Morphodynamics of superimposed bed forms in a cellular automaton dune model

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Asymptotic behaviours of different sets of partial differential equations

$$\frac{\partial h}{\partial t} = F(q, S, ...)$$
$$\frac{\partial q}{\partial x} = G(h, S, ...)$$

Scaling

A cellular automaton approach

- A grid of specified shape.
- A collection of cells with different states.
- Rules based on the states of neighboring cells.

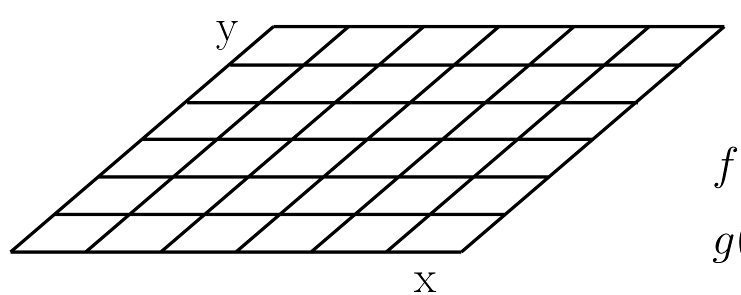
Emergence

Traditional cellular automatons in geophysics

Emergence ?

Scaling ?

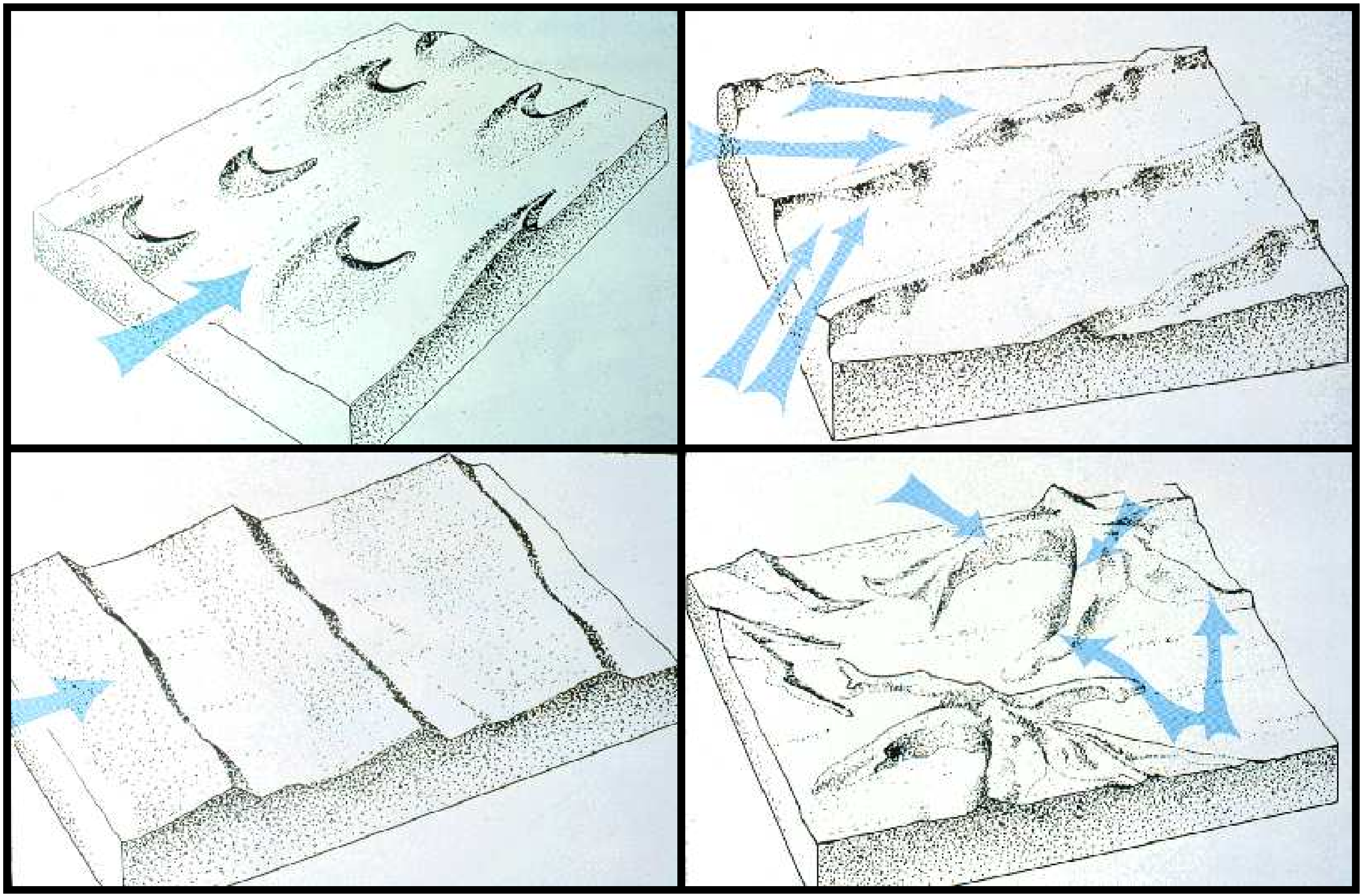
$x \in \mathbb{N}$
$y \in \mathbb{N}$
$\Delta t = cte$

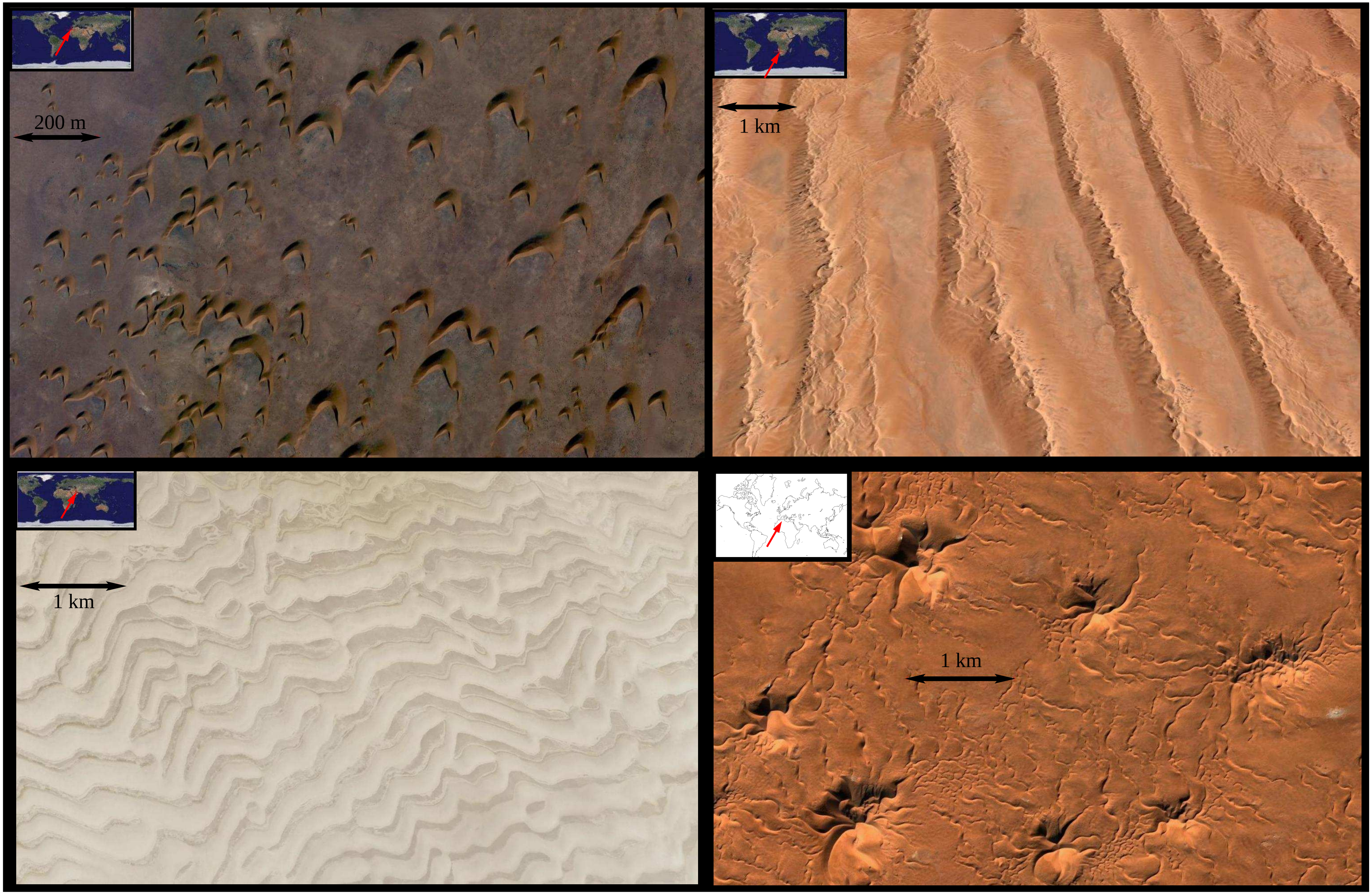


$$f(h(x,y,t), q(x,y,t)) \longrightarrow h(x,y,t+\Delta t)$$
$$g(h(x,y,t), q(x,y,t)) \longrightarrow q(x,y,t+\Delta t)$$

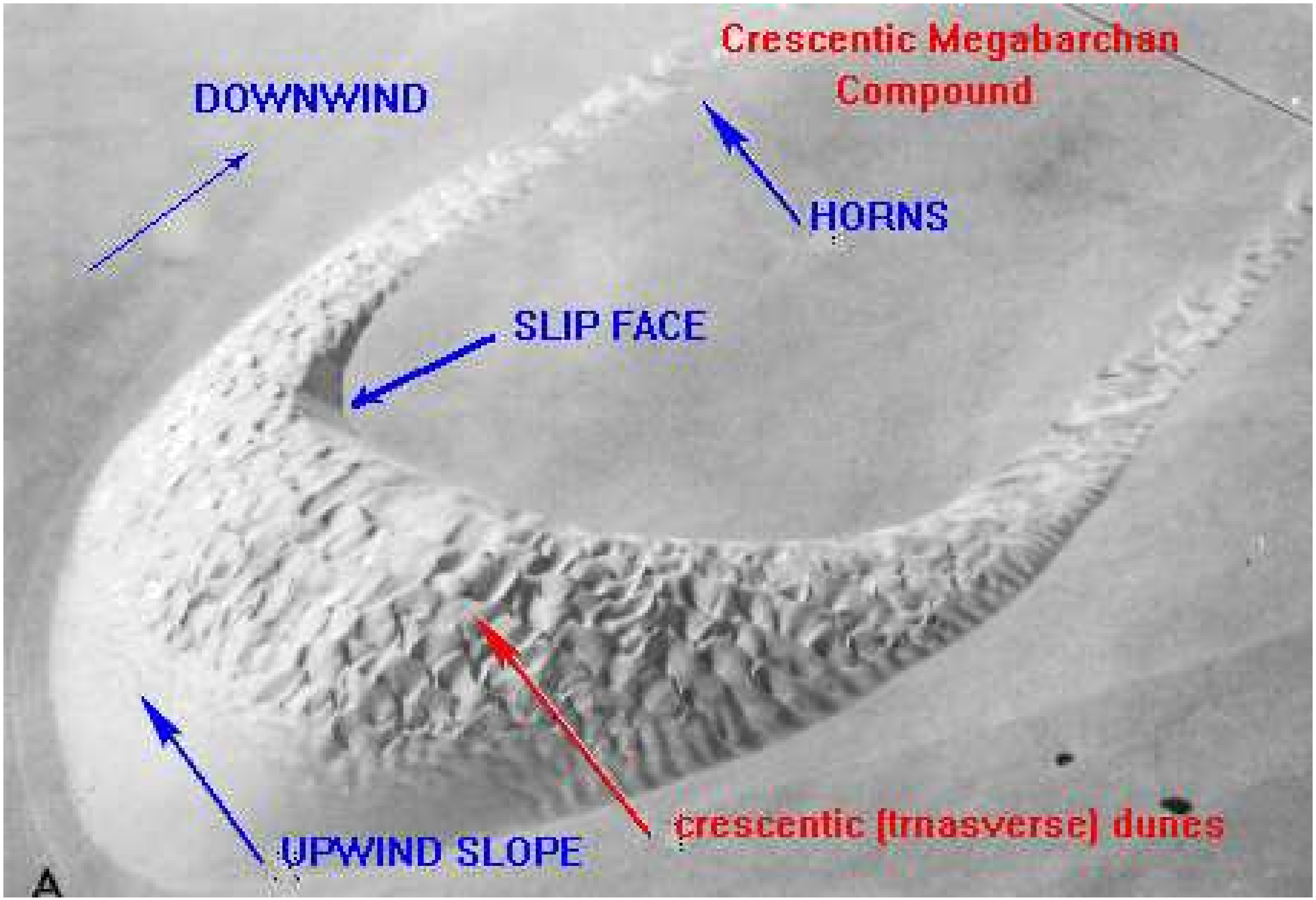
Let us develop new and more theoretical CAs to combine more efficiently scaling and emergence

