On the resolution of regional archaeomagnetism: untangling directional geomagnetic oscillations and data uncertainties using the French archaeomagnetic database for dates between AD 1000 and 1500 as a guide

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Abstract: A complexity is emphasized in the distribution of French archaeomagnetic directions during the thirteenth and the fourteenth centuries AD. Data uncertainties, and the smoothing introduced when estimating an average secular variation curve, prevent scrutiny of the very nature of this complexity. It might correspond to a directional yaw, the nature of which would be compatible with the recent geomagnetic field evolution as traced by the gufm1 model. In order to emphasize this indeterminacy, a reference secular variation curve was constructed for dates between AD 1000 and 1500, including the yaw in question, and synthetic databases that mimic the accuracy and density characteristics of the true French archaeomagnetic database were considered for some of these. The synthetic curves hence obtained show that the dating accuracy of archaeomagnetic data is the crucial parameter for constructing a detailed secular variation path. The significant impact of the experimental data accuracy is also illustrated. Even more crucial is the fact that the precision of the data dating required to describe the directional variability over the century timescale largely exceeds the precision of the archaeological dates available for the structures generally studied. This highlights the intrinsic limitation of archaeomagnetism for regional reconstruction of century-scale geomagnetic field variations.

Supplementary material: Figures describing the British and Welsh directional database between AD 1000 and 1500, and synthetic secular variation curves deriving from it are available at https://doi.org/10.6084/m9.figshare.c.4728287 ***Fig. S1 and S2 at the end of this file***

Magnetization measurements carried out on fired archaeological structures, such as kilns and hearths, found *in situ* over a more or less extensive region allow the construction of millennial-scale records of geomagnetic directional secular variation (DSV). Crucial parameters for such reconstructions are the dating of the studied structures – in this case, the dating of their last firing – and the precision of the archaeomagnetic directions determined. Together with archaeointensity determinations, all these results make up the archaeomagnetic database for a given region. With the strong development of archaeomagnetism worldwide in the past two decades, DSV curves have been proposed for different regions, most of them located only in Europe (e.g. Gallet *et al.* 2002; Gómez-Paccard *et al.* 2006; Tema *et al.* 2006; Hervé *et al.* 2013; Batt *et al.* 2017). A discussion of the reliability of the statistical methods used for their construction was not the objective of this study. Instead, its aim was more to present it from the perspective of the experimenter, in wondering whether an archaeomagnetic dataset is able to accurately constrain and reproduce all the complexity of the regional DSV.

This study arose from our attempts to update the French DSV reference curve for the last two millennia, following a significant increase in the number of data collected by our laboratory since those published in Thellier (1981) and Bucur (1994). Despite a regular overall refinement, there still exists a recurrent complexity around the thirteenth and fourteenth centuries that manifests itself as a greater dispersion of individual archaeomagnetic directions: that is, those obtained at the level of the different burned structures. It may be recalled that at a time when archaeomagnetic results were still relatively rare, Aitken (1970) and Clark *et al.* (1988) drew a loop in the British DSV of this period. Similarly, Bucur (1986) proposed a complexity with a 'V' shape in the French curve. On the other hand, more recent DSV curves estimated for France and neighbouring countries appear to have favoured a more regular path (e.g. Bucur 1994; Le Goff *et al.* 2002; Batt *et al.* 2017). Independently, Pavón-Carrasco *et al.*

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(2011) emphasized the need for precisely dated results between AD 1100 and 1400, based on a statistical analysis of the European database.

Based on the observations made above, the question arises as to the extent to which a set of archaeomagnetic data is able to reveal loops, hooks or laces in a DSV path. Because of the non-dipole field component, the gufm1 model (Jackson *et al.* 2000) shows, over the last three centuries, highly variable geomagnetic field evolutions at the latitude of Paris around the Earth (north and south), with large or reduced fluctuations, with or without loops or hooks (Fig. 1) (see also Thompson & Barraclough 1982). For example, the large almost semicircle well known in western Europe (e.g. Le Goff & Gallet 2017), with a length of *c.* 17° (Fig. 1a), is reduced to a yaw of *c.* 11° in eastern Canada (Fig. 1b) and to a *c.* 5° linear segment in the Aleutian Islands (Fig. 1c) or a loop towards the Prince Edward Islands in the southern hemisphere (Fig. 1d). It is safe to consider that

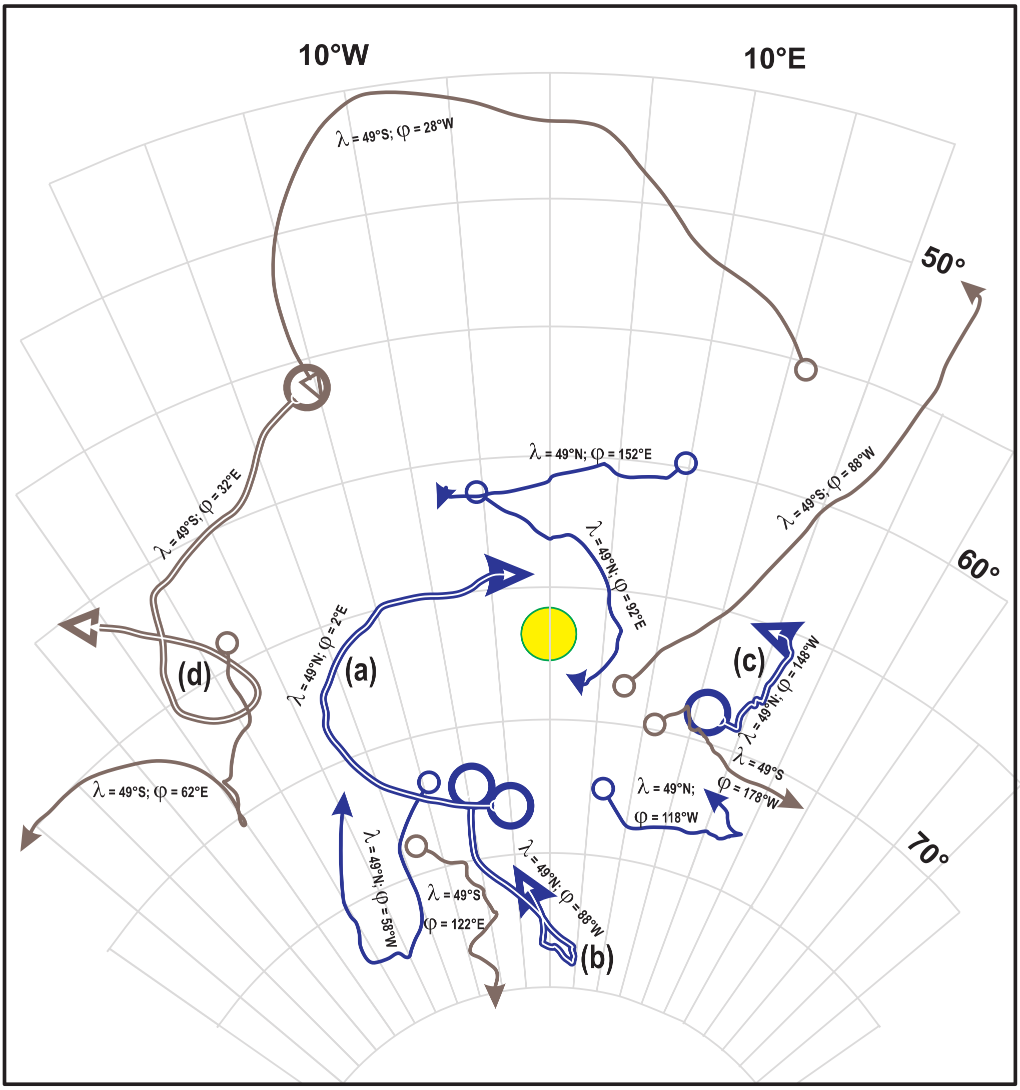

Fig. 1. Examples of directional segments, curves, loops and yaws over the past few centuries (AD 1700–1990) observed from the gufm1 model (Jackson *et al.* 2000) for different sites located in the northern (in blue) and southern (in brown) hemispheres at the same latitude as Paris (see details on the figure and in the text). The evolutions from four locations are discussed in the text (double lines): (a) Paris, (b) eastern Canada, (c) Aleutian Islands and (d) Prince Edward Islands. The yellow circle indicates the direction of the axial-dipolar geomagnetic field at the latitude of Paris.

these various paths of the local DSV, observed at the same time in different places, may also have occurred at the same place over past centuries. Given the characteristics of an archaeomagnetic database – that is, the precision and density of the data available over time – would such fluctuations be recovered and, if so, how?

