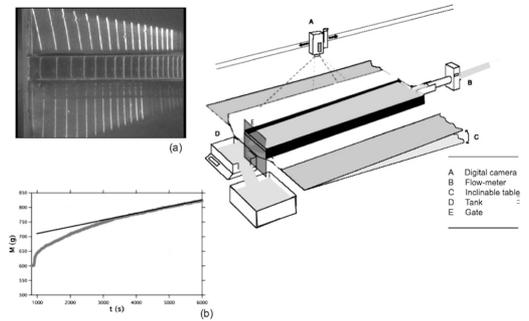


Figure 1. Schematics of the experimental setup. (a) Measurements of bed elevation by mean of a set of laser sheets projected on the river. (b) Cumulated sediment mass as a function of time at the flume outlet: about ten minutes after initiation of the flow, sediment discharge reaches a stationary state.

rapidly in regards of the timescale of channel evolution so that it can be considered as instantaneous.

The river response to this sudden perturbation was measured by mean of a set of laser sheets projected onto the river bed. A digital camera positioned at the vertical of the bed recorded images of the laser sheets which deviation allowed us to measure the variations of bed elevation within an accuracy of 6%. The size of the region imaged by the camera was about 40 cm.

Note that there was no sediment input at the river inlet. As a result an erosion wave progressively propagated slowly from the inlet towards the outlet of the flume. All our experiments were stopped before this degradation wave had reached the region of interest so that it never interfered with our results.

### 2.2 Experimental results

Several series of experimental run were conducted with initial bed slope ranging from  $0.3^\circ$  to  $4^\circ$ , water discharge from 0.5 L/minute to 2.5 L/minute and offset from 0.5 cm to 1 cm. Experiment duration ranged between 30 minutes and 1 hour. Figure 2 displays typical variations of bed elevation observed for three different experimental run. The river responds to the offset by spreading a diffusive erosion wave. The upstream face of the scarp is eroded so that the scarp progressively smooths.

## 3 THEORETICAL ANALYSIS

### 3.1 Governing equations

In this section, we develop an aggradation-degradation model to account for the experimental observations. The model we developed is very similar to the one

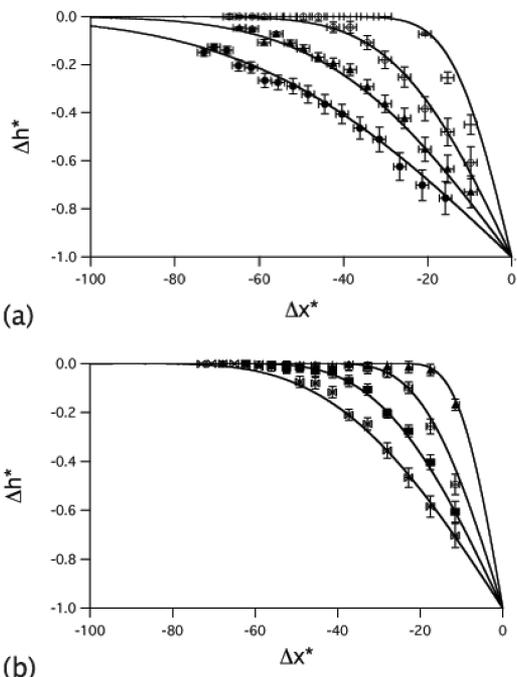


Figure 2. Dimensionless bed elevation  $\Delta h^* = (h(x,t) - h_i(x)) / A$  as a function of  $x^* = x/A$  for three different experimental run. Crosses, triangles, circles and squares correspond to experimental data respectively acquired at  $t = 50s, 200s, 500s$  and  $1000s$ . Straight lines correspond to the model predictions. (a)  $S_i = 0.035, S_c = 0.003$  and  $A = 0.5$  mm and (b)  $S_i = 0.007, S_c = 0.006$  and  $A = 0.5$  mm.

already developed by Gary Parker and described in his Morphodynamics Ebook<sup>1</sup>.

We are interested in the evolution of the longitudinal bed elevation  $h(x,t)$  of our experimental river as a function of time  $t$  and streamwise distance  $x$ . Before being perturbed by faulting, the river is assumed to be at steady state with a profile of constant slope  $S_i$ . The input sediment flux at the upstream end of the river therefore compensates exactly the erosion of the bed.

At time  $t = 0$ , an offset of magnitude  $A$  is created at the flume outlet located in  $x = 0$  as illustrated on Figure 3. The initial bed profile right after perturbation is therefore:

$$h(x,t=0) = \begin{cases} -S_i x + A & \text{for } x < 0 \\ 0 & \text{for } x = 0 \end{cases} \quad (1)$$

<sup>1</sup> Gary Parker's ebook can be freely downloaded from the following URL: <http://cee.uiuc.edu/people/parkerg/>.