Different types of sand dunes

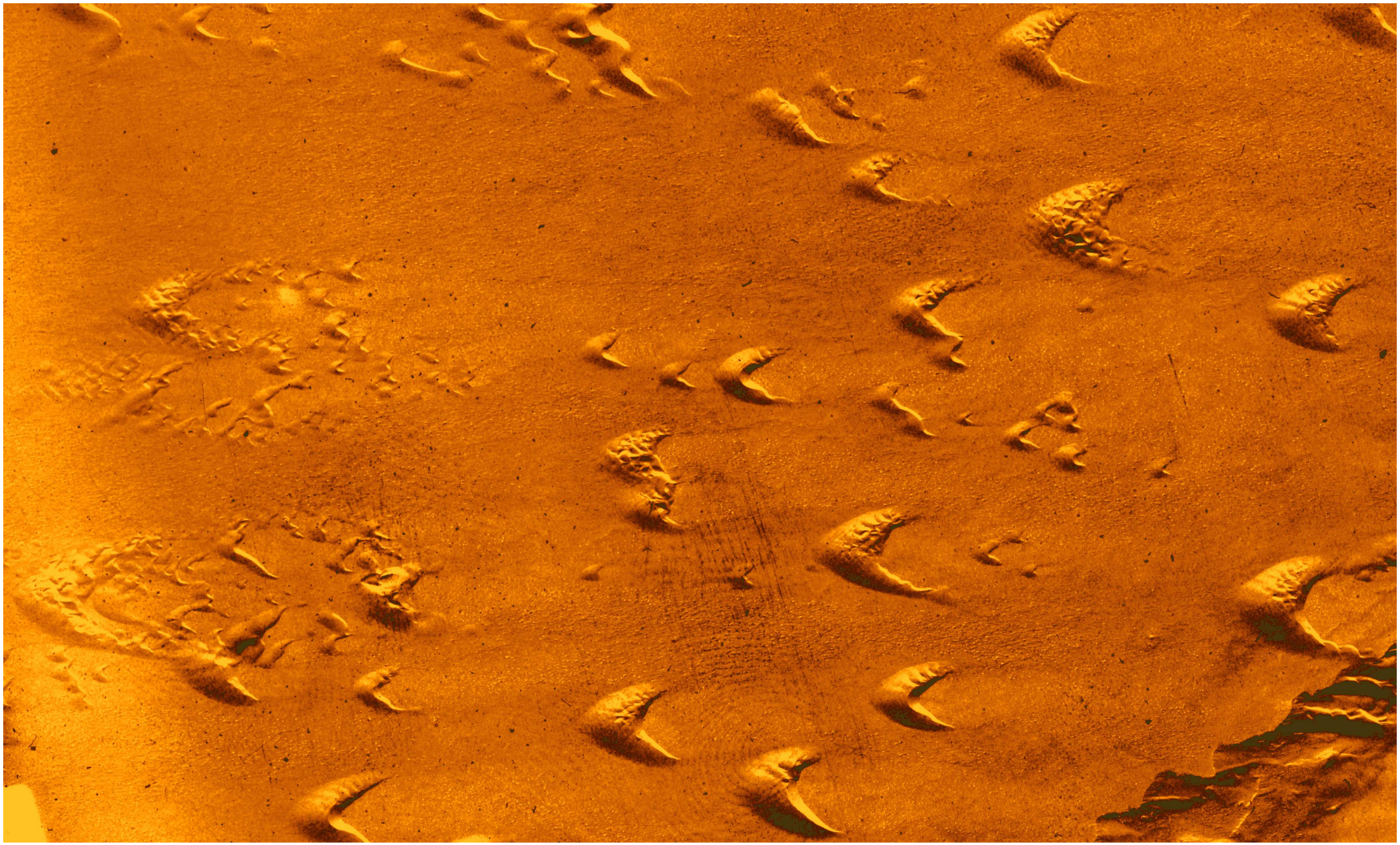


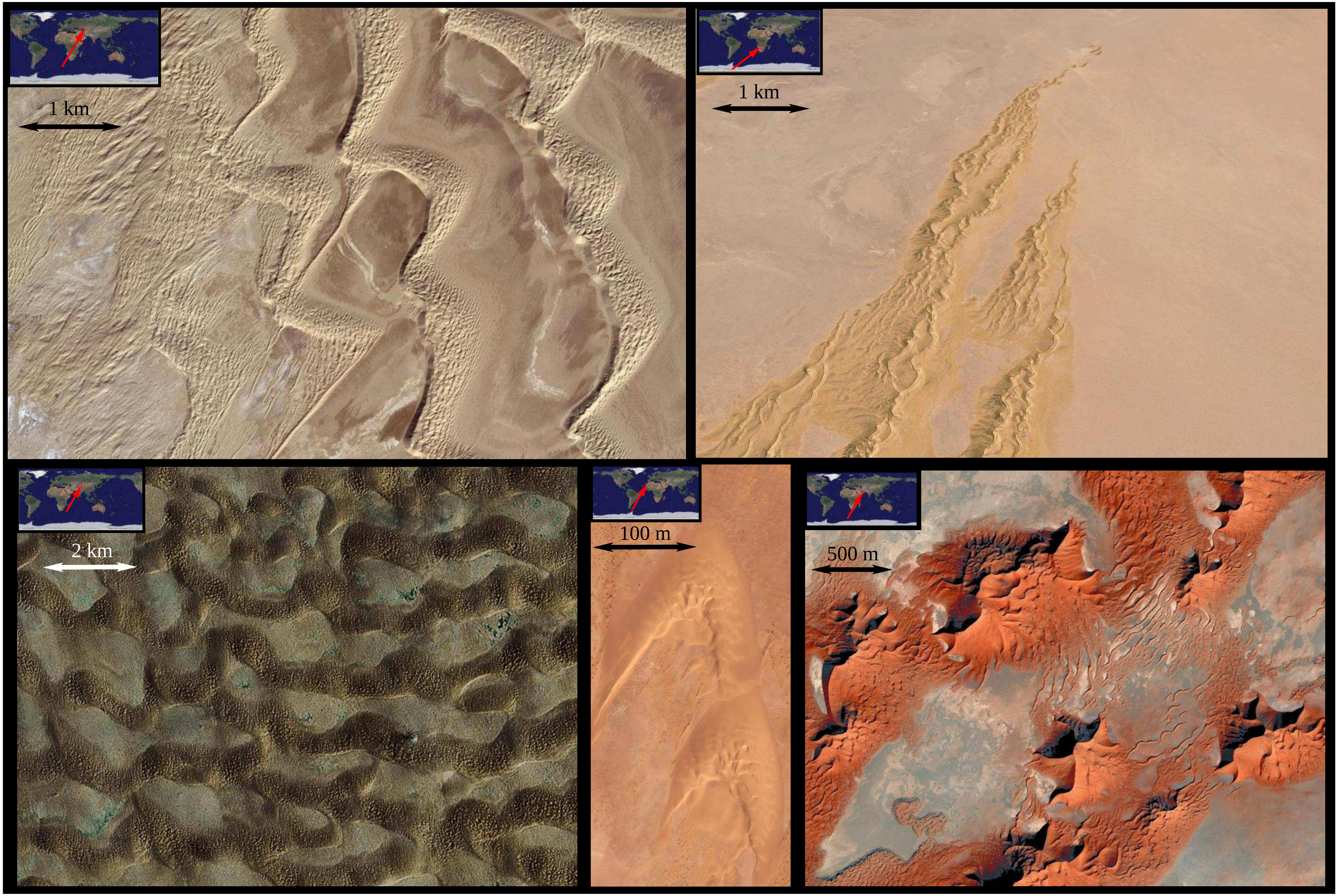

200 m
1 km
1 km
1 km

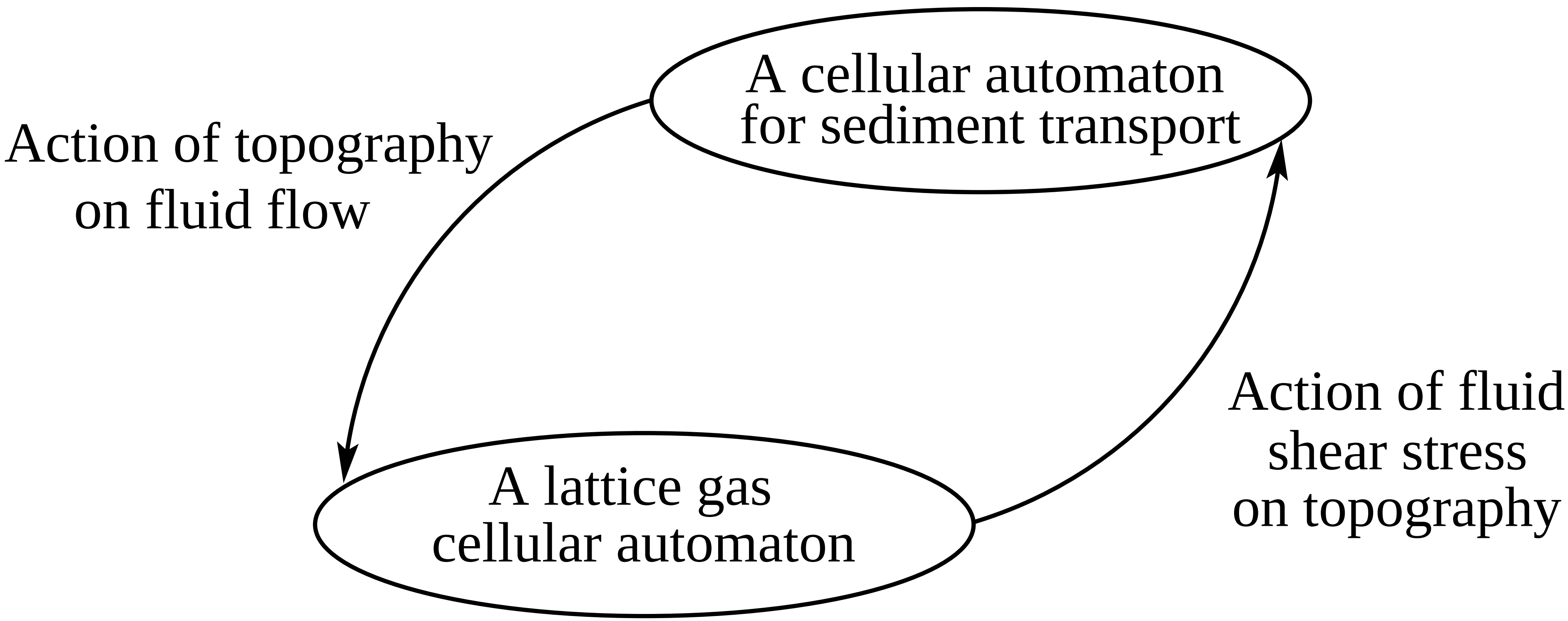
Superimposed dune patterns

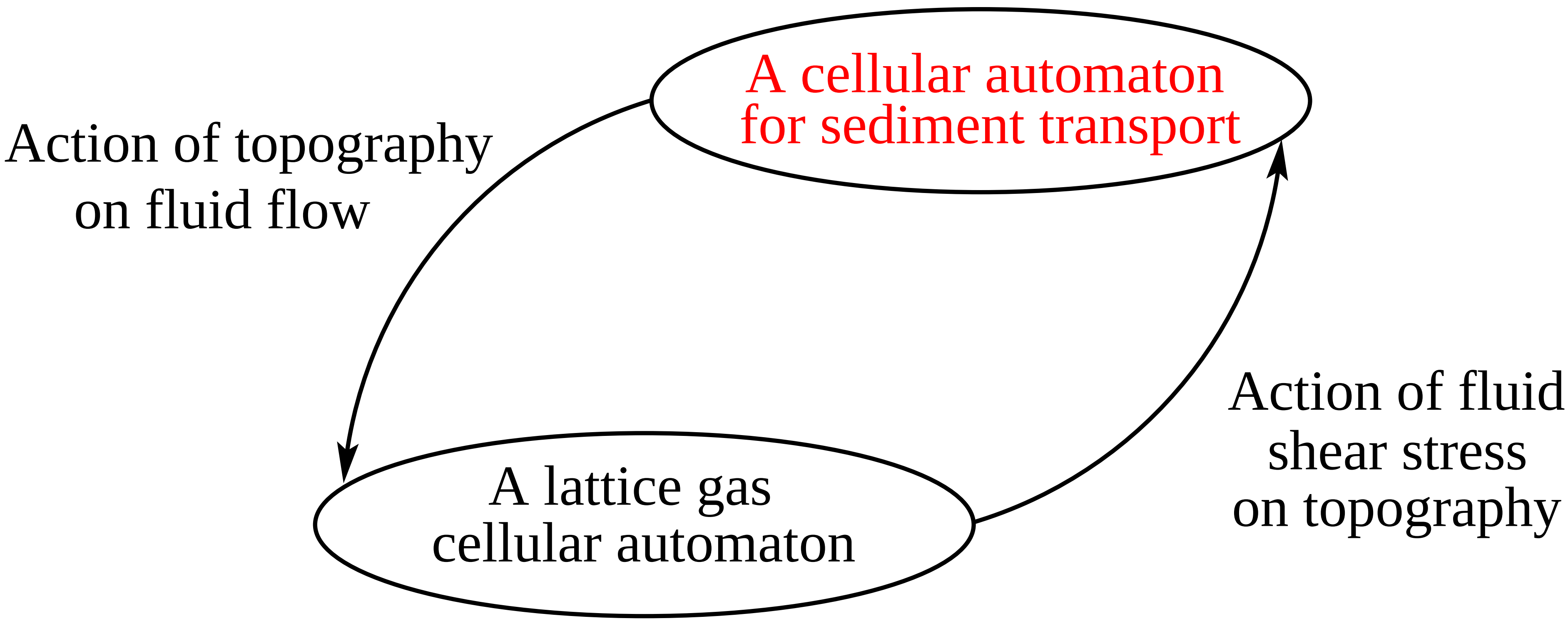


Superimposed dune patterns




1 km
1 km
2 km
100 m
500 m


A cellular automaton
for sediment transport
Action of topography
on fluid flow
A lattice gas
cellular automaton
Action of fluid
shear stress
on topography



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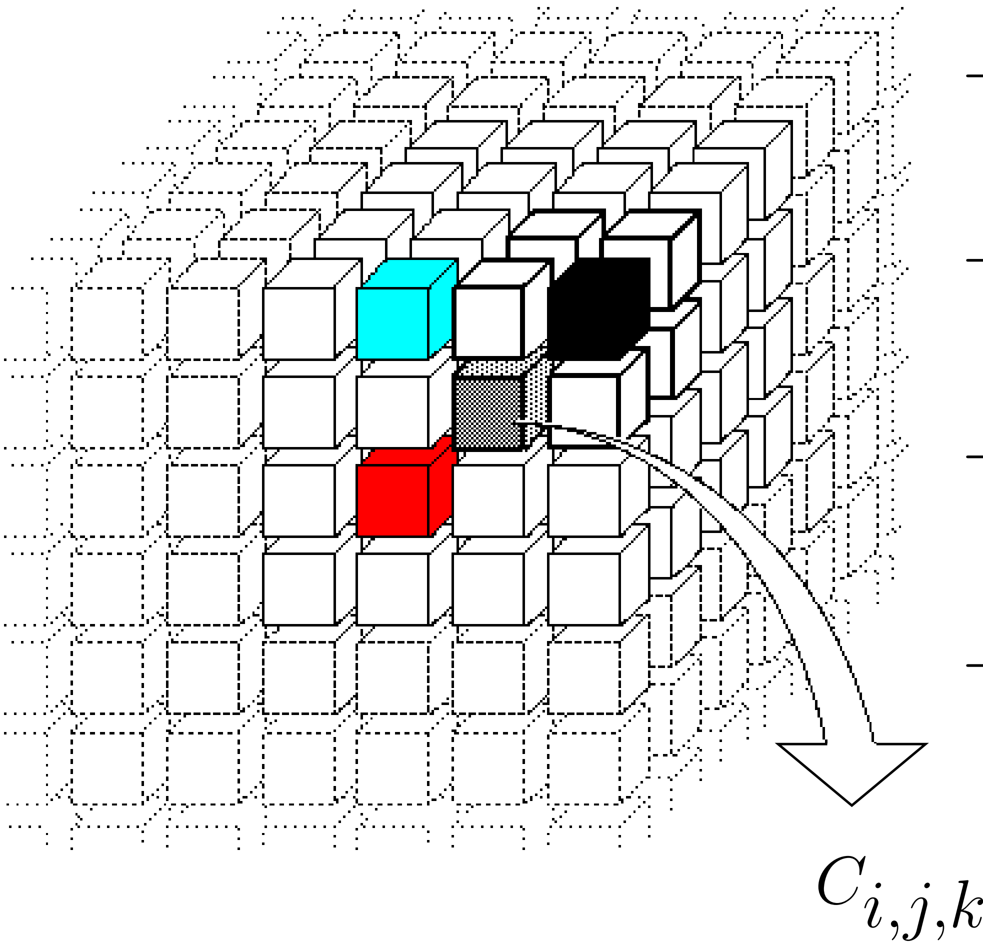
Cellular automaton for sediment transport



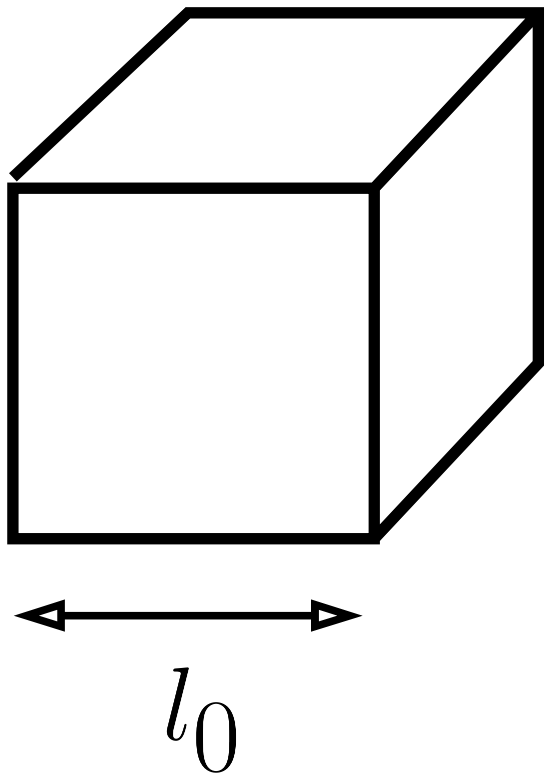
→ A 3 dimensional model.

→ State variables.