To our knowledge, the preliminary question of the precision of the archaeodirectional results necessary to recover an accurate DSV evolution has not been fully debated, with regard to the known complexity of the geomagnetic field. In this study, we have chosen to illustrate this question empirically by constructing synthetic databases from a tortuous, but realistic, DSV path, and from the concrete case of the French archaeomagnetic database currently available for dates between AD 1000 and 1500. The main objective of this exercise was, therefore, to provide constraints on the dating and experimental precision of the archaeomagnetic data that are required to trace a DSV path as accurately as possible.

A possible secular variation path in France for dates between AD 1000 and 1500

Description of the French archaeomagnetic database

It is worth pointing out that the purpose of this study was not to describe new archaeomagnetic data obtained in France, but rather to highlight the global characteristics of such data (number, precision of their determination and dating). First, however, we briefly recall some information related to their acquisition.

The data considered in this study were obtained using the experimental method developed by Emile Thellier (e.g. Thellier 1981). The samples were collected using the so-called plaster cap method, which allowed them to be precisely orientated with respect to geographical north. These samples were extracted from decimetre-sized blocks of baked clay, taken from the best-preserved parts of the structures, and consolidated with plaster strips to ensure their cohesion during sampling and subsequent preparation in the laboratory. After sawing in the laboratory, each sample associated with a plaster cap bearing its orientation was embedded in plaster according to a standard format, with a square base of 12 cm on each side. The remanent magnetization was measured in the palaeomagnetism laboratory at Saint-Maur (Institut de Physique du Globe de Paris) using an inductometer especially designed for large samples (Le Goff 1975).

In the case of fired archaeological structures, the remanent magnetization of the samples is generally the simple superposition of a thermal remanent magnetization (TRM) acquired during cooling following the last heating of the structure and a viscous remanent magnetization (VRM) (e.g. Thellier 1981; Dunlop & Özdemir 1997). To estimate the importance of the VRM relative to the TRM, and to largely remove its effect, the samples were kept for several weeks (at least 1 month) in a position parallel to the direction of the ambient geomagnetic field. After this interval of time, the remanent magnetization of the samples was measured for the first time. The samples were then placed in a position reversed with respect to that of the ambient field and again left for several weeks. A second measurement of their remanent magnetization then made it possible to estimate, by vector subtraction, the importance for each sample of the VRM relative to the TRM (and its direction), as well as the TRM direction. Samples with viscosity rates (expressed by the VRM/TRM ratio, in %) that were too high (>12%) were eliminated because their TRM direction might be slightly biased by a certain fraction of VRM (although, in fact, the magnetic viscosities rarely exceeded 12%).

For the first five centuries of the second millennium AD (central dates between AD 1000 and 1500), the currently available French database comprises 113 directional data points, distributed irregularly, and corresponding to an average of about 23 results per century. Among these data, 59 were described by Bucur (1994), including those of Thellier (1981), while 54 data were acquired over the last 25 years and have not yet been published.

Figure 2 presents the angular (α_{95}) and dating (Δt) uncertainties of all the directions contained in the currently available French database. Concerning the directional accuracy (Fig. 2a), *c.* 90% of the data are defined with α_{95} values <2.0°, while *c.* 70% of the data have α_{95} values <1.5°. Only *c.* 5% of the data have α_{95} values >3.0°. On average, the mean α_{95} value for the archaeomagnetic directions of the French database for dates between AD 1000 and 1500 is *c.* 1.3°. Concerning the age uncertainties (Fig. 2b), *c.* 75% of the data have a date known to lie within a time interval of 200 years ($\Delta t = 100$ years). This proportion decreases to *c.* 60% for a time interval of 150 years ($\Delta t = 75$ years) and to *c.* 35% for a time interval of 100 years ($\Delta t = 50$ years). For this study, we chose to eliminate seven data points with Δt values >150 years (i.e. age ranges >300 years). For the remaining 106 data points, the dating interval of the available results has a mean duration of *c.* 110 years ($\Delta t = 55$ years).

Estimated and schematized DSV curve

The set of 106 archaeomagnetic directions was reduced to a single site, Paris (latitude 48.9° N, longitude 2.3° E), using the virtual geomagnetic poles. Figure 3 shows all the declination and inclination

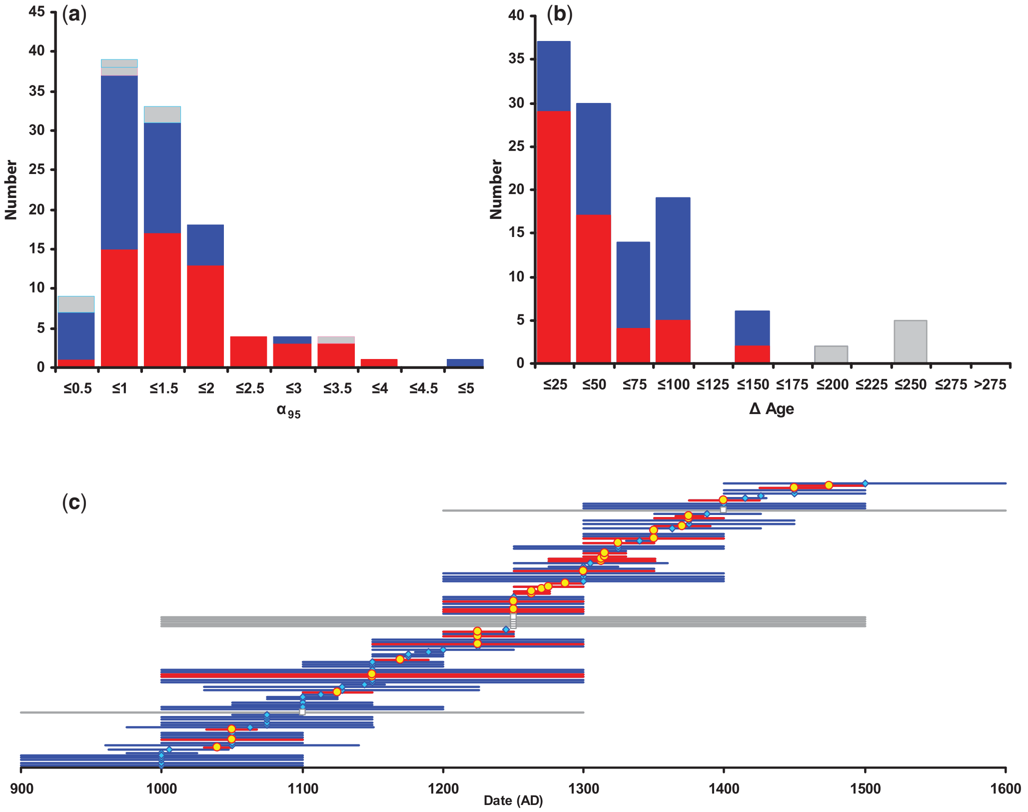

Fig. 2. Description of the 113 directional data points from the French archaeomagnetic database currently available for dates between AD 1000 and 1500. (**a**) Histogram of directional uncertainties (α_{95}). (**b**) Histogram of age uncertainties (Δt). (**c**) Temporal distribution of age intervals of the available data. In red, data reported in Bucur (1994); in blue, new archaeomagnetic data obtained in the laboratory since 1994. Seven results with age uncertainties of >150 years (in grey, in the different panels) were discarded.