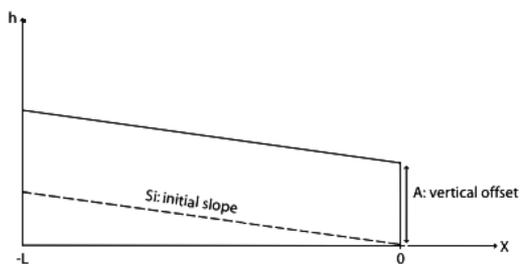


Figure 3. Representation of initial river bed which undergoes a vertical offset of magnitude  $A$ .  $h$  represents the bed elevation,  $x$  is the position along the bed river and  $L$  is the length of the flume.

The evolution of the bed of our micro-scale river (alike natural rivers) is governed by the Exner sediment mass conservation equation:

$$(1 - \lambda) \frac{\partial h}{\partial t} + \frac{\partial q_s}{\partial x} = 0 \quad (2)$$

where  $\lambda$  is the porosity of the bed and  $q_s$  is the volumetric sediment discharge per unit width. The width of the experimental river is constant so that conservation of water reads:

$$q_l = UH \quad (3)$$

where  $q_l$  is the discharge per unit river width,  $U$  is the average flow velocity and  $H$  is river depth.

The Reynolds number in our experimental river ranges between 100 and 500 so that our river can be considered laminar (Malverti (submitted)). As a result both the sediment transport equation and the equation linking  $U$  and  $H$  are likely to differ in our experiments and in a natural river. Several experimental run were performed in order to calibrate the relationship between sediment transport rate and basal shear stress in our experimental flume. The results of these series of experiments have already been reported in another publication (Malverti (2007)) so that we only recall the main result of interest for the present paper. Sediment transport rate  $q_s$  was measured with the scale and the average flow velocity  $U_m$  was estimated by tracking the motion of a drop of dye injected in the river. A large number of experimental run were performed for different bed slopes ranging from  $0.3^\circ$  to  $4^\circ$  and water discharges ranging from  $0.5$  L/minute to  $2.5$  L/minute. Assuming normal flow conditions, the basal shear stress exerted by the flow  $\tau_b$  was estimated by

$$\tau_b = \rho_f gHS \quad (4)$$

where  $\rho_f$  is the density of water and  $S$  is the bed slope.

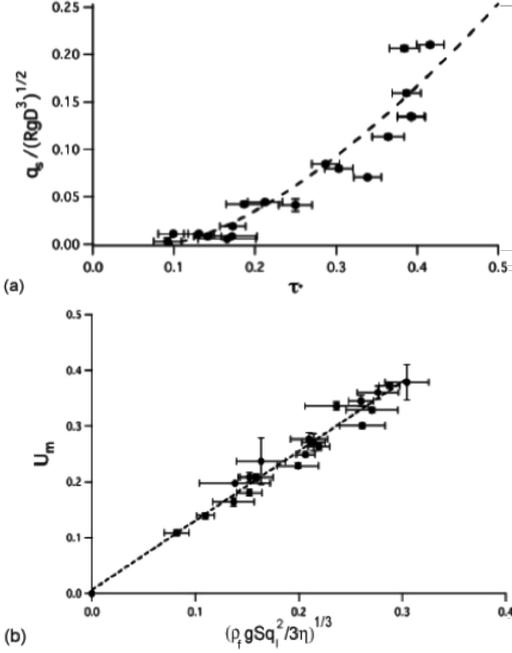


Figure 4. (a) Dimensionless sediment transport rate  $q_s/(RgD^3)^{1/2}$  versus shields number. (b) Average flow velocity  $U$  as a function of  $(\rho_f g S q_i^2 / 3 \eta)^{1/3}$ .

The experimentally measured sediment transport law is shown on figure 4a. Surprisingly sediment transport in our experimental river follows the same law than in a natural one. It is indeed governed by the Meyer-Peter and Müller law:

$$\frac{q_s}{\sqrt{RgD^3}} = \alpha (\tau^* - \tau_c^*)^{3/2} \quad (5)$$

where  $\rho_s$  is the density of the sediment,  $R = (\rho_s - \rho_f)/\rho_f$ ,  $g$  is gravity,  $D$  is the diameter of the glass beads forming the bed and  $\alpha = 0.965$  is a dimensionless coefficient.  $\tau^* = \tau_b/(\Delta\rho g D)$  is the Shields number (Shields (1936)),  $\tau_b$  is the streamwise bed shear stress and  $\tau_c^* = 0.09$  is a dimensionless threshold shear stress.