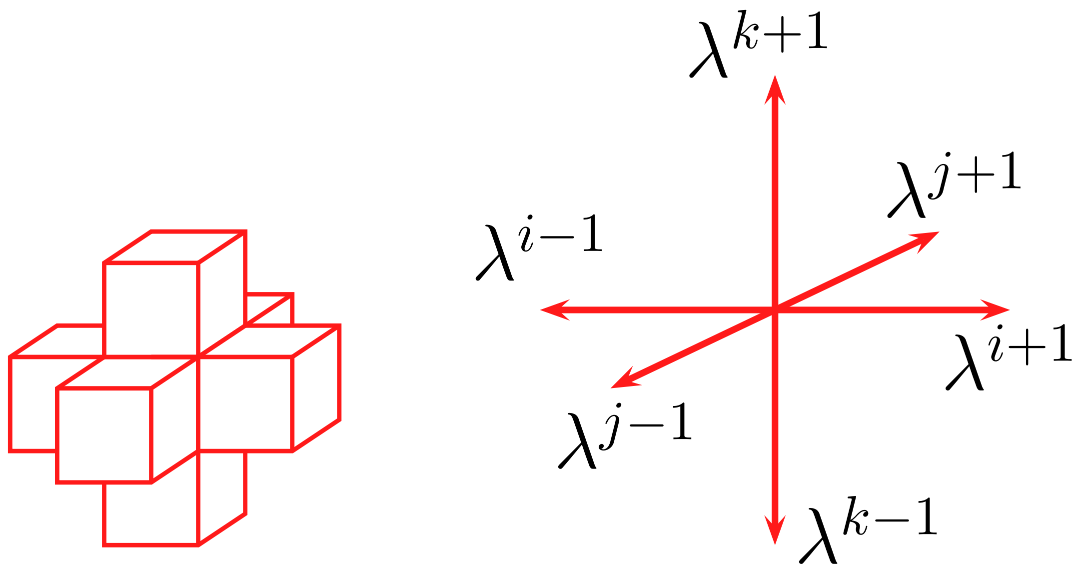
→ Nearest neighbour interactions.

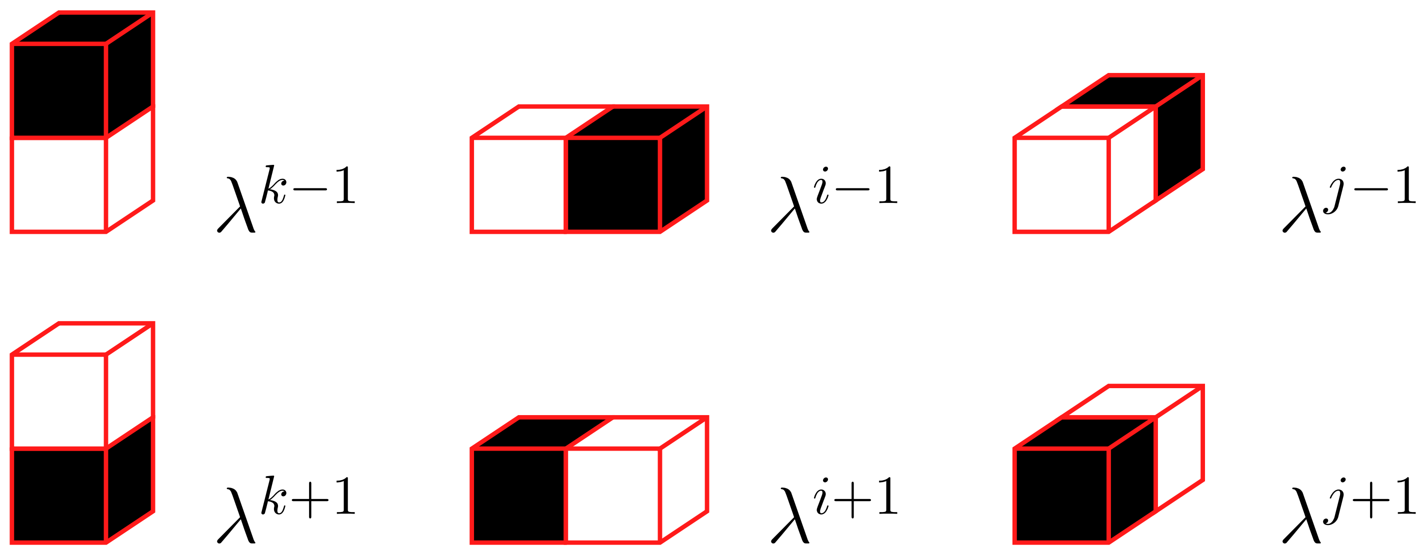
→ Individual physical processes.

An elementary length scale

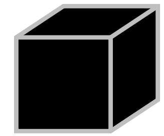
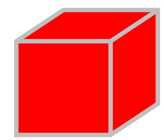
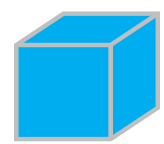


Nearest neighbour interactions

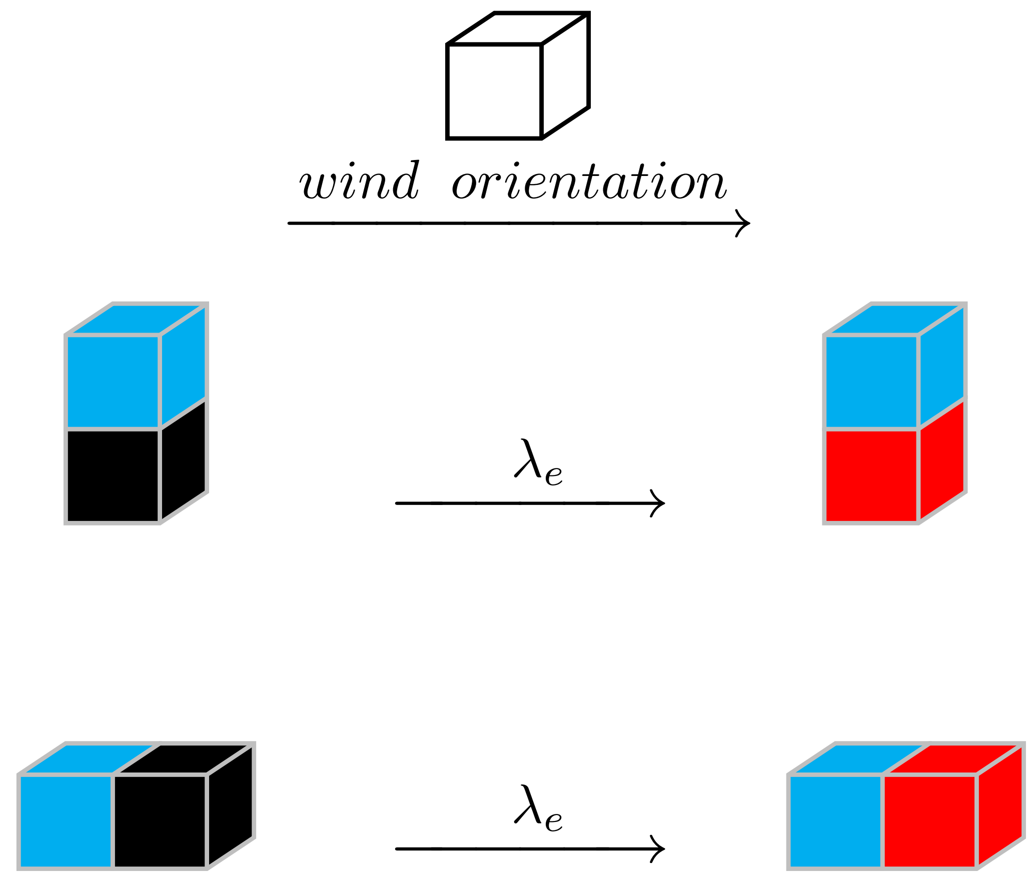




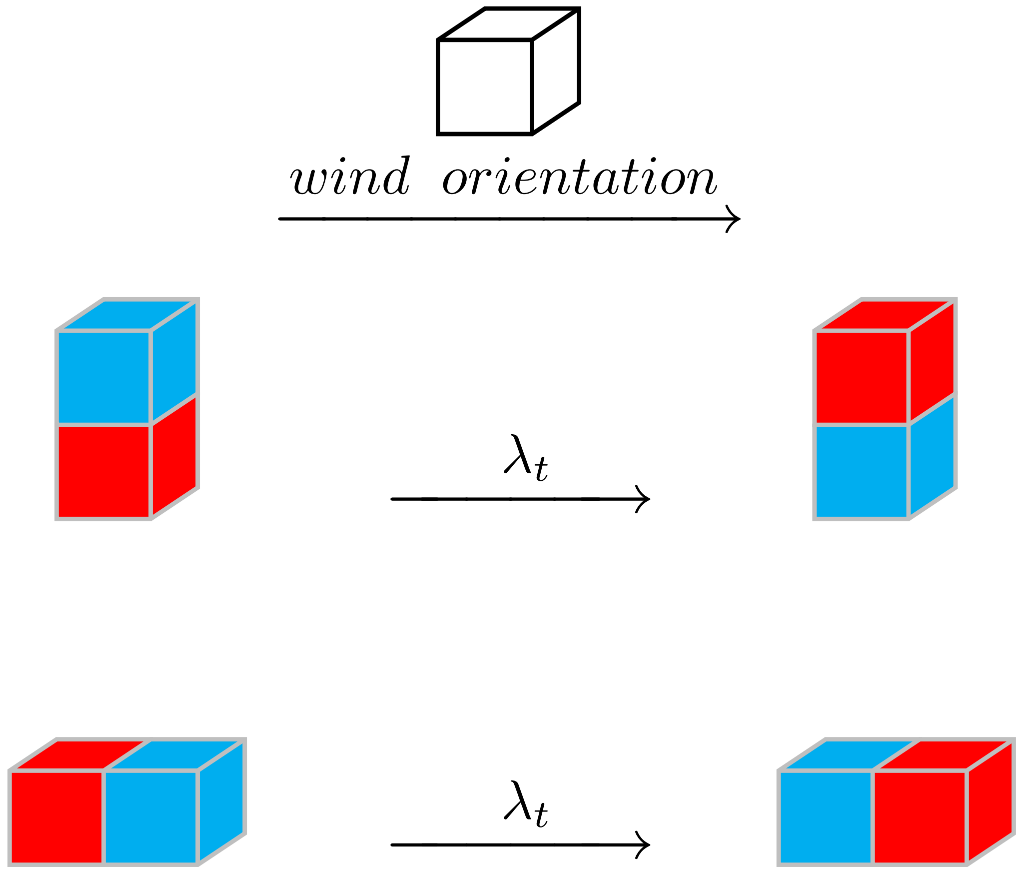
Physical states

Granular material	 Grain
Mobilized granular material	 Mobilized grain
Fluid flow	 Fluid

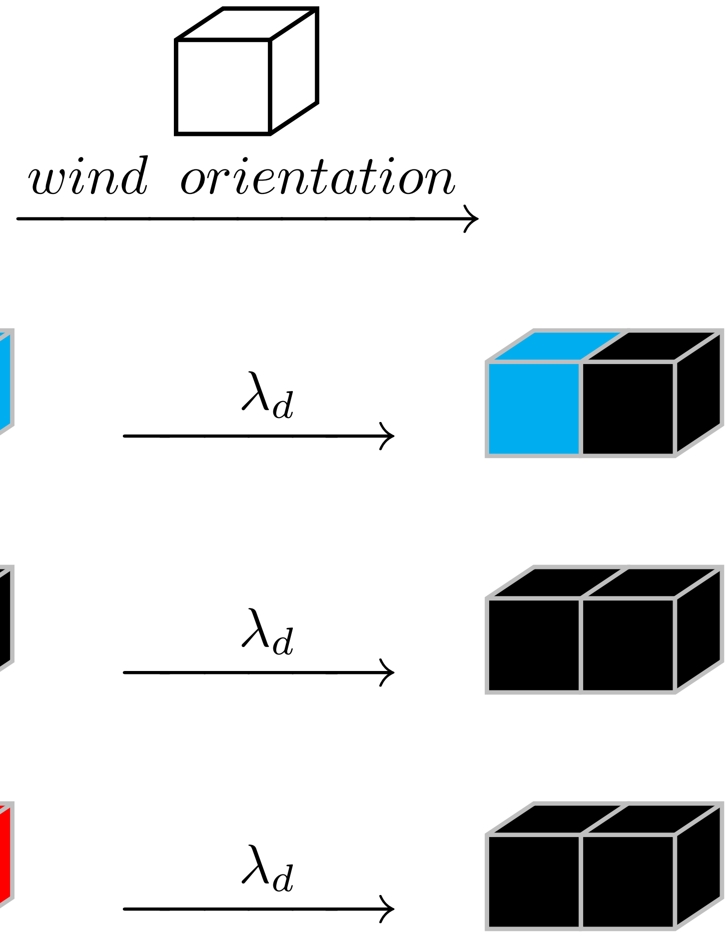
Erosion



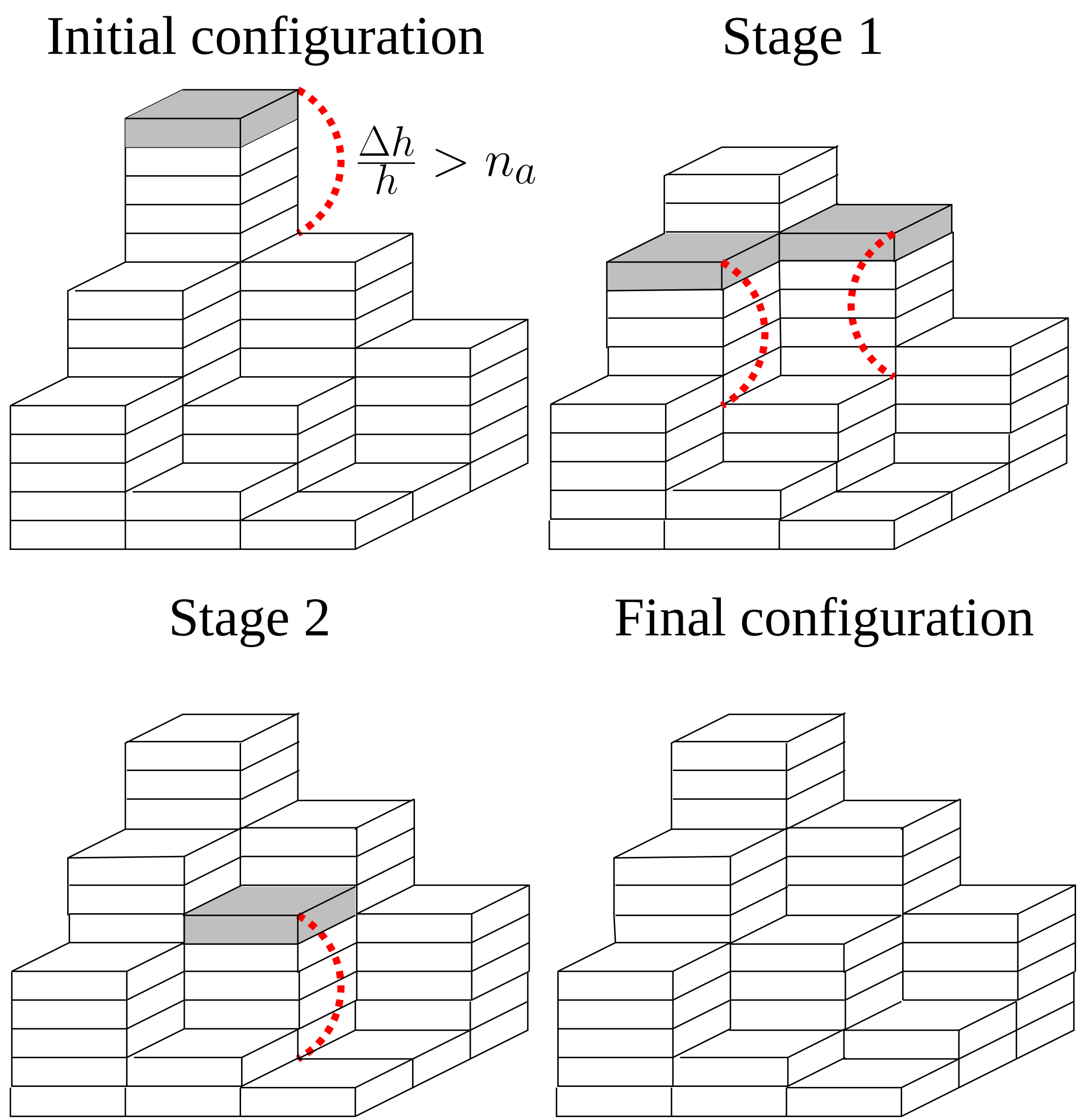
Transport



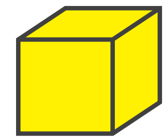
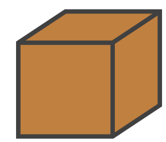
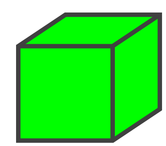
Deposition