data after their reduction to Paris. These data were then used to estimate an average secular variation curve, using the method that relies on sliding windows of variable durations developed by Le Goff *et al.* (2002). The duration of each window was automatically adjusted, following the temporal distribution of the available data, by fixing a minimum density (or weight) of the data in each window (here eight), each datum being weighted by the portion of time intercepted in the window. The minimum duration of the windows was set at 20 years. The windows followed one another with a minimum step of half the duration of the previous (older) one (see the discussion in Le Goff *et al.* 2002). The mean directions of the different time windows were estimated using the bivariate extension of Fisher's (1953) statistics (see Le Goff 1990; Le Goff *et al.* 1992). This method, which illustrates the DSV path as a succession of ovals, is particularly well suited to the problem of secular variation because, in the ideal case of almost no directional and temporal uncertainties, the elongation of each oval is an indicator of the drifting direction of the geomagnetic field through the course of the window's duration. When placed over a longer segment of the DSV curve (i.e. over several centuries), it is therefore expected that the elongation directions of the ovals correspond to the drifting directions of the geomagnetic field. Conversely, a transverse elongation, with respect to the geomagnetic evolution, or even small elongations over a segment of the curve with a significant directional evolution can be seen as either reflecting a problem in the data for the considered period or indicative of geomagnetic complexity during this period.

Such complexity was observed around the second half of the thirteenth century AD in the secular variation paths obtained by Bucur (1994) and Le Goff *et al.* (2002), both studies relying on the same data selection but with the use of sliding windows

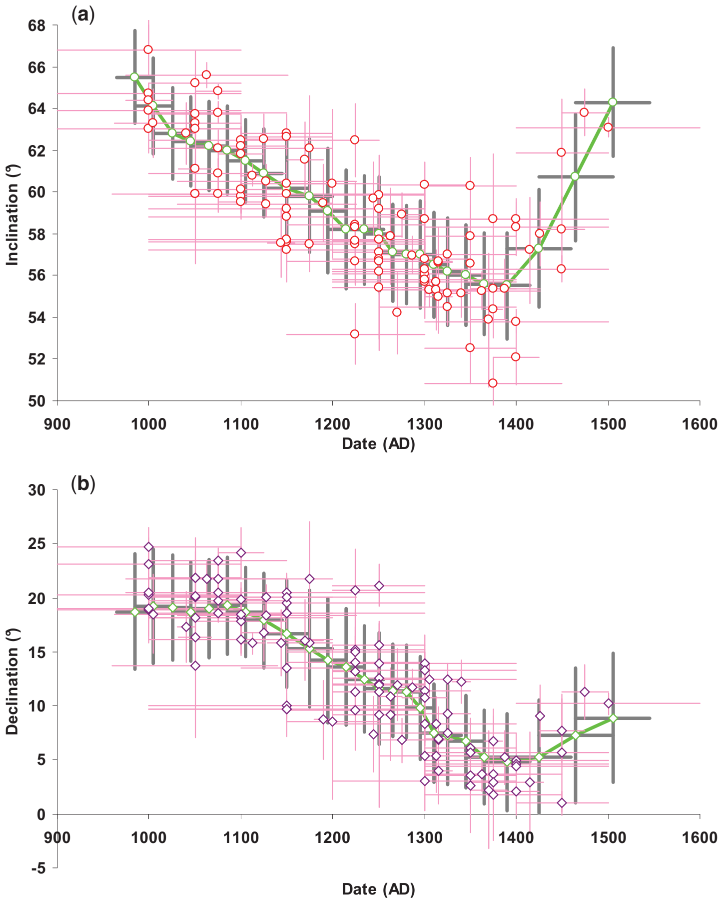

Fig. 3. Declination (**a**) and inclination (**b**) data available from the French archaeomagnetic database for dates between AD 1000 and 1500. The declination and inclination averages (green line) were estimated using the Le Goff *et al.* (2002) method. The associated uncertainties (in grey) are given as values of α_{95} obtained for the different sliding windows.

of the same duration (80 years) in the former and of variable durations in the latter (with a minimum weight of 2.5 and a minimum duration of 20 years). These DSV paths were estimated only on the basis of data reliably dated by the archaeologists (referred to as ‘PC’ in Bucur 1994), thus retaining less than half the available results (i.e. 26 data points between AD 1000 and 1500, instead of the 57 results considered in the present study). We also retained the data dated with less certainty by the archaeologists, as they asked for confirmation of the archaeological dating using archaeomagnetism (note that only the archaeological dating was retained). All individual archaeomagnetic directions were obtained with the same care and, whatever their dating, they equivalently represent the direction of the geomagnetic field that prevailed when the magnetization was acquired in the baked clay.

Logically (and ideally), the representation of directions on a spherical diagram should draw the path of the DSV, which would then only need to be anchored in time. This is fortunately what can be observed in Figure 4, which illustrates all the directions, with the confidence ovals depicting a relatively coherent evolution despite some clearly identifiable remote data. In this figure, the DSV curve was estimated using a minimum weight of 8 and a minimum duration of 20 years for each time window. Note that the choice of the weight was rather delicate (and so could be challenged) but the value used gives enough detail (about five windows per century) without oversmoothing the DSV evolution. In order to better illustrate this point, black and white bars, marking both the direction and ellipticity (the length of the bars) of the ovals, have been added to Figure 4 at the centres of the different confidence

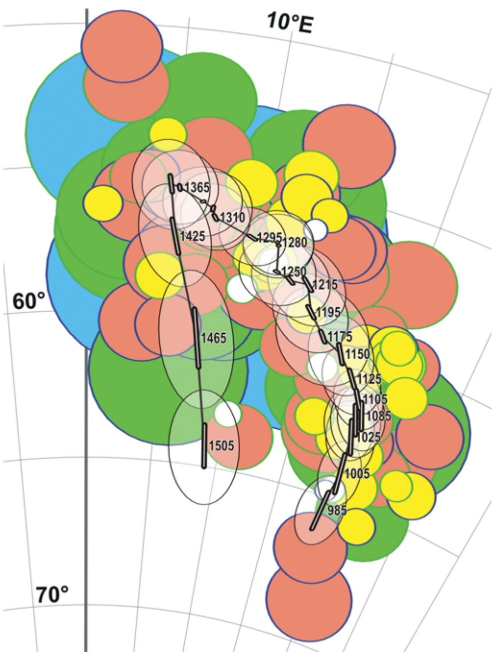

Fig. 4. Directions dated for dates between AD 1000 and 1500 from the French archaeomagnetic database and the associated average secular variation curve, estimated using bivariate Fisher statistics over sliding windows of varying durations (Le Goff *et al.* 2002) (see the text for details). All directions were reduced to Paris. A colour code corresponds to the α_{95} values of the individual directions: white, $\alpha_{95} \leq 0.5°$; yellow, $0.5° < \alpha_{95} \leq 0.9°$; peach, $0.9° < \alpha_{95} \leq 1.5°$; green, $1.5° < \alpha_{95} \leq 2.6°$; blue, $\alpha_{95} > 2.6°$. The corresponding circles are superimposed from largest to the smallest. The 95% confidence ovals of the average directional curve are shown in transparent white. The lengths of the bars at the centres of the ovals give the direction and ellipticity of the different ovals.

ovals. We observe that in the time interval between the thirteenth and fouteenth centuries, unlike the intervals before and after, the elongation of the confidence ovals does not follow the geomagnetic directional drift. This time interval, however, is not marked by an absence of directional variations. This remark is further illustrated in Figure 5a, b, in which two directional branches (for dates between AD 1000 and 1350, and between AD 1350 and 1500) are shown separately, the first being characterized by decreasing inclinations (Fig. 5a) and the second by increasing inclinations (Fig. 5b). In these two diagrams, each individual direction is recorded by a diamond, orientated along the tangent to the curve taken at its central date (here the curve was estimated using a weight of 10), coloured according to the age range assigned by an archaeologist (see the colour code in the figure). The distribution of the directions shows that the thirteenth century period is characterized by more scattered data than the other periods, in a direction transverse to the overall evolution of the secular variation (double arrow in Fig. 5a). Similarly, there is more scatter in the inclinations of directions dated around the end of the fourteenth century (Fig. 5b) where a directional kink occurs (Bucur 1994; Le Goff *et al.* 2002), with individual inclinations around AD 1380 in particular being significantly lower than those of the mean curve (thick white arrow in Fig. 5b).