For a pure laminar flow, a straightforward calculation leads to the following relationship between depth-averaged flow velocity and slope:

$$U = \left( \frac{\rho_f g S q_i^2}{3\eta} \right)^{1/3} \quad (6)$$

where  $\eta$  is the dynamical viscosity of water. A fit of the experimental data led to a slightly different expression (see figure 4b):

$$U_m = 1.24 \left( \frac{\rho_f g S q_i^2}{3\eta} \right)^{1/3} \quad (7)$$

The difference between equations (6) and (7) is discussed by Malverti (submitted). To summarize, we believe that it might be due to the fact that the velocity at the bed is not 0 as assumed in equation (6).

Combining equations (2), (3), (4), (5) and (7) leads to:

$$(1-\lambda) \frac{\partial h}{\partial t} + C_D \frac{\partial}{\partial x} \left( \left( -\frac{\partial h}{\partial x} \right)^{2/3} - S_c^{2/3} \right)^{3/2} = 0 \quad (8)$$

$C_D$  is homogeneous to a diffusion coefficient. It is defined by:

$$C_D = \alpha \varepsilon^{3/2} \left( \frac{\eta q_i}{R^2 \rho_f} \right)^{1/2} \quad (9)$$

where  $\varepsilon = 1.1631$ .

$S_c$  is a critical slope defined by:

$$S_c = \left( \frac{R^3 D^3 \rho_f g \tau_c^{*3}}{\varepsilon^3 \eta q_i} \right)^{1/2} \quad (10)$$

If the slope of the river falls below  $S_c$ , no sediment is transported.

Let us now write the boundary conditions. First of all, as already stated above, we consider that, before perturbation, the river is at steady state with a profile of constant slope  $S_i$ . The input sediment flux at the upstream river end ( $x = -L$ ) therefore compensates exactly the bed erosion which reads:

$$q_s(x = -L, t) = C_D \left[ (S_i)^{2/3} - S_c^{2/3} \right]^{3/2} \quad (11)$$

This boundary condition holds of course as long as the incision wave generated by the offset has not reached the downstream end of the reach investigated.

The second boundary condition states that the elevation of the bed at the end of the flume remains constant in time:

$$h(x = 0, t) = 0 \quad (12)$$

### 3.2 Scaling

In order to exhibit the relevant control parameters governing the evolution of the river bed, we choose to make the equation dimensionless by defining the following dimensionless variables:

$$\begin{aligned} h^* &= \frac{h}{A} \\ x^* &= \frac{x}{A} \\ t^* &= \frac{C_D}{(1-\lambda)A^2} t \end{aligned} \quad (13)$$

With other words, distances are measured in units of the offset  $A$  and time is measured in units of the characteristic diffusive time  $\tau_D = (1-\lambda)A^2/C_D$ . With these new variables, equation (8) becomes:

$$\frac{\partial h^*}{\partial t^*} + \frac{\partial}{\partial x^*} \left( \left( -\frac{\partial h^*}{\partial x^*} \right)^{2/3} - S_C^{2/3} \right)^{3/2} = 0 \quad (14)$$

whereas the initial and boundary conditions read:

$$h^*(x, t=0) = \begin{cases} -S_i x + 1 & \text{for } -\frac{L}{A} \leq x^* < 0 \\ 0 & \text{for } x^* = 0 \end{cases} \quad (15)$$

$$\begin{aligned} q_s^*(x^* = -\frac{L}{A}) &= (S_i^{2/3} - S_C^{2/3})^{3/2} \\ h^*(x^* = 0) &= 0 \end{aligned} \quad (16)$$

The set of equations (14), (15) and (16) are controlled by three dimensionless parameters  $S_i$ ,  $S_C$  and  $L/A$ . This last parameter however is very large as the amplitude of the offset is small compared to the typical flume dimension. We therefore do not expect  $L/A$  to exert any influence. Only  $S_C$  and  $S_i$  are expected to control the response of the river to the perturbation. In our micro-scale river,  $S_i$  ranged between  $10^{-3}$  and  $10^{-2}$  and  $S_C$  varied between  $10^{-3}$  and  $10^{-2}$ .

The set of equations (14), (15) and (16) were solved numerically using a finite differences scheme for various values of  $S_C$  and  $S_i$ . The evolution of dimensionless bed elevation versus dimensionless time is displayed on figure 5 for different values of  $S_C$  and  $S_i$ . The river responds to the offset by spreading a diffusive erosion-deposition wave. The upstream face of the scarp is eroded downstream so that the scarp progressively smoothes around a knickpoint. Note that this latter is not advected. In the frame of this simple model, the river response is purely diffusive.