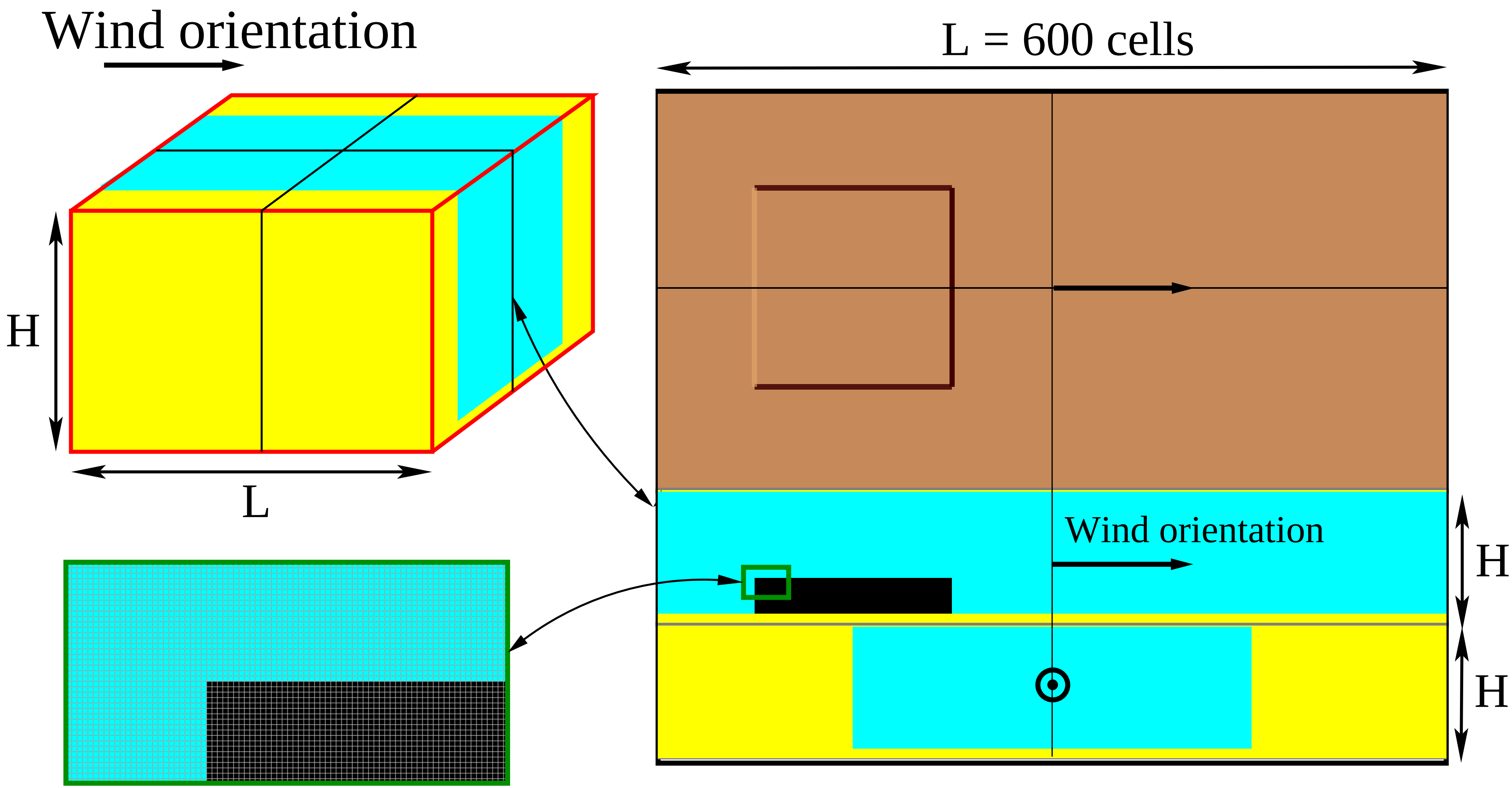


Avalanches

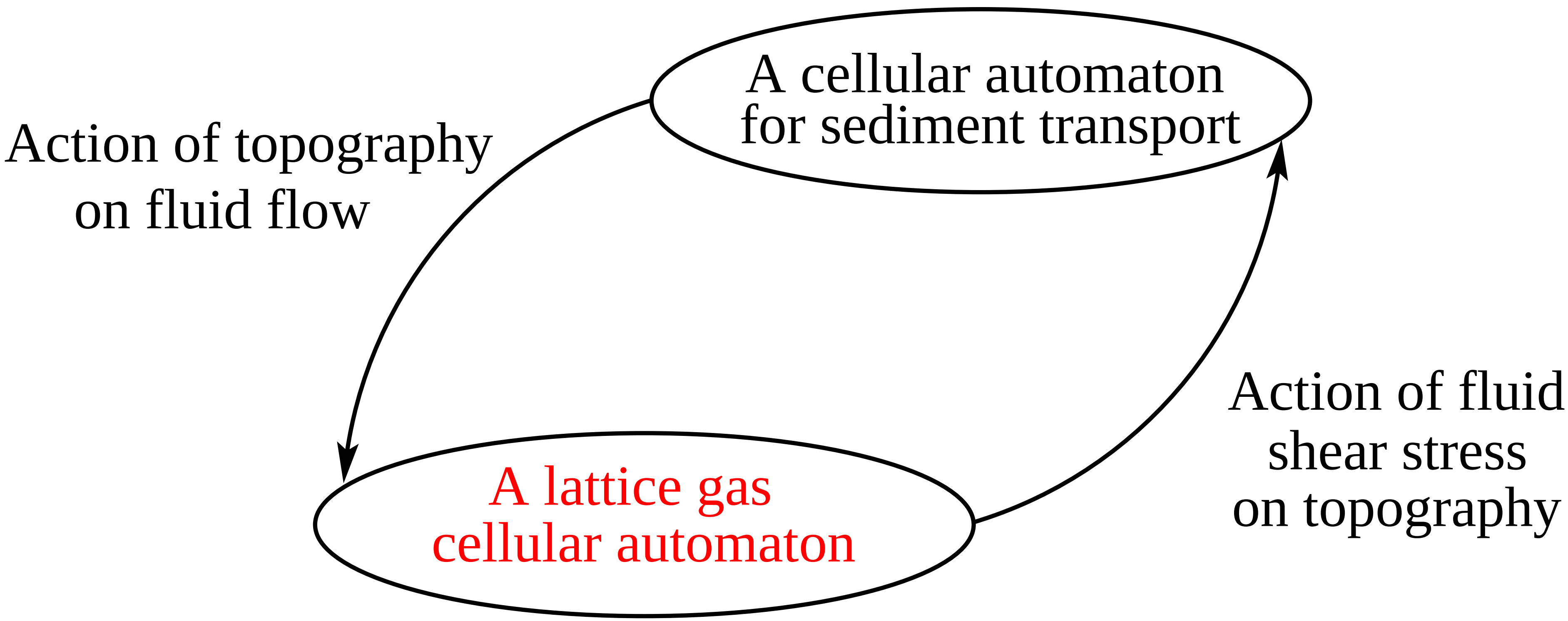

Initial configuration
$\frac{\Delta h}{h} > n_a$
Stage 1
Stage 2
Final configuration

Boundary conditions

Neutral boundaries	 Solid
Removal boundaries	 Out
Injection boundaries	 In



Wind orientation
H
L
L = 600 cells
Wind orientation
H
H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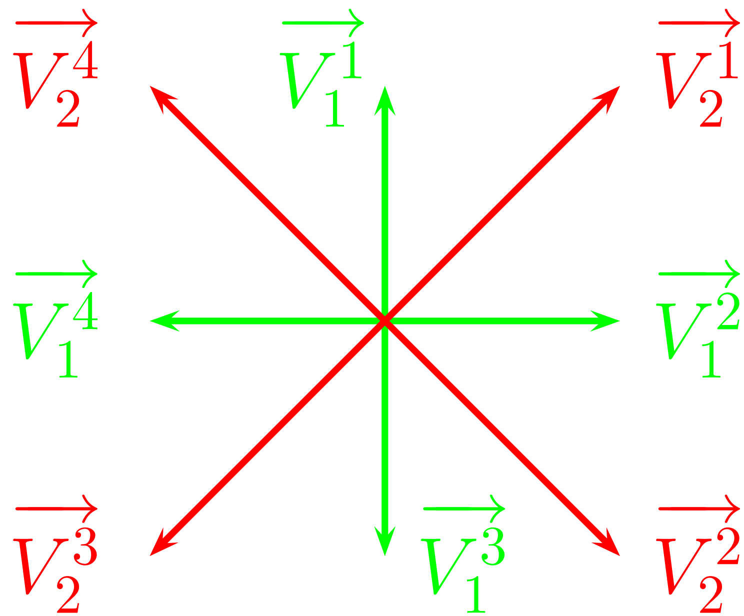
A lattice gas model

A lattice gas model is composed of a set of fluid particles flying from one lattice node to its neighbour in one unit of time. Possible velocity vectors are



with

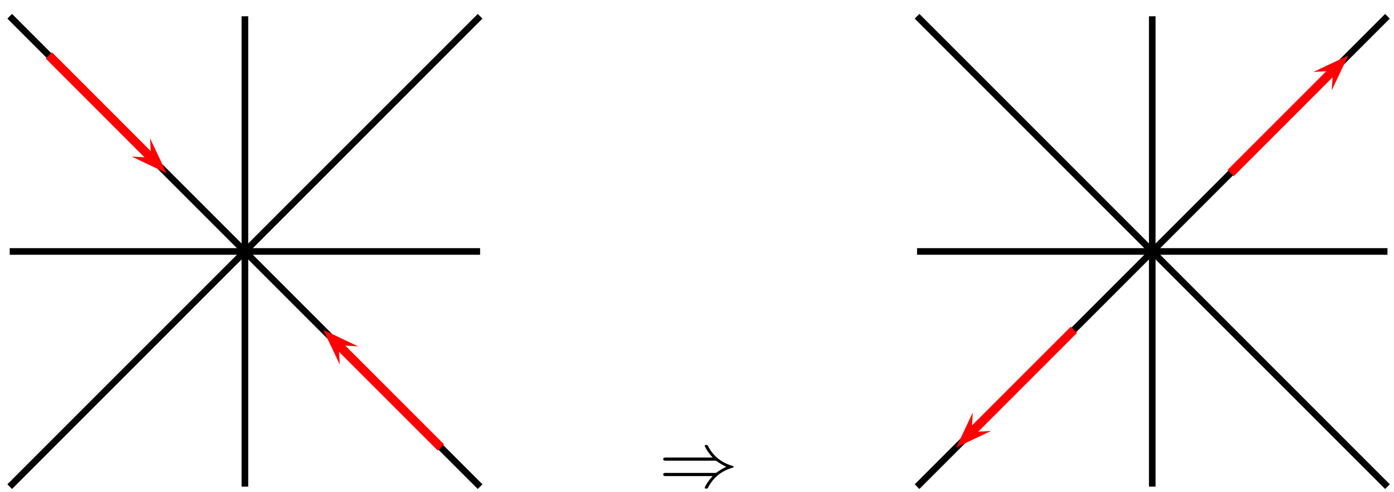
$$\|\overrightarrow{V_2}\| > \|\overrightarrow{V_1}\|.$$

A lattice gas model

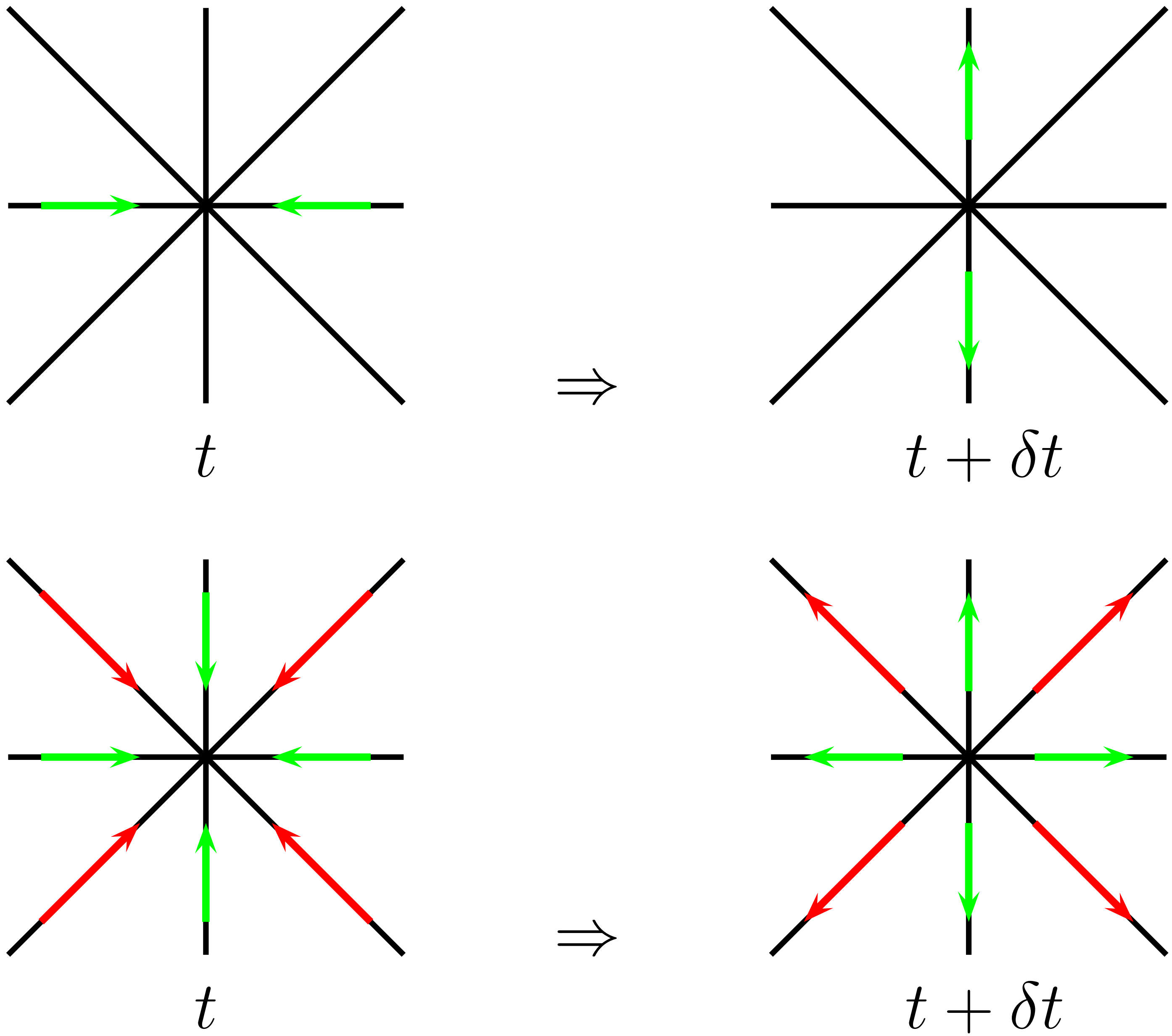
Two particles cannot sit simultaneously on the same node.

• Propagation: the particles move from their node to the nearest neighbour in the direction of their velocity vector.