Because of the absence of archaeological arguments allowing the rejection or correction of individual archaeomagnetic directions dated from the thirteenth and fourteenth centuries AD, which could have resulted from a slight tilting of the structures due to soil compaction, for example, or could have been affected by poorly-established dating, it appears, from the previous observations, that the ‘true’ secular variation path during this time interval might be significantly more complex than the simple and large directional curvature observed between dates of about AD 800 and 1400 (Fig. 4) (see also Bucur 1994; Le Goff *et al.* 2002). The uncertainties associated with the available archaeomagnetic data and those related to the temporal smoothing induced by the computation of the reference DSV curve (as is also the case for all other methods developed so far) make it so that there is no statistical tool available to robustly reveal the exact nature of this (still potential) complexity. The latter should, nevertheless, be included in the 95% confidence intervals of the reference curve. The only way to achieve this is to empirically and approximately estimate it in such a way that as many individual archaeomagnetic directions as possible lie close to a segment of the secular variation path.

In our study, we thus constructed (by hand) a yaw during the thirteenth and fourteenth centuries AD (green curve in Fig. 6), somewhat similar to that previously proposed by Bucur (1986) and which was also built manually, which is discussed below. This yaw was obtained by stretching the mean DSV curve using Bézier curves. This is a reasonable solution but it could just as easily be a loop. In both cases, these evolutions are as possible, in terms of amplitude and duration of variations, as those based on the constraints provided by the gufm1 model (see examples in Fig. 1). Note that this indeterminacy is not critical to the subject of this study.

Method used to analyse the sensitivity of a database

Our approach aimed to illustrate how the accuracy of the reconstruction of the secular variation

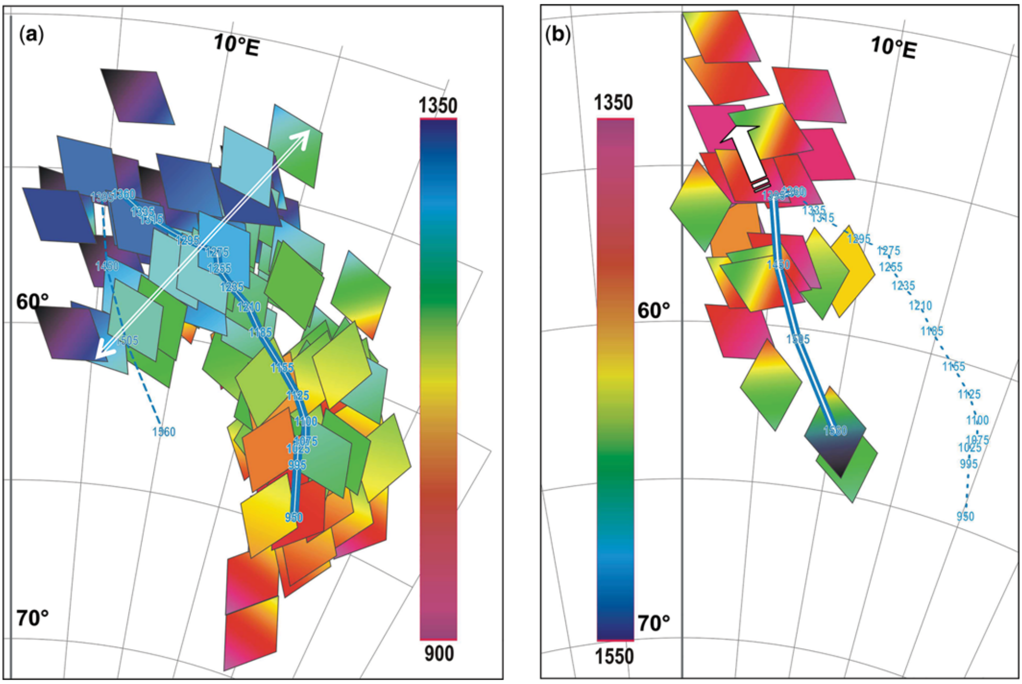

Fig. 5. An alternative representation of the individual archaeomagnetic directions available for the AD 1000–1500 time interval. A distinction was made between two periods: (**a**) AD 1000 and 1350, during which the inclinations decrease; and (**b**) AD 1350 and 1500, during which they increase. Each direction is reported as a diamond orientated along the tangent to the average (blue–white) curve taken at its central date and coloured according to the age range assigned by archaeologists (see the colour codes given in the figure).

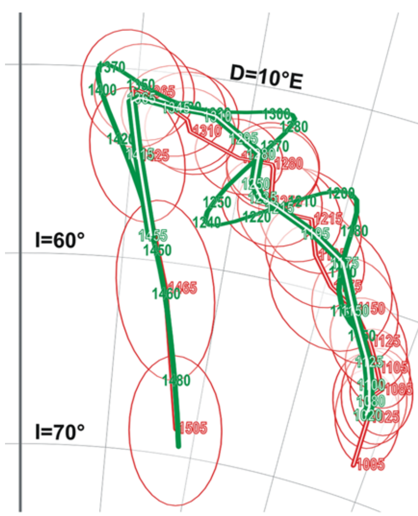

Fig. 6. Estimated and schematized average directional secular variation curves for dates between *c.* AD 1000 and 1500. Red–white curve and associated 95% confidence ovals: secular variation curve estimated using the French archaeomagnetic database (same as in Fig. 4). Green curve: hand-drawn directional evolution,

path was conditioned by: (i) the precision parameters in the direction and age of the individual archaeomagnetic data; and (ii) the parameters of abundance, distribution and temporal homogeneity of the relevant dataset.

To do this, synthetic databases were extracted from the reference DSV curve that included the geomagnetic complexity discussed in the previous section. This consisted of taking sets of directions with realistic age intervals and directional accuracies (i.e. based on those from the available database), or that were arbitrarily fixed. The number of data and their temporal distributions for each dataset were two other parameters that could be adjusted. Synthetic databases were constructed in several ways, by taking directions (declinations, inclinations) from a number

Fig. 6. *Continued.* as considered in the present study, which is compatible with both the available data and the expected field variability (see the discussion in the text). Green–white curve: mean secular variation curve obtained from a dataset directly given by the green curve itself with the same parameters (experimental and dating uncertainties, number and temporal distributions) as those of the real French archaeomagnetic database (see the text).

of dates existing in the real database (i.e. having the same mean ages), or from a number of dates that were regularly or randomly distributed, to which dating and experimental uncertainties were assigned, either according to the real database or in a homogeneous way (i.e. with fixed and constant parameters). The first option based on the French, as well as the British, database was the most realistic case. Other 'ideal' databases were composed of directional data all having identical α_{95} and Δt values evenly distributed over the 500 year interval.

From these synthetic databases, renewed by random draws, DSV curves were calculated by setting, in advance and once and for all, the minimum density of the data in each self-adjusting time window. With a few dozen draws of datasets, a distribution of curves and an average curve were obtained that could then be directly compared to the original DSV curve. In the tests described in the next section, we started with initial parameters close to those of the French database for dates between AD 1000 and 1500 (excluding the data with $\Delta t > 150$ years). We then carried out computations using the characteristics of the British database, with only English and Welsh data, from the same time interval (see Fig. S1 in the Supplementary material). The two databases differed in their mean α_{95} (1.3° for the French database and 3.2° for the British database) and the number of data (about 20 results per century, on average, for the French database and 29 per century for the British database).