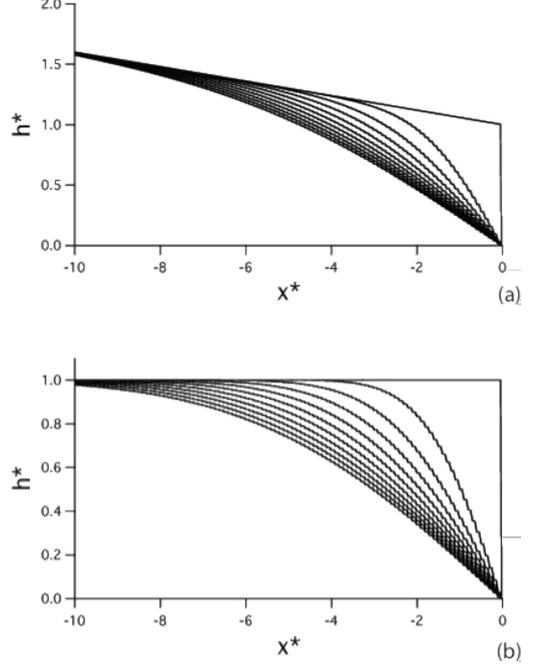


Figure 5. Dimensionless bed elevation  $h^*(x^*, t^*)$  from  $t^* = 0$  to 10. The dimensionless time step between two successive profiles is  $\Delta t^* = 1$ . (a)  $S_i = 0.03$  and  $S_C = 0.001$  (b)  $S_i = 0.0001$  and  $S_C = 0.0001$ .

### 3.3 Comparison between experimental data and model predictions

The model predictions display a very good agreement with the experimental data as illustrated on figure 2. To be complete, it is however important to note that the agreement between our theoretical modeling and the experiments remains good as long as the amplitude of the offset  $A$  remains smaller or of the order of the water depth  $H$ . When  $A$  becomes much larger than  $H$ , normal flow assumption is not fulfilled any more so that the model fails to capture the first instants of the erosion of the scarp.

## 4 UPSCALING THE MODEL TO NATURAL RIVERS

The model developed in the preceding section can easily be adapted to the case of natural gravel bed rivers. Indeed, in the case of natural rivers, the flow is generally turbulent and flow velocity is linked to water flow depth  $H$  through Chézy formula so that equation (7) needs to be replaced by :

$$U = C_z \sqrt{gHS} \quad (17)$$

where  $C_z$  is the Chézy coefficient.

Equations (2), (3), (4) and (5) are unchanged. Combining them with equation (17) leads to the equation governing the evolution of the bed of a natural gravel bed river:

$$(1-\lambda)\frac{\partial h}{\partial t} + C_D \frac{\partial}{\partial x} \left( \left( \frac{\partial h}{\partial x} \right)^{2/3} - S_c^{2/3} \right)^{3/2} = 0 \quad (18)$$

where the expressions of the parameters  $C_D$  and  $S_c$  are now:

$$C_D = \frac{\alpha q_l}{C_z R} \quad (19)$$

and

$$S_c = \frac{\sqrt{R^3 D^3 g C_z \tau_c^{*3/2}}}{q_l} \quad (20)$$

The equations governing the evolution of a natural or an experimental river are therefore formally identical. The differences of time and length scales at work in experimental and natural river are encoded in the expression of  $C_D$  and  $S_c$ . The expressions of these parameters provide therefore the key to extrapolate our experimental results to a natural river and justify the use of a laminar microscale river.

## 5 CONCLUSION

We have investigated the response of an alluvial river to a vertical offset of its bed generated by faulting. The problem is first addressed experimentally by investigating the response of a laminar micro-scale river to a vertical offset of its bed.

We then developed a model by solving sediment mass conservation together with transport law and shallow-water equations. This allows us to predict the evolution of the bed elevation. The use of dimensionless equations reveals that the initial slope  $S_i$  and the critical slope  $S_c$  are the only two relevant parameters governing the response of the river. Predictions of the model are found to be in very good agreement with the experimental observations thus justifying our theoretical approach.

Calibration of the experimental sediment transport law and of the velocity-water depth relationship together with scaling considerations reveal that, in their dimensionless form, the equations governing the

evolution of a natural or an experimental river are identical. The differences of time and length scales at work in experimental and natural river are encoded in the expression of the two parameters  $C_D$  and  $S_c$ . This result justifies the use of a microscale river and provides the key to (and the extent up to which) extrapolate our experimental results to the field scale.

The next step would be now to apply the model to a set of natural data. This work, which is in progress, is however difficult because of the need of reliable hydrologic and granulometric data together with precise measurements of a vertical offset generated along the bed.

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