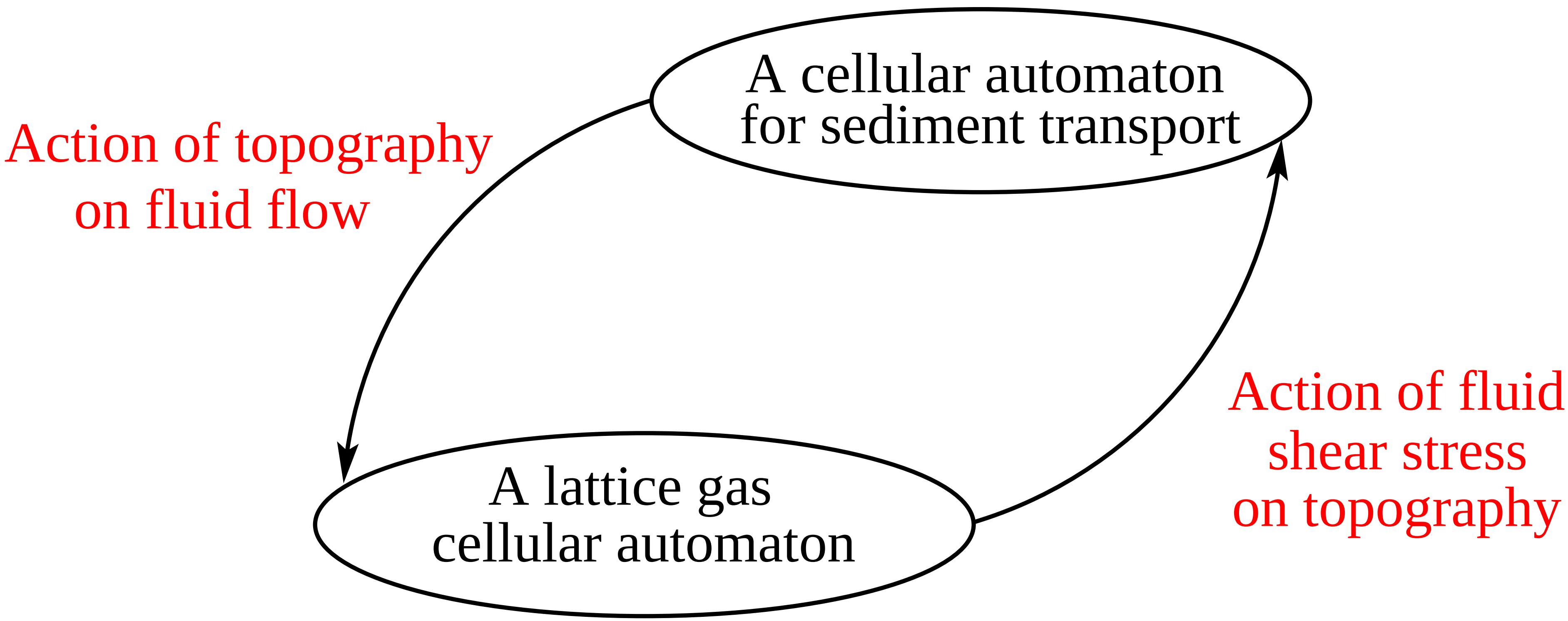
• Collision: particles on the same node may exchange momentum according to the imposed collision rules.



$t \quad \Rightarrow \quad t + \delta t$



⇒
t
t + δt
⇒
t
t + δt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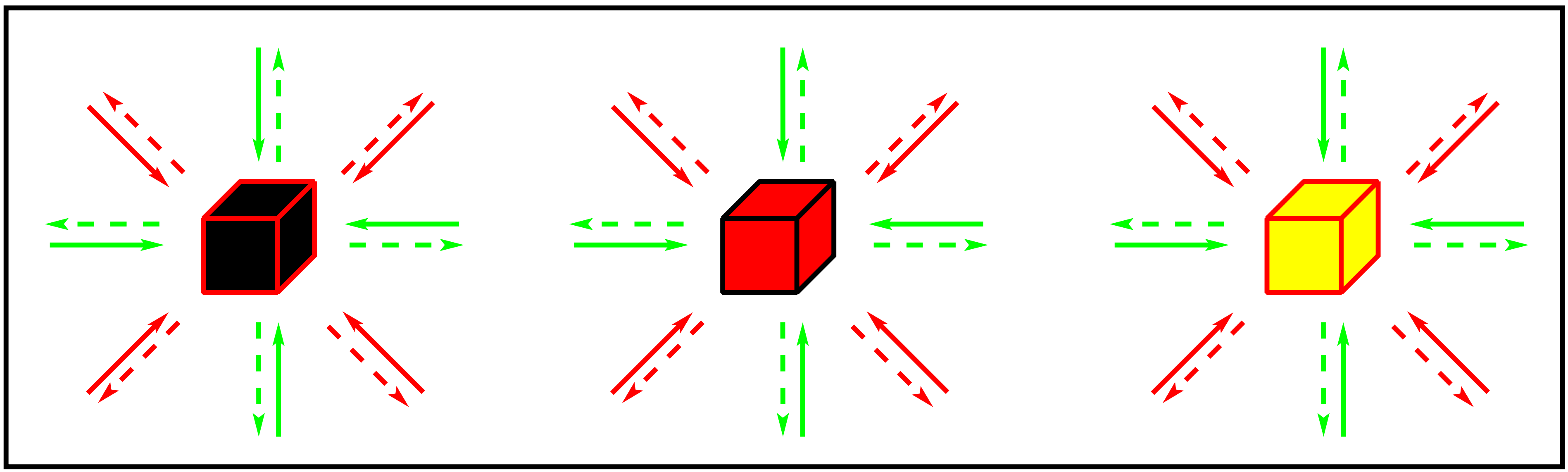
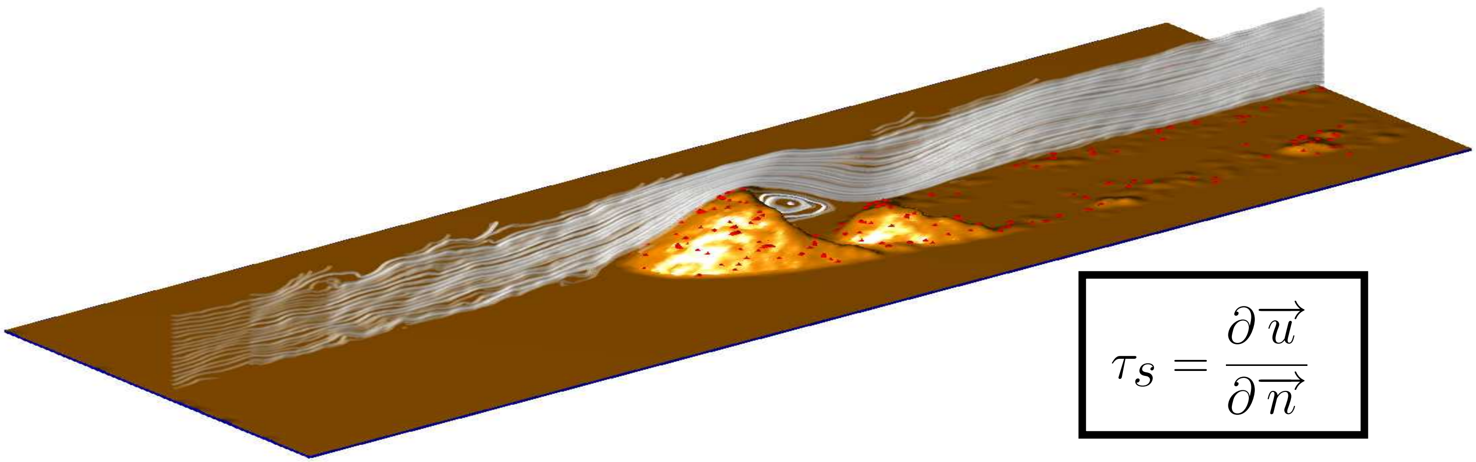
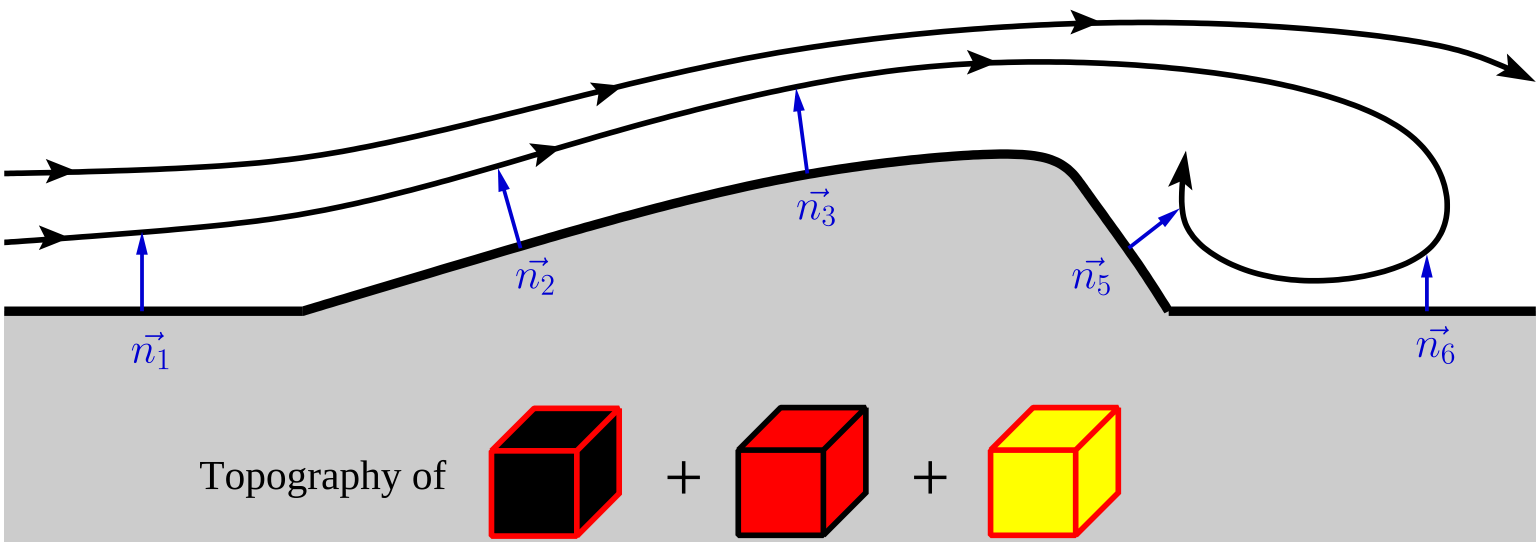
Boundary conditions



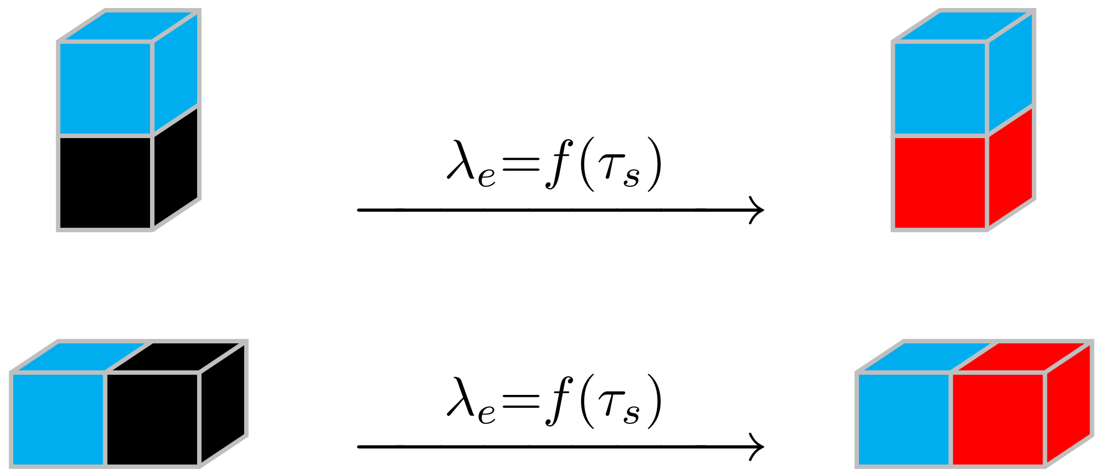
The bed shear stress



$$\tau_S = \frac{\partial \overrightarrow{u}}{\partial \overrightarrow{n}}$$

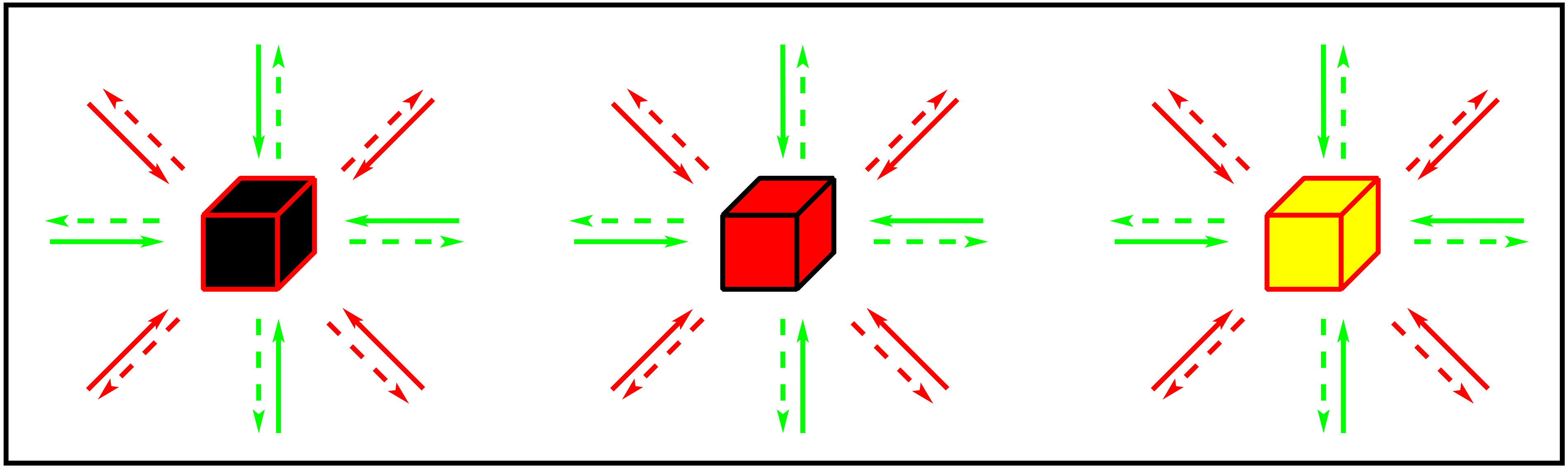


Erosion



$$\lambda_e = \begin{cases} 0 & \text{for } \tau_s \leq \tau_{min}, \\ \lambda_0 \dfrac{\tau_s - \tau_{min}}{\tau_{max} - \tau_{min}} & \text{for } \tau_{min} \leq \tau_s \leq \tau_{max}, \\ \lambda_0 & \text{else.} \end{cases}$$