Results of the synthetic tests

Effect of dating and experimental uncertainties

The first series of computations relied on the same number (106) of directions, and on the exact same time distribution to those of the French data for dates between AD 1000 and 1500. The synthetic data were produced using a uniform random drawing of a date in each of the dating ranges of the true data, and determination of mean directions derived from the reference DSV curve at the dates previously obtained, randomly drawn from a uniform distribution within the α_{95} confidence circle of the true directions at the considered time intervals. The corresponding α_{95} values and age ranges were then assigned to these new directions. We could thus scrutinize the impact on the reliability of the recalculated curve of an overall improvement or degradation of the parameters α_{95} and Δt by applying a multiplicative factor of <1 or >1, respectively.

The results in Figure 7 are reported in two different ways. First, their spherical projection (upper panels) gives a direct view of the synthetic curves (in blue), their variability and their average behaviour (curve in yellow). Second, results are also presented using a more quantitative approach by considering the mean angular departures of the synthetic curves from the reference DSV path as a function of time (lower panels). Note that positive and negative values refer to mean departures outside and inside the reference curve. Figure 7a shows that the realistic yaw introduced into the secular variation curve was practically smoothed out by the process of calculating the mean directions. More generally, only a small inflection is observable around AD 1250, as is also the case when the computations are carried out using the exact parameters of the real French archaeomagnetic database (green–white curve in Fig. 6). Similarly, the extremity of the directional kink at around AD 1350 was almost never reached. These differences lead to significant mean angular departures up to 2° between the synthetic curves and the reference curve. The Δt values had to be significantly reduced (by a factor of 5), until they reached an average of 11 years ($0 < \Delta t < 30$ years), to faithfully and almost systematically reproduce the western extremity of the yaw, but the short undulation around AD 1200 remained undetected (Fig. 7b). When the α_{95} values were chosen to be higher on average (value of 3.0°), the directional curves appeared more scattered and irregular (Fig. 7c, d). Their average (Fig. 7) led to the same observations as previously, but the most important element here is the variability between the individual curves, each estimated using a selected dataset. We observed that increasing the α_{95} mean value can result in a curve rather different from the reference path, even when a mean Δt of 11 years was considered (Fig. 7d). To first order, this observation is also attested by the averaging of the one-sigma errors of the angular departures estimated for dates between AD 1000 and 1500: (a) 0.34°, (b) 0.30°, (c) 0.62° and (d) 0.68°. This illustrates the fact that, although the impact of age-related uncertainties on the reliability of the estimated secular variation curve is strong, the impact of experimental errors cannot be neglected either.

In a second series of computations (see Fig. S2 in the Supplementary material), we showed that the previous observations remained unchanged when the time distribution and number (145) of the English and Welsh archaeomagnetic data were considered (Batt *et al.* 2017). The only difference concerns the undulation around AD 1200, which appeared to be better resolved with a mean Δt of 11 years than in the French case but only when the directional accuracy was decreased, on average, to 1.0°, thus close to the French value (see Fig. S2d in the Supplementary material). This also underlines the effects due to the number and age distribution of the data (see below).

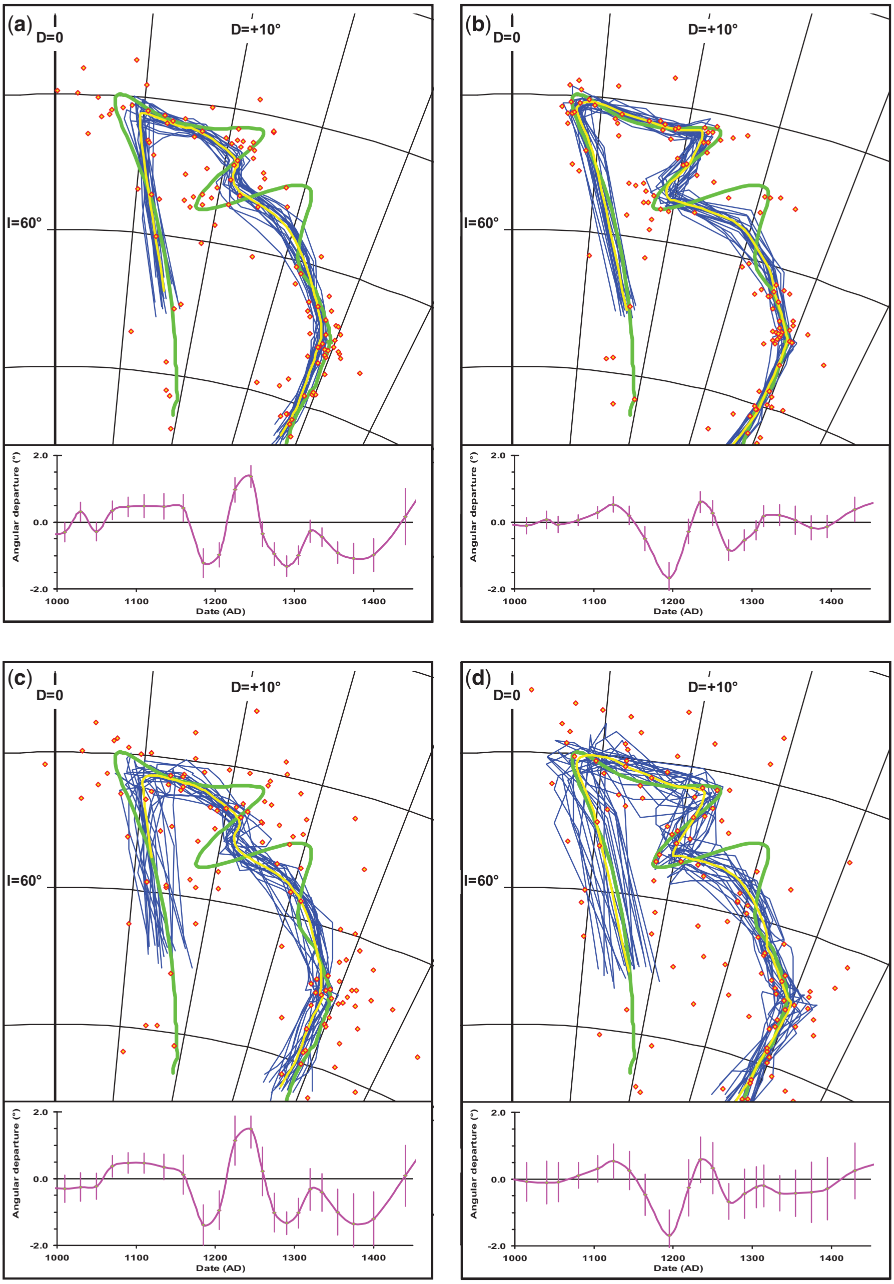

Fig. 7. Effects of the experimental and age uncertainties on the recovery of the directional secular variation curve. Twenty synthetic datasets were randomly selected from the real French archaeomagnetic database and used to sample the reference secular variation curve that exhibits a directional yaw in the thirteenth and fourteenth centuries AD. (Upper panels) Spherical projections of the results and of the reference DSV curve (in green). The red dots show the randomly selected directions of one of the 20 datasets. The directional curves obtained using the Le Goff *et al.* (2002) method are reported in blue, their average is in yellow. (**a**) Mean $\alpha_{95} = 1.3°$, mean $\Delta t = 55$ years; (**b**) mean $\alpha_{95} = 1.3°$, mean $\Delta t = 11$ years; (**c**) mean $\alpha_{95} = 3.0°$, mean $\Delta t = 55$ years; and (**d**) mean $\alpha_{95} = 3.0°$, mean $\Delta t = 11$ years (see the text for further explanations). (Lower panels) Corresponding mean angular departures between the synthetic curves and the reference DSV path. Positive (negative) values indicate when the differences are, on average, outside (inside) the reference curve. Vertical bars show the one-sigma errors of the mean angular departures.