A fully coupled system



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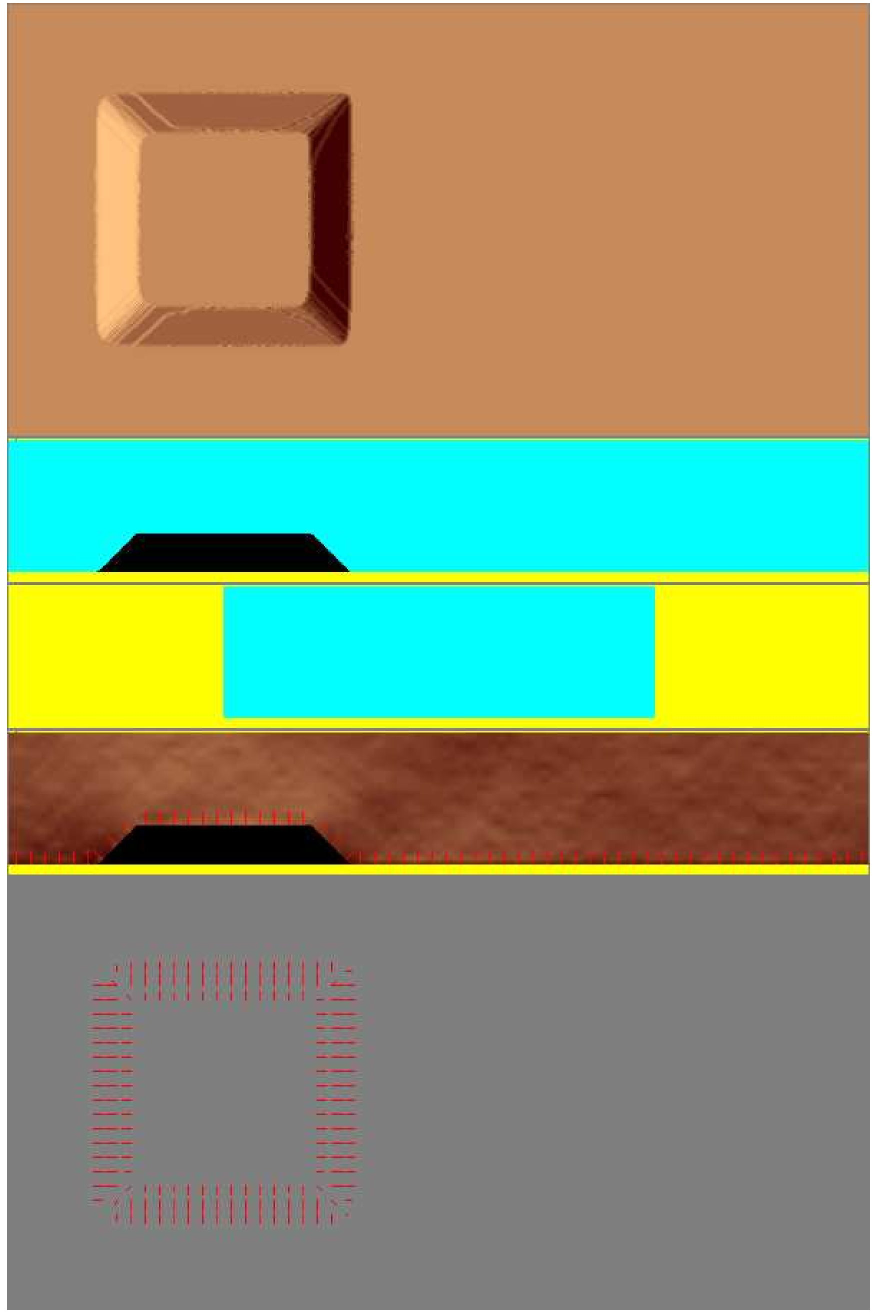
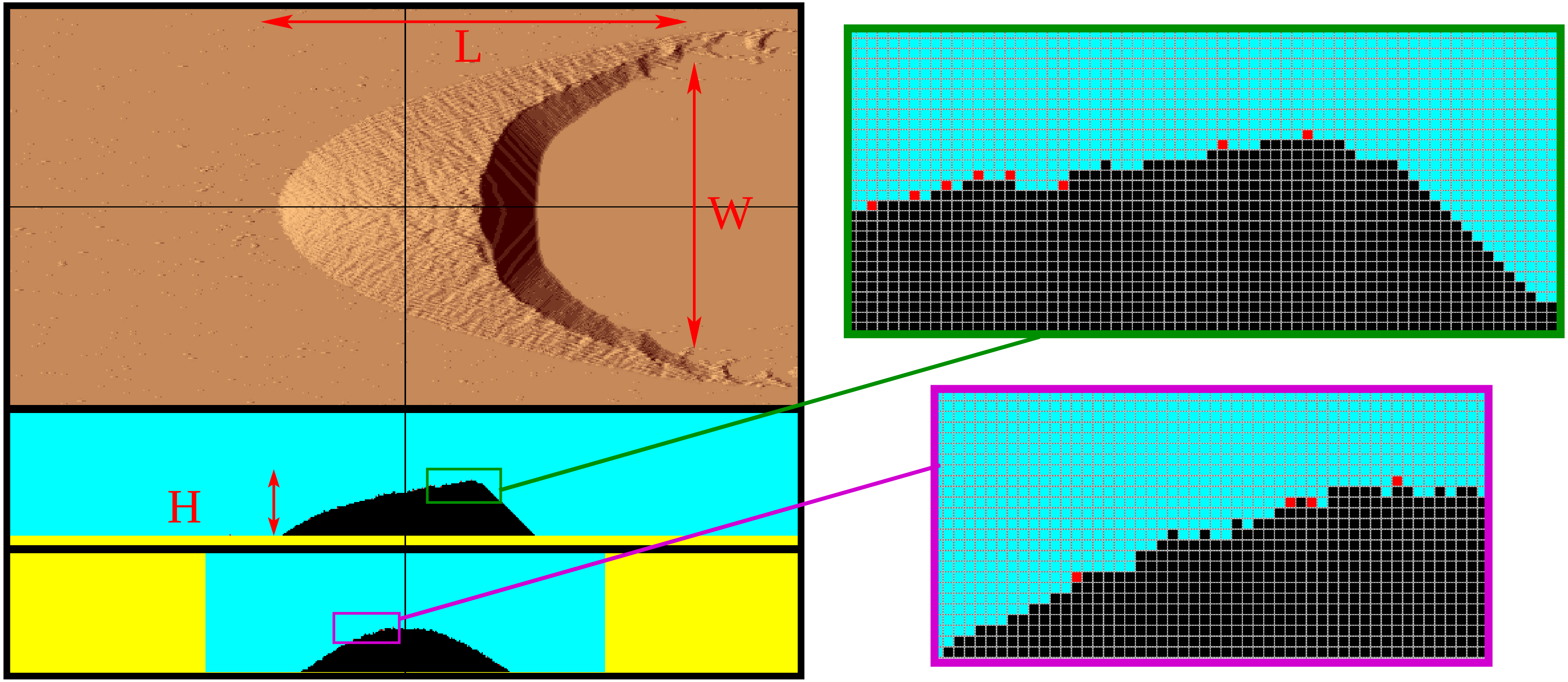
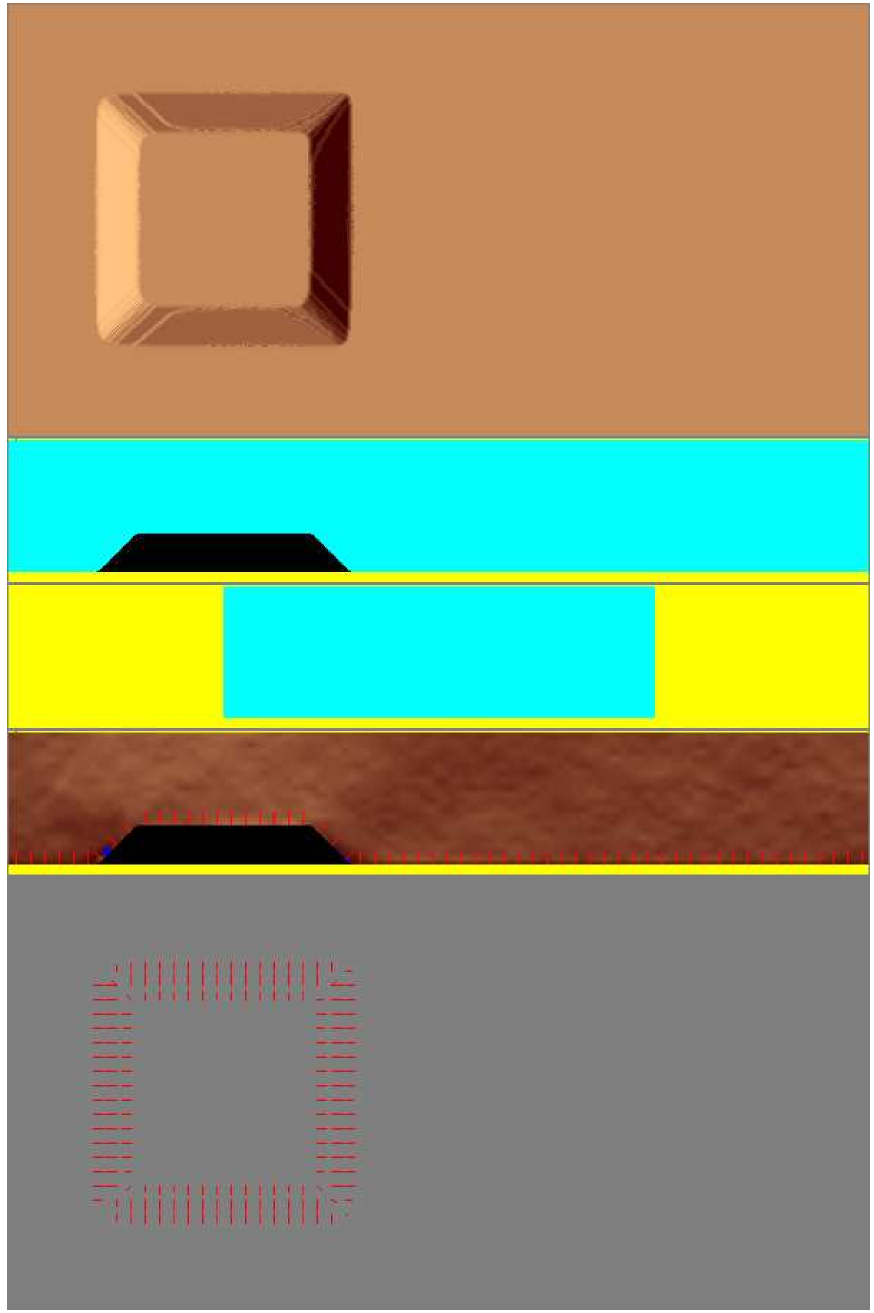
High wind speed $\Longleftrightarrow$ low τ_{min}-value