We then considered a more idealized situation with, on the one hand, a uniform distribution of the mean dates of the data over 500 years and, on the other hand, constant values of α_{95} and Δt for all data. In a first case, we considered a distribution of 170 data or 34 data per century (Fig. 8). When all Δt were 55 years, the yaw in the secular variation was completely smoothed out, whether the α_{95} values were fixed at 1.3° (Fig. 8a) or 3.0° (Fig. 8c). As in the previous calculations, Δt had to be reduced by a factor of 5 to reveal the complexity in the secular variation path, including the undulation around AD 1200. In the same way, Figure 8c, d also indicates that a better data accuracy increases the reliability of the DSV reconstructions (note that the averaged one-sigma errors of the angular differences are: (a) 0.30°, (b) 0.25°, (c) 0.41° and (d) 0.55°).

Effect of data density

To illustrate this aspect, an idealized situation was again considered, with a uniform temporal distribution of the data, but varying their number between 170 and 50, and by fixing their mean α_{95} and Δt at 1.3° and 11 years, respectively. The first case with 170 values was identical to the one shown in Figure 8b, where the yaw in the secular variation path is accurately depicted (Fig. 9a). When this number was reduced to 100 values (20 data per century), the yaw was always observable but its amplitude was less than that of the reference curve (Fig. 9b). By reducing the number to 50 data (Fig. 9c) (i.e. 10 data per century), the yaw became completely hidden, and most of curves showed a smooth evolution similar to that observed when considering the parameters of the French archaeomagnetic database (106 data, with a variable temporal distribution of the data, a mean α_{95} of 1.3° and a mean Δt of 55 years). This indicates that, in this case, the number of data and the values of Δt acted in a very similar way on the recovery of the secular variation path. As before, Figure 9d further shows that this observation was not fundamentally modified when the accuracy of the individual data was reduced (with a mean α_{95} of 3.0°). As previously, the accuracy of the data had an impact on the regularity and variability of the individual curves, more than on the overall evolution of the secular variation that the curves collectively defined (yellow curve in Fig. 9c, d).

Discussion and concluding remarks

The present study involved an experimental illustration to demonstrate the characteristics that an archaeomagnetic database should possess to recover detailed directional variations of the regional geomagnetic field. Our synthetic tests underlined the fact that the crucial parameter for constructing a detailed secular variation curve is the precision in the dating of the data. Although less critical, the experimental data accuracy (defined by a mean α_{95}) has, nevertheless, a significant and obvious impact on the reliability and precision of DSV reconstructions.

Another fundamental element is the intrinsic limitation of archaeomagnetism for DSV reconstruction. Using the gufm1 historical geomagnetic field model (Jackson *et al.* 2000), which in our case provided realistic information on possible secular variation paths, we showed that the dating accuracy of the archaeomagnetic data required to describe the directional variability largely and very generally exceeded the accuracy of the archaeological dates available for the studied archaeological structures (i.e. the dating of the last use of the kilns). A dating accuracy of *c.* ±10 years, on average, for the archaeomagnetic data appeared necessary to correctly recover the directional geomagnetic field variations. More in detail, however, it should be recalled that even with such a good mean accuracy, the French database would still not allow the recovery of a short undulation like the one simulated around AD 1200 mainly because of the age distribution of the available data. In any case, this dating precision is not realistic, given the possibilities offered by archaeology. Furthermore, our synthetic tests also indicated that a larger number of data would not compensate for and/or significantly improve the reconstructions based on results that were not precisely dated (even with a mean Δt of *c.* 50 years: Fig. 9).

From our tests, we found that, given the nature of the archaeomagnetic data used to calculate a mean secular variation curve (i.e. a process that necessarily generates a temporal smoothing), it is very difficult, if not impossible, to precisely constrain the secular variation path that may have occurred over a timescale of one or two centuries. The apparent absence of geomagnetic complexity observed over this timescale does not prejudge its actual absence or occurrence. In other words, our knowledge of the regional secular variation, based on archaeomagnetic data, is necessarily limited, with an incompleteness that cannot be estimated even if the data are precisely experimentally determined and available in large numbers (several dozen data points per century). Our study, nevertheless, indicates that the use of the bivariate Fisher statistics for constructing secular variation curves (Le Goff *et al.* 1992, 2002) gives some clues about possible undetected, rapid directional fluctuations.

These findings should not be seen as a negative surprise (see the discussion in Pavón-Carrasco *et al.* 2011). Our tests have mainly illustrated a situation that is often neglected, and which is not without echo with the spatial and temporal resolution of the

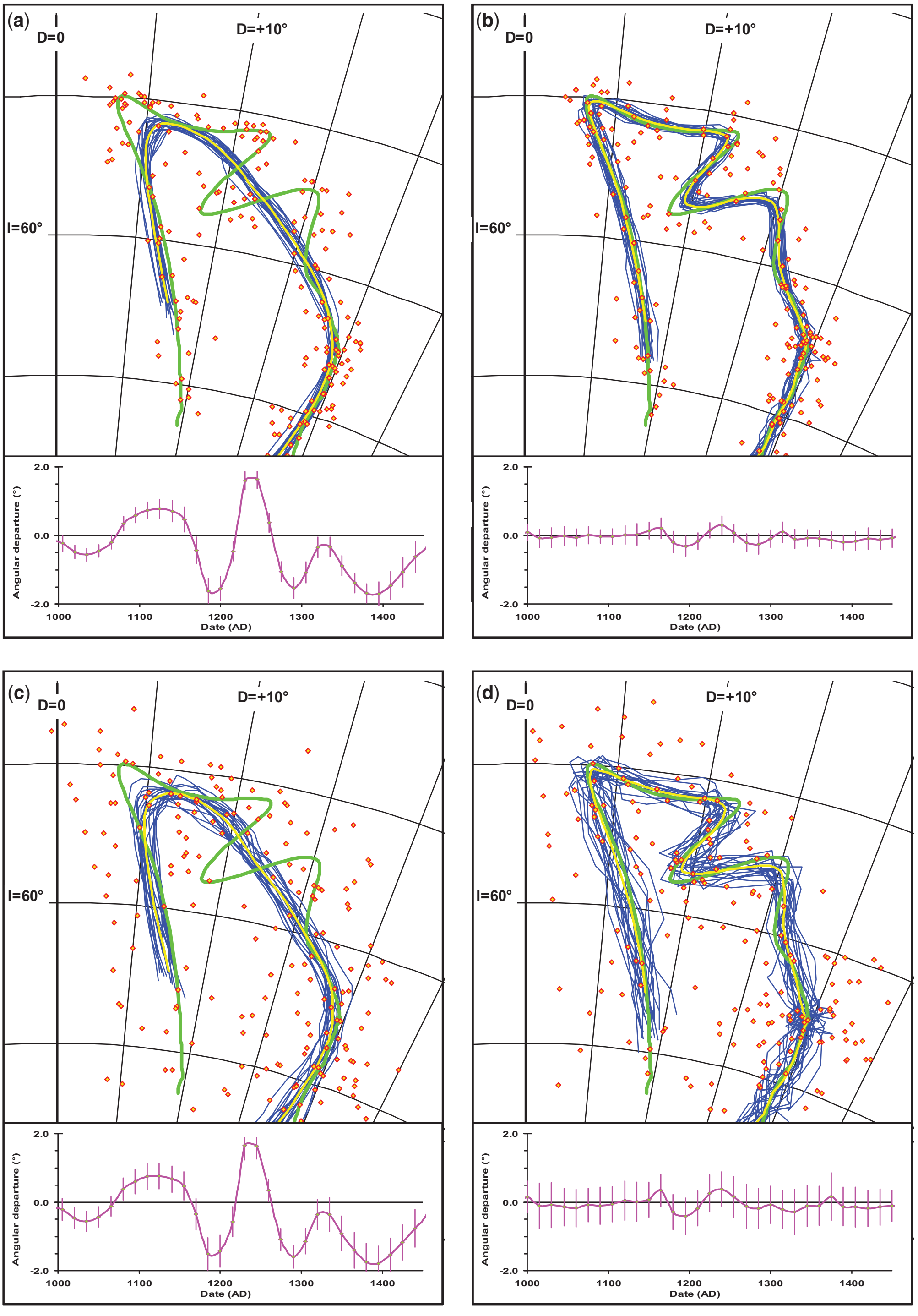

Fig. 8. Effect of the time distribution of the data on the recovery of the directional secular variation curve. Twenty synthetic sets of 170 data points with uniform time distributions were considered inside the AD 1000–1500 interval. Constant values of α_{95} (**a** & **b**: 1.3°; **c** & **d**: 3.0°) and Δt (**a** & **c**: 55 years; **b** & **d**: 11 years) were considered for each diagram. The same legend as in Figure 7.