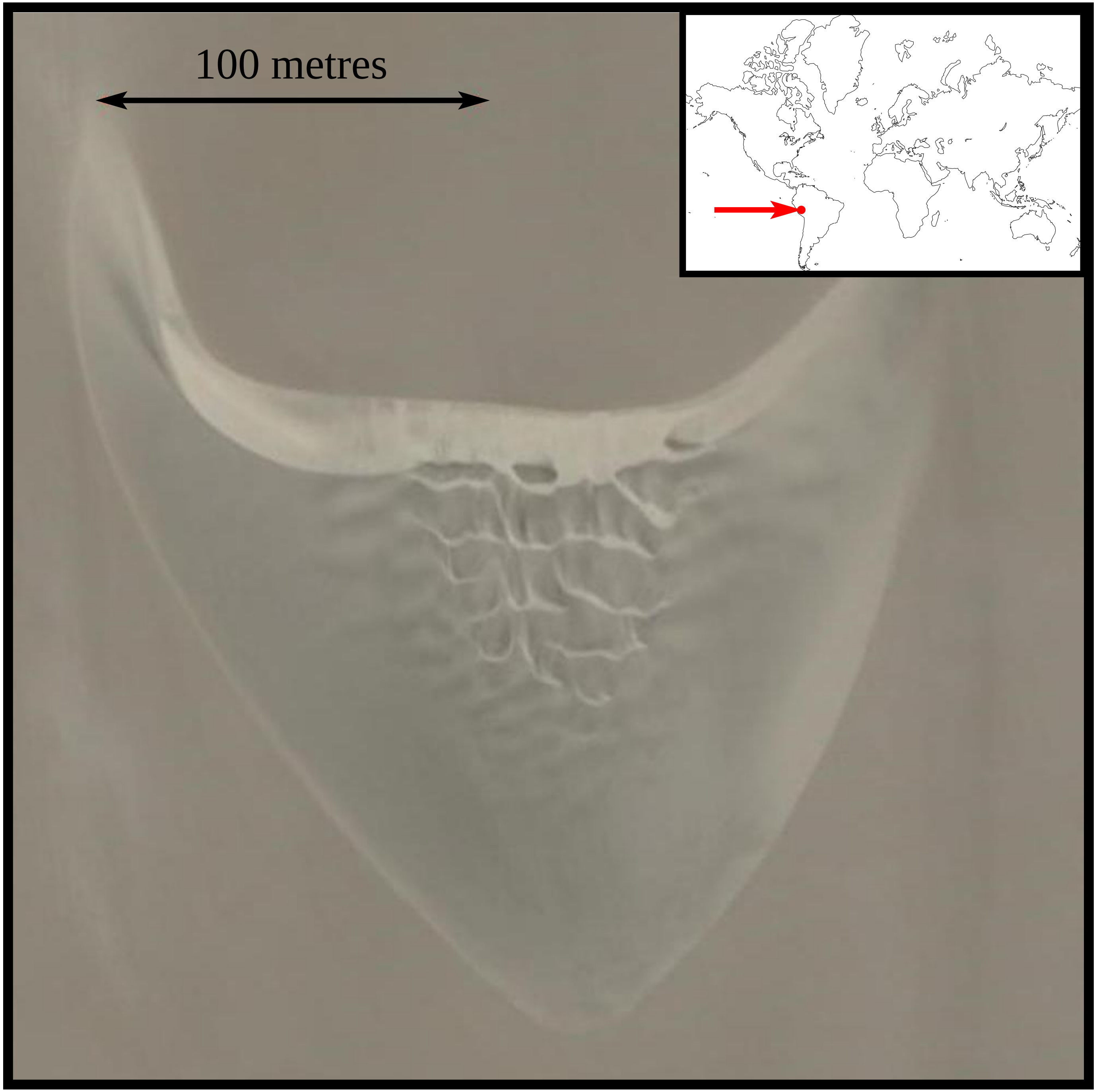


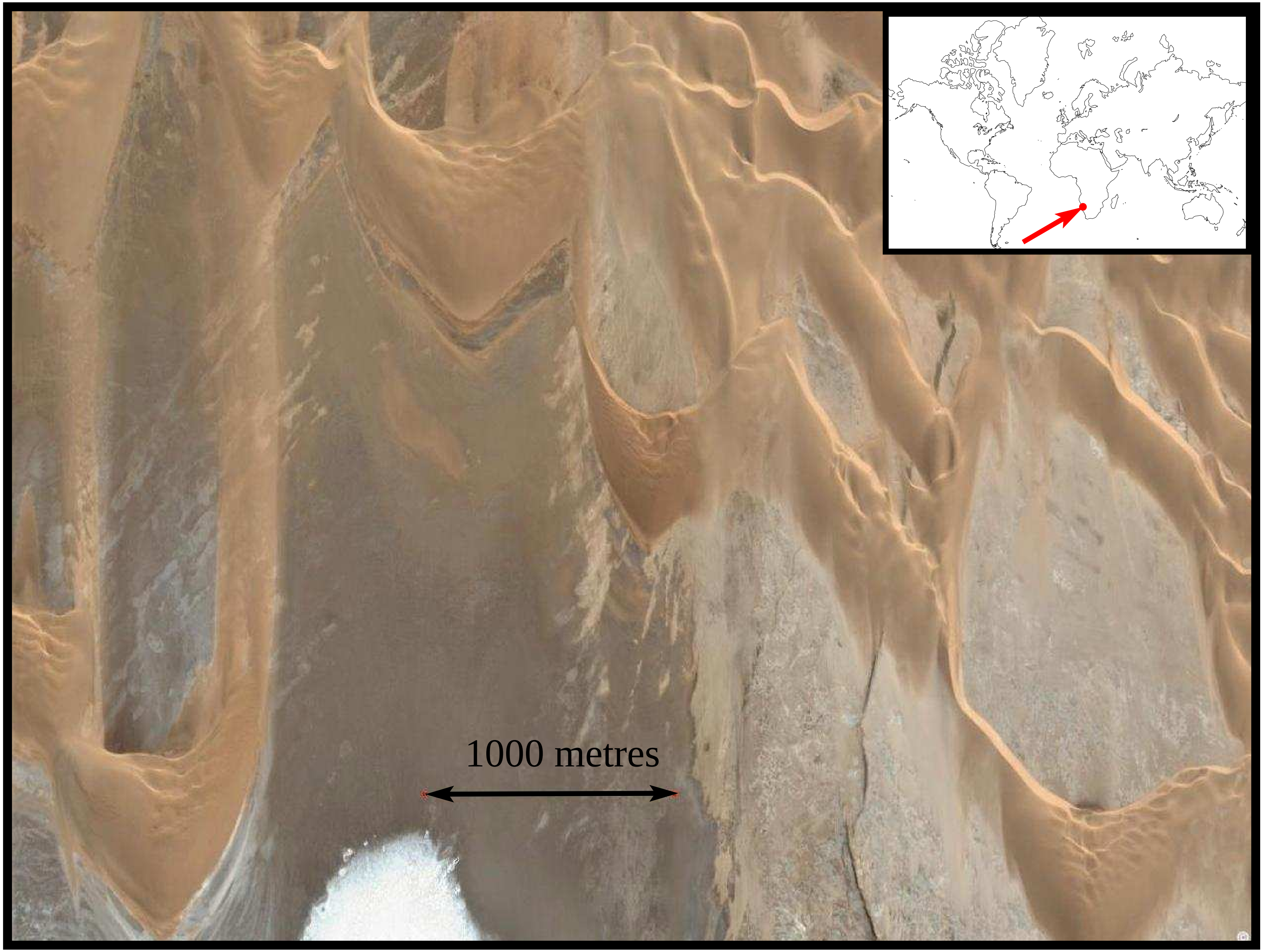


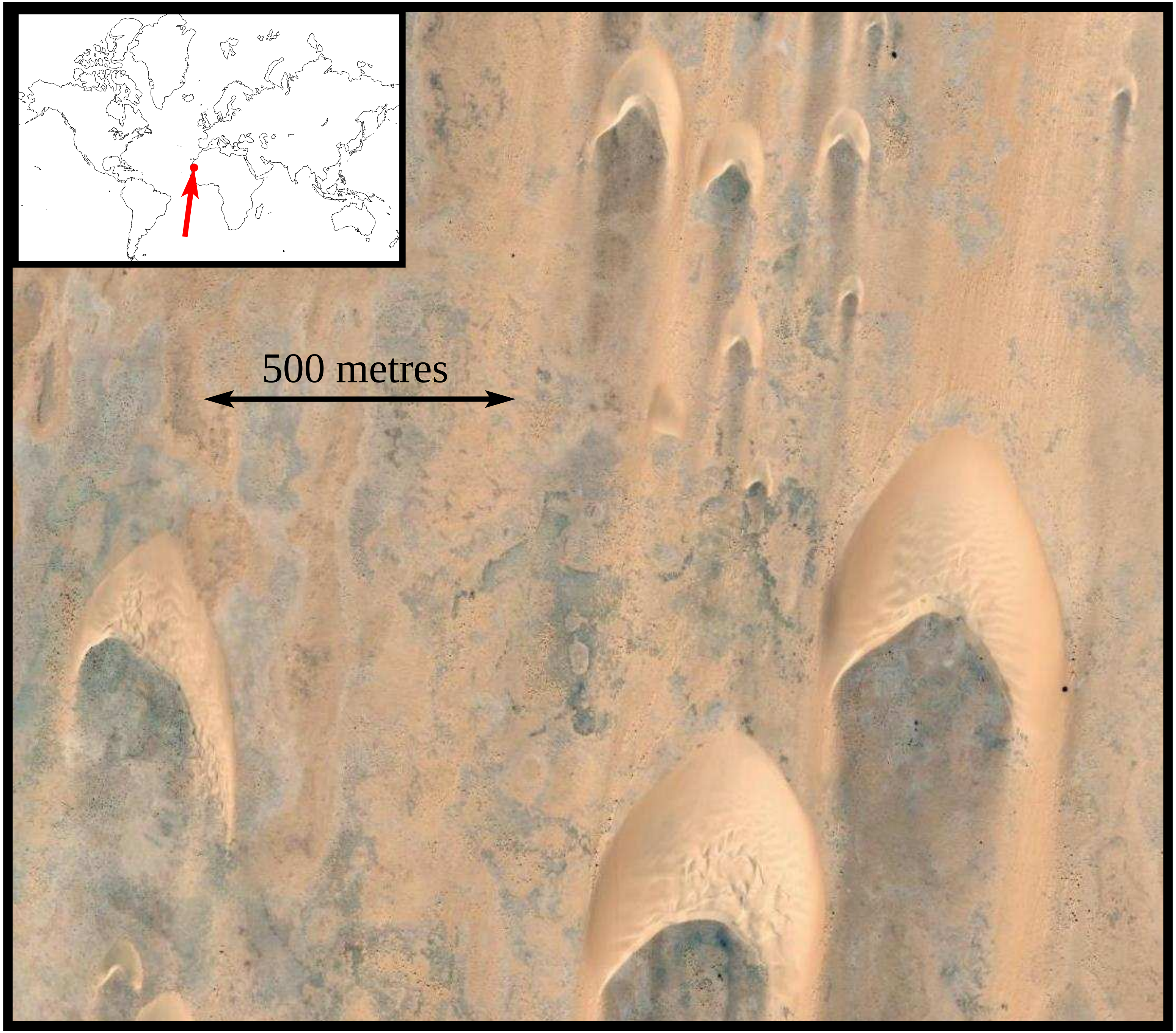
L
W
H

Low wind speed $\Longleftrightarrow$ high τ_{min}-value



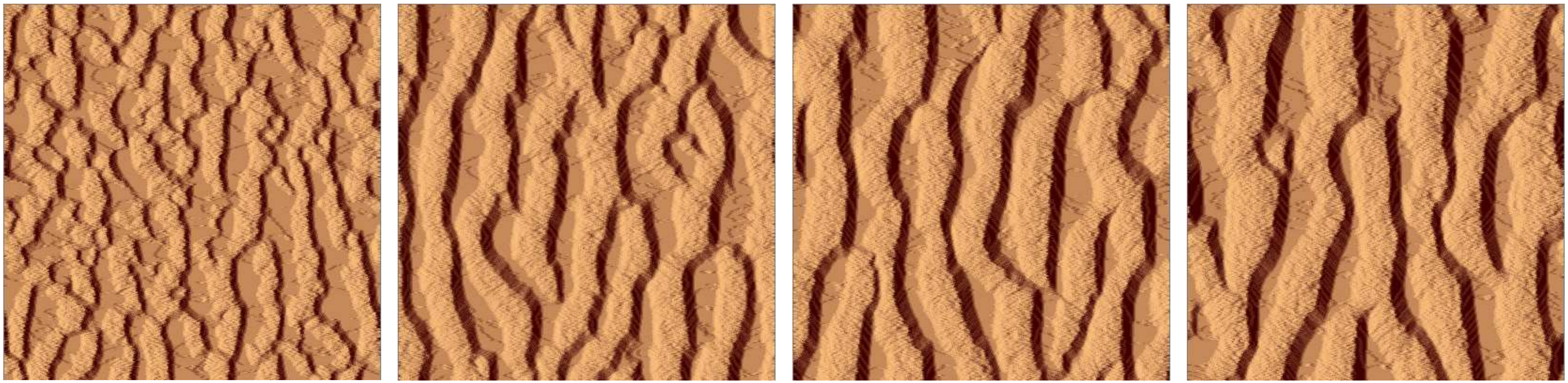

100 metres


1000 metres

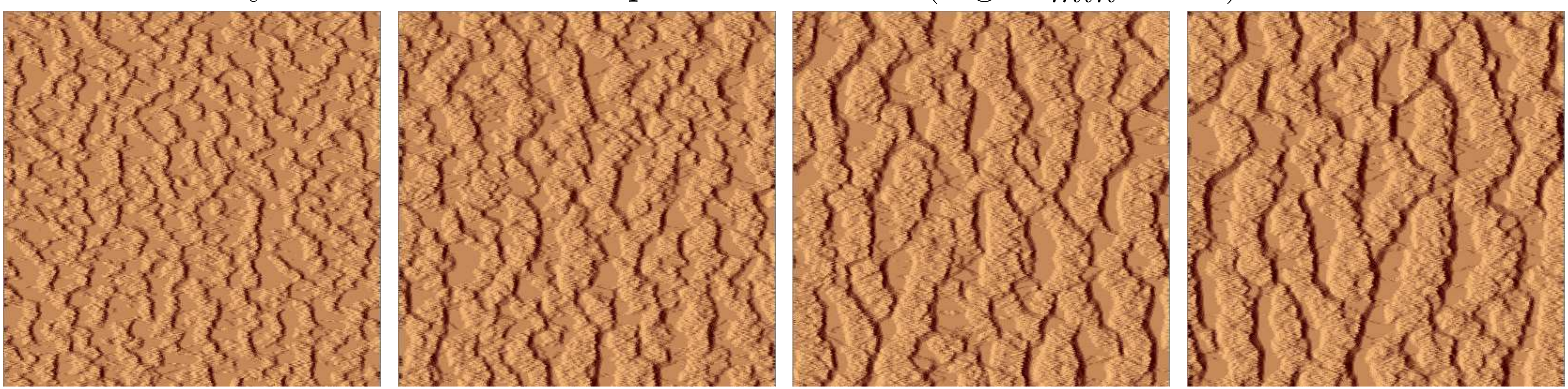

500 metres

Transverse dunes

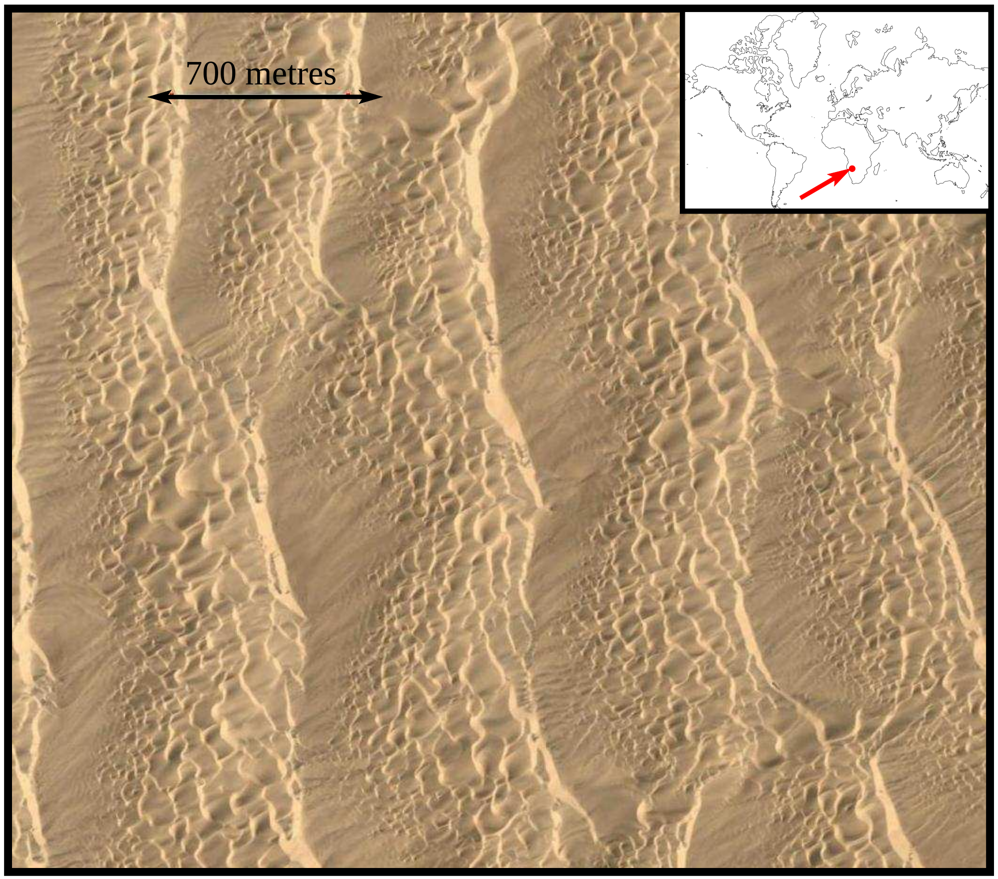
Thick sand layer under high wind speed conditions (low τ_{min}-value)

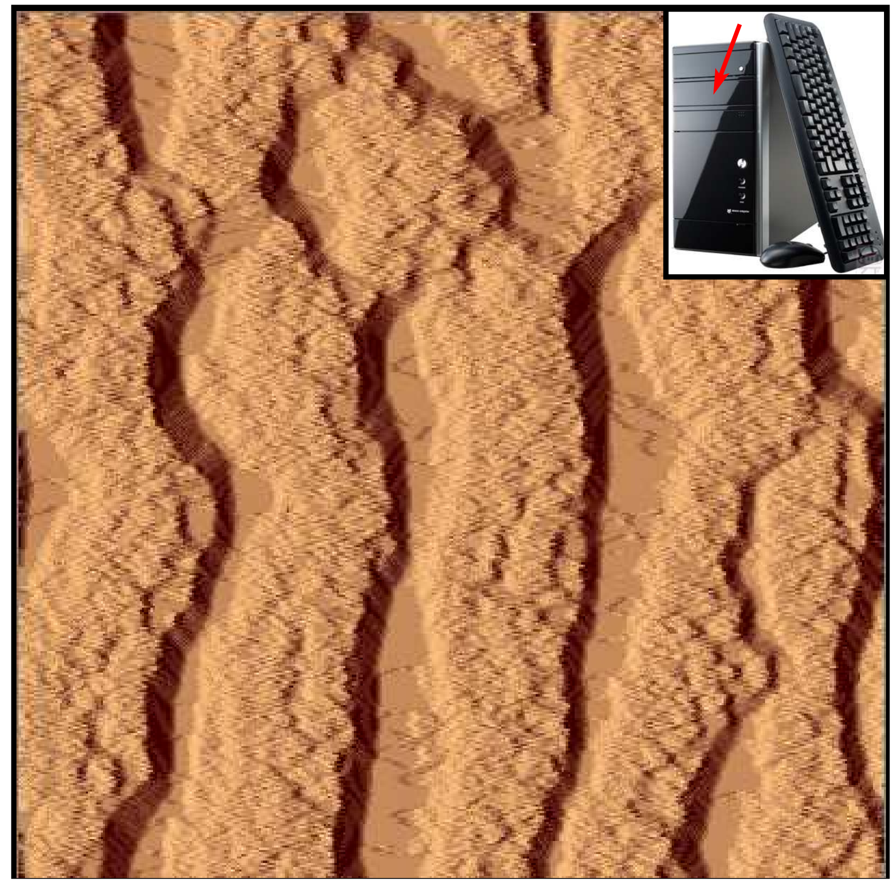


Thick sand layer under low wind speed conditions (high τ_{min}-value)



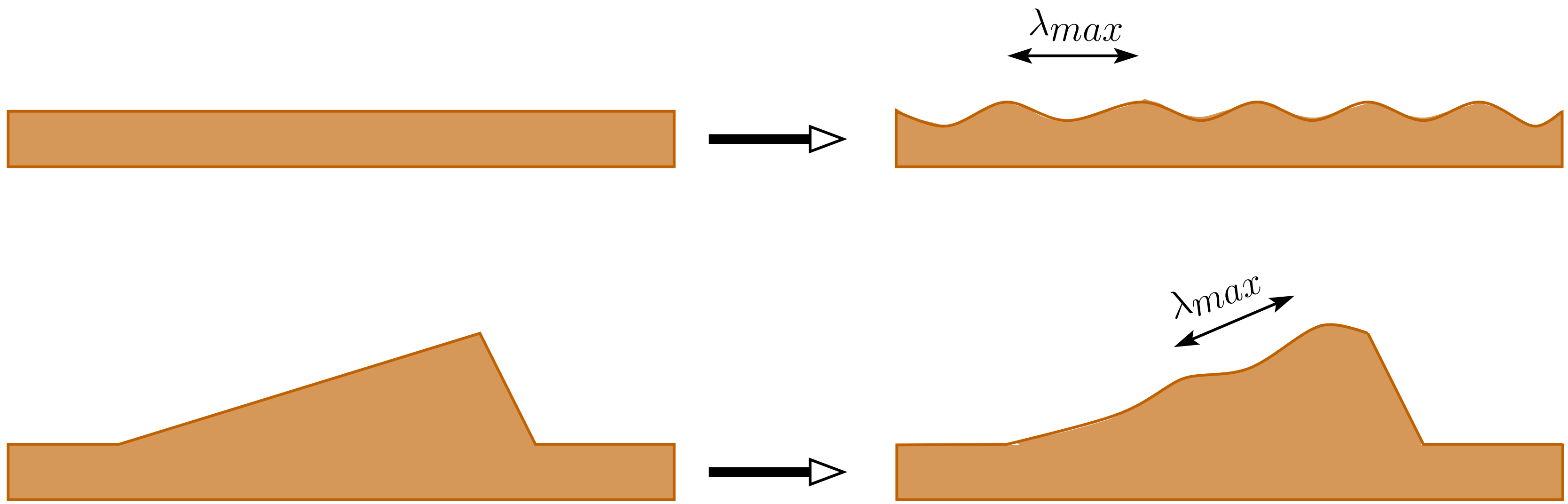
Transverse dunes





- Length and time scales of the model.
- Dune propagation speed.
- Phenomelogy of dune-fields.
- Control parameters for the dune instability.
- Role of the top boundary layer.
- Role of lateral sediment motions.
- Role of dune-dune interactions.
- Multiscaling in dune-fields.
- etc ...