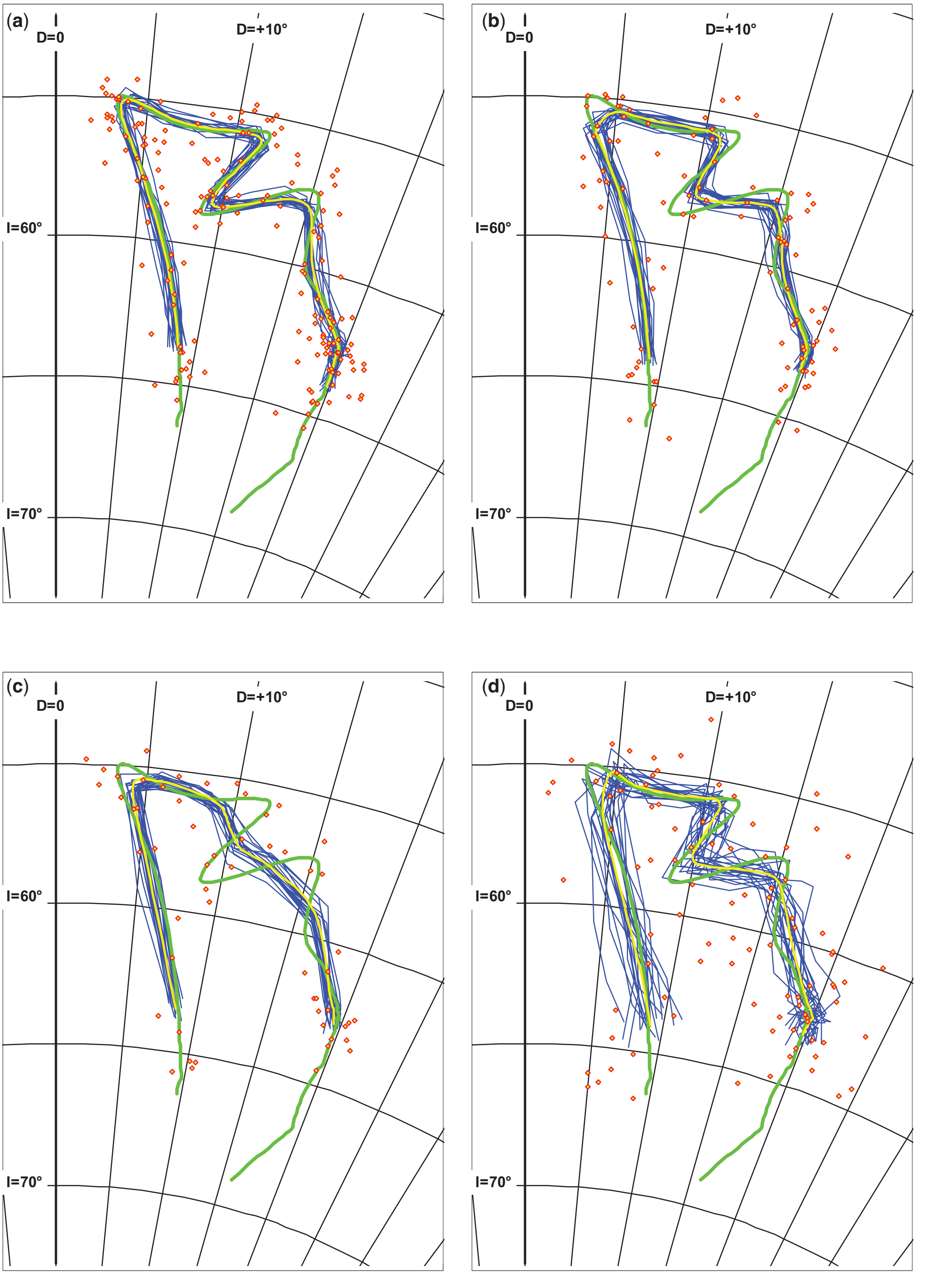

Fig. 9. Effect of the number of data (N) on the recovery of the directional secular variation curve. For each dataset considered, the time distribution of the N data are uniform inside the AD 1000–1500 interval, and they have constant values of α_{95} and Δt (the latter is 11 years for the four diagrams). Same conventions as in Figure 7. (**a**) $\alpha_{95} = 1.3°$, $N = 170$; (**b**) $\alpha_{95} = 1.3°$, $N = 100$; (**c**) $\alpha_{95} = 1.3°$, $N = 50$; and (**d**) $\alpha_{95} = 3.0°$, $N = 100$.

global archaeomagnetic field reconstructions (e.g. Hongre *et al.* 1998; Korte *et al.* 2009 and references therein). Licht *et al.* (2013) and Sanchez *et al.* (2016) showed that, given the precision of the data and their spatial and temporal distributions, the global archaeomagnetic database (including both archaeological and volcanic data) available for the last three millennia does not allow reliable global field reconstructions beyond spherical harmonics of degrees 4–5. Note that this corresponds to the optimal case, for which there are no outliers in the archaeomagnetic database, otherwise the resolution would be less. It seems likely that the Gauss coefficients above degrees 3–4 could generate fluctuations of the same nature as the yaw analysed in our study, which would provide first-order information on the geomagnetic resolution of regional archaeomagnetism.

Finally, the implications of our results on the archaeomagnetic dating method should be stressed (see also the discussions in Le Goff *et al.* 2002 and Pavón-Carrasco *et al.* 2011). This method is based on a reference secular variation curve that is therefore limited in resolution, and is possibly incomplete. The basic principle of an archaeomagnetic dating relies on the comparison between a snapshot of the geomagnetic field (as provided by an individual archaeomagnetic direction), revealing all its possible complexity, and a smoothed secular variation curve, the error bars of which give relatively little information on the detailed, century-scale secular variation path. This leads to the ambiguities inherent in the archaeomagnetic dating method, which require further methodological developments that exceed the aim of the present study.

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Author contributions **MLG**: conceptualization (equal), formal analysis (equal), writing – original draft (equal); **YG**: conceptualization (equal), formal analysis (equal), writing – original draft (equal).

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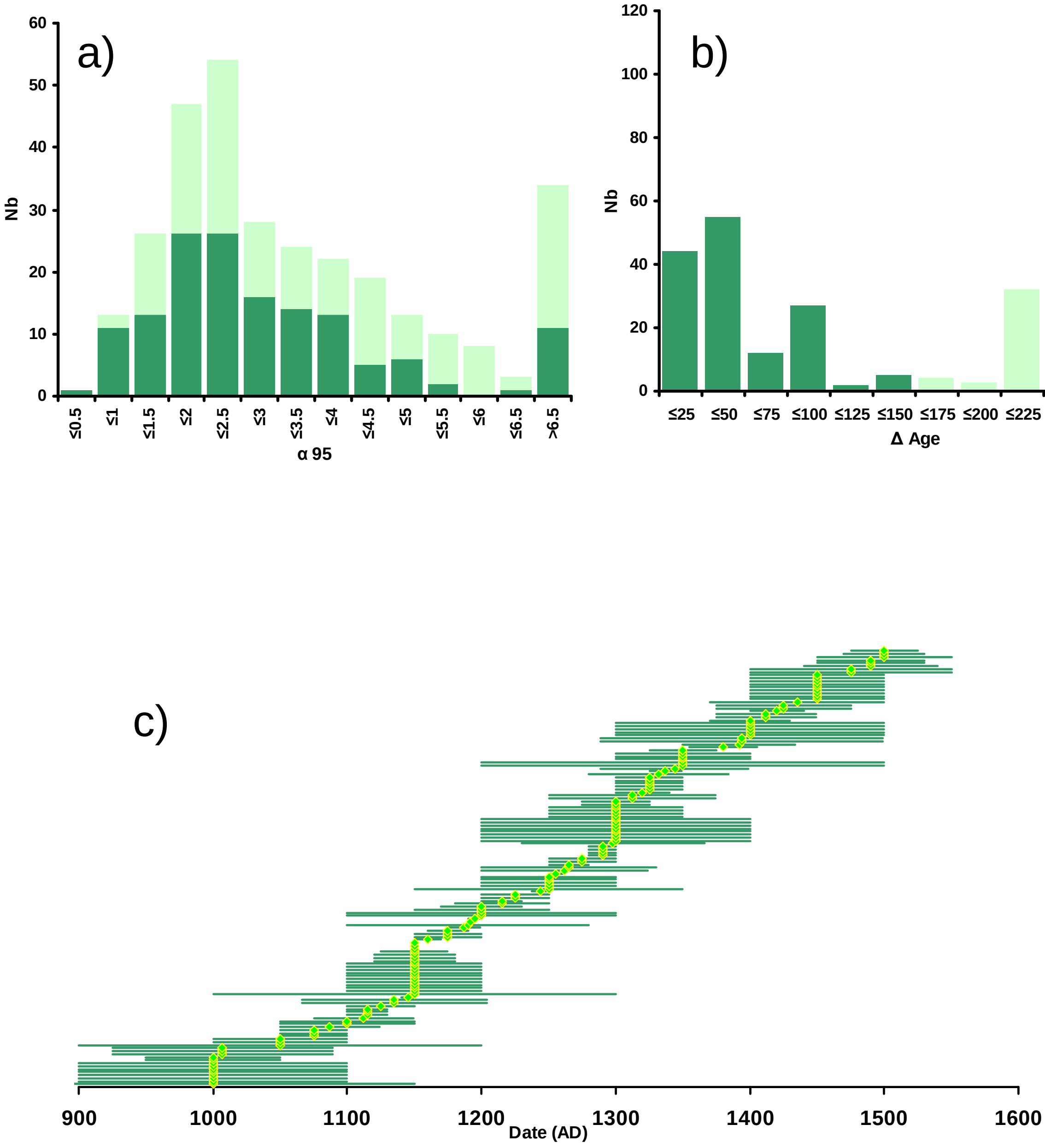

Fig. S1. Description of the 303 directional results dated between 1000 and 1500 AD obtained from England and Wales (Batt *et al.* 2017).
a) Histogram of experimental uncertainties (α_{95}). Pale green, all data; dark green, only data with Δt ≤ 150 years.
b) Histogram of age uncertainties (Δt) for all available data. Pale green, data with Δt > 150 years; dark green, data with Δt ≤ 150 years.
c) Temporal distribution of age intervals of the available data with Δt ≤ 150 years.

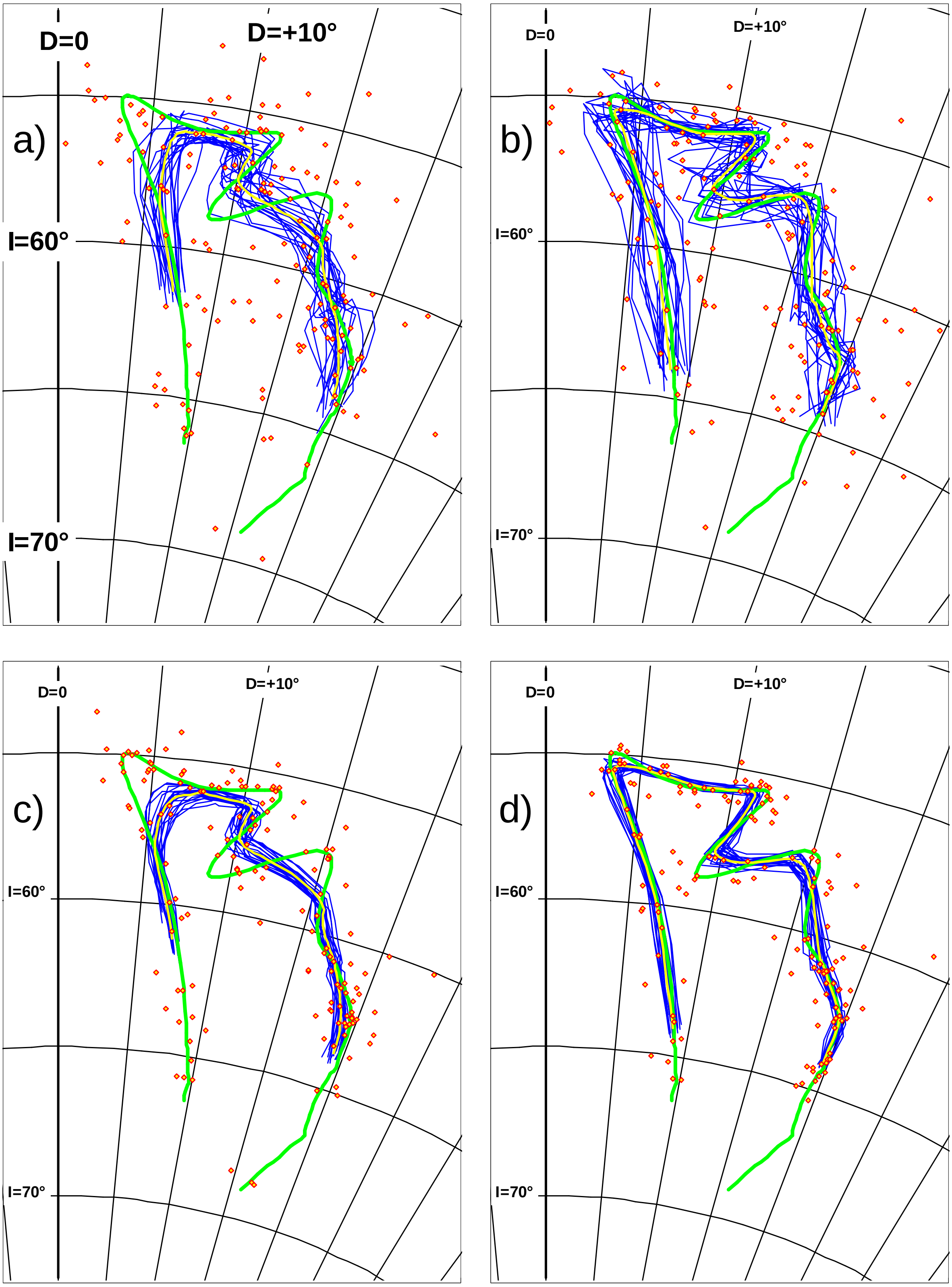

Fig. S2. Secular variation curves (in blue) obtained from 20 synthetic directional datasets randomly selected from the real British (England and Wales) archaeomagnetic database (Batt *et al.* 2017). All directions were transferred to the latitude of Paris. (a) Mean α_{95}=3.2°, mean Δt=53 years, (b) Mean α_{95}=3.2°, mean Δt=11 years, (c) Mean α_{95}=1.0°, mean Δt=53 years, (d) Mean α_{95}=1.0°, mean Δt=11 years. Same legends as in Fig. 7.