Initiation of dunes patterns

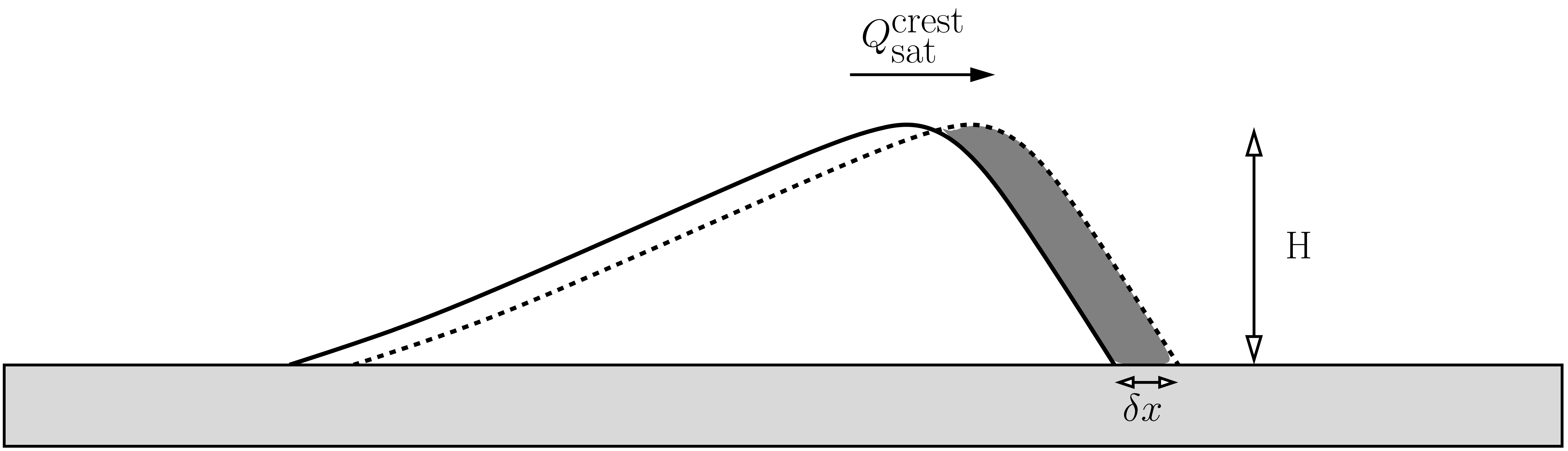


In the model:

$$\lambda_{max} \approx 40 \ \ l_0$$

In arid desert on Earth:

$$\lambda_{max} \approx 20 \ \ m$$


$Q_{\mathrm{sat}}^{\mathrm{crest}}$
H
δx

Propagation speed of dunes

$$c = \frac{Q_{\rm sat}^{\rm crest}}{H + H_0} = \frac{a\, Q_{\rm sat}^{\rm flat}}{H + H_0},$$

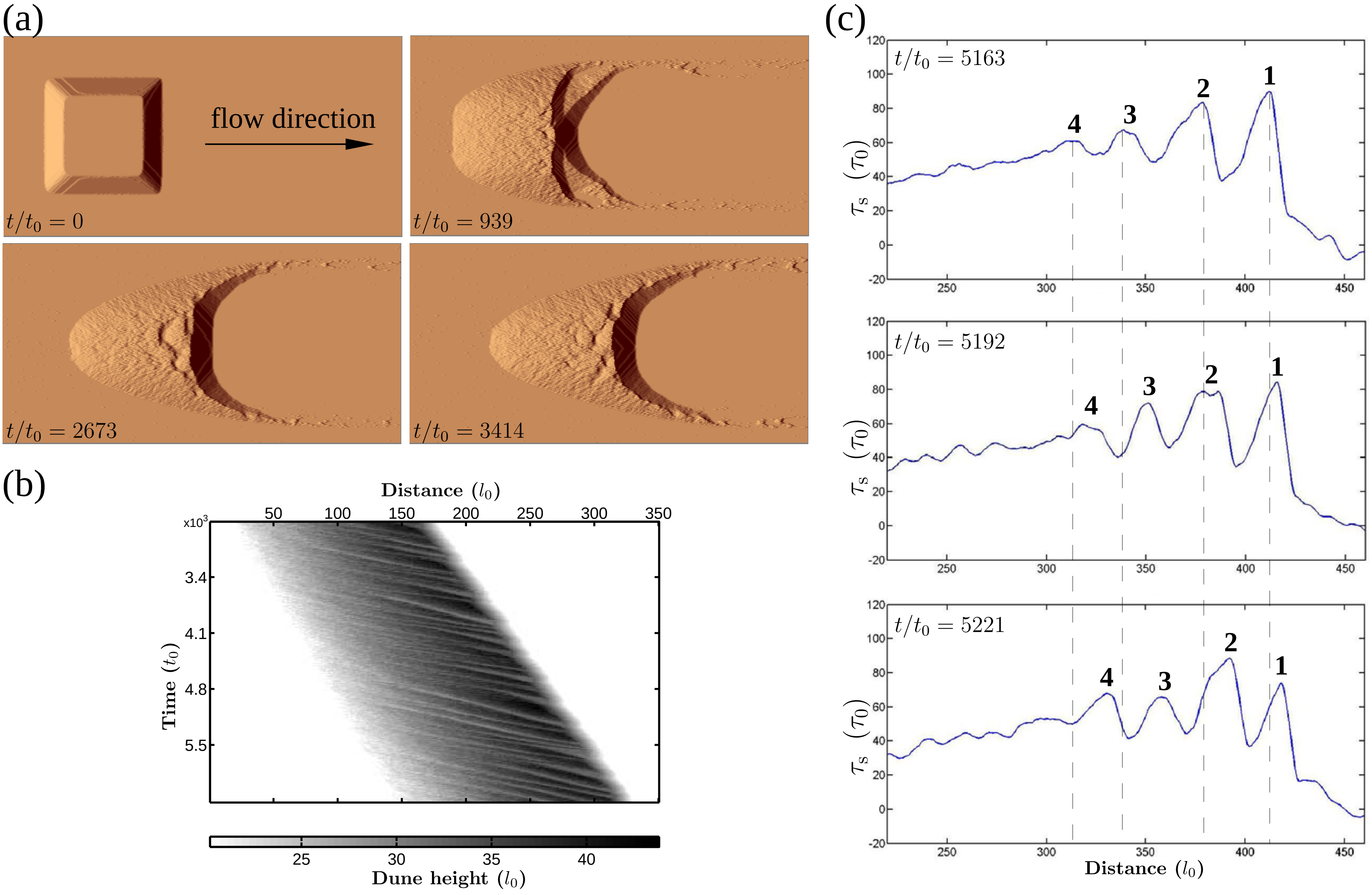
H is dune height.

H_0 is a minimum dune size.

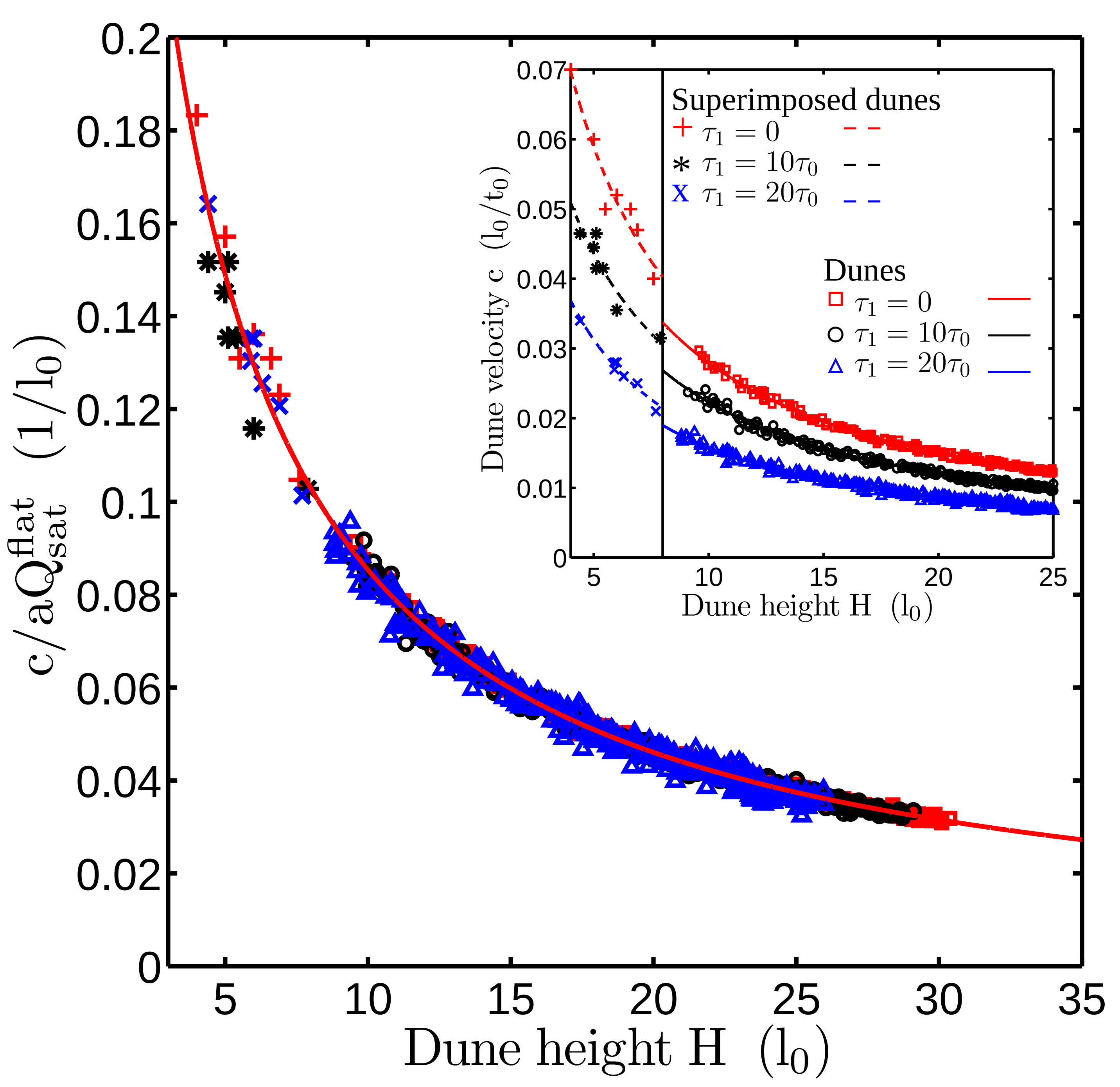
$Q_{\rm sat}^{\rm crest}$ is the saturated flux at the crest.

$Q_{\rm sat}^{\rm flat}$ is the saturated flux upstream of the dune.

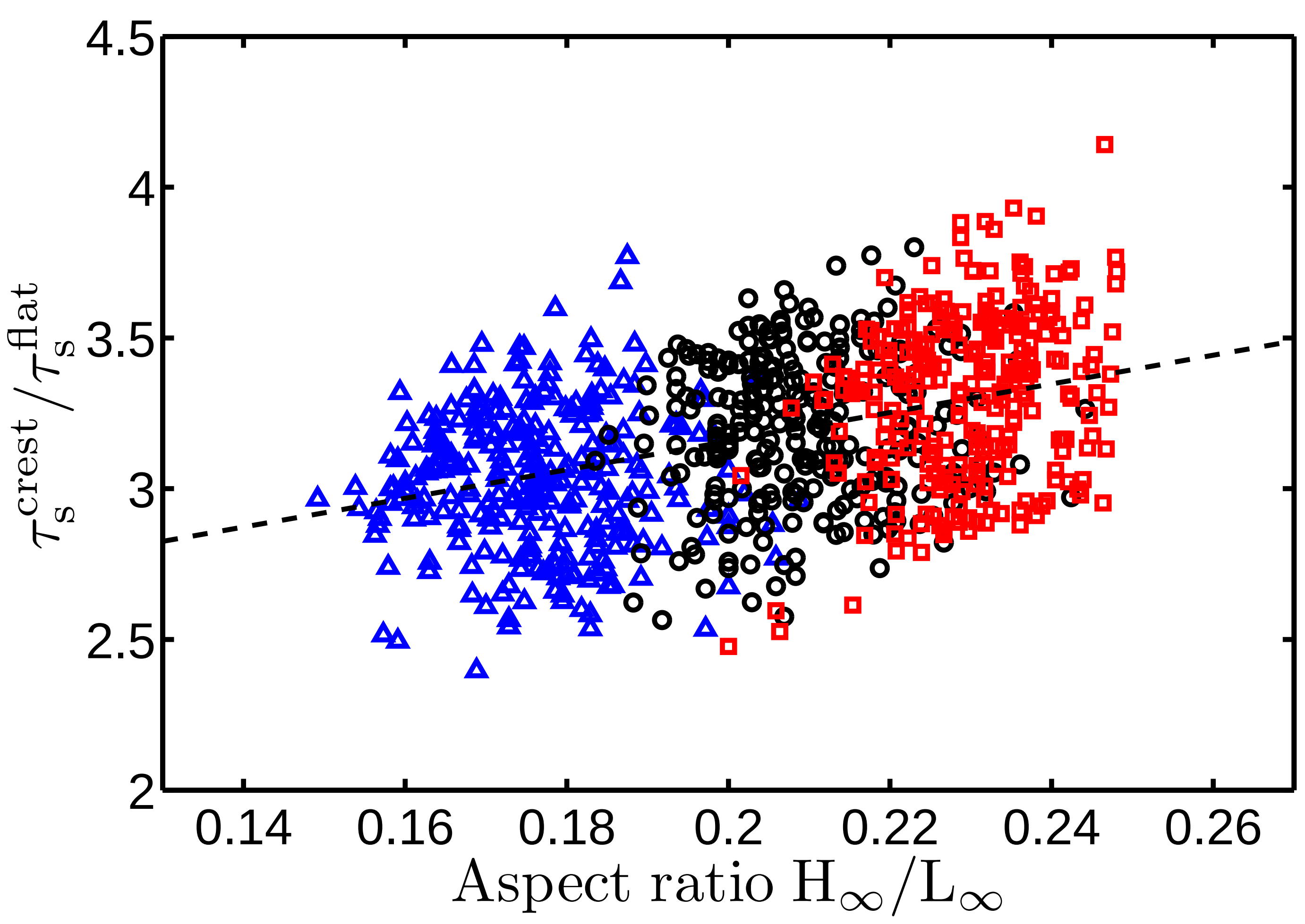
$$a = \frac{Q_{\rm sat}^{\rm crest}}{Q_{\rm sat}^{\rm flat}}$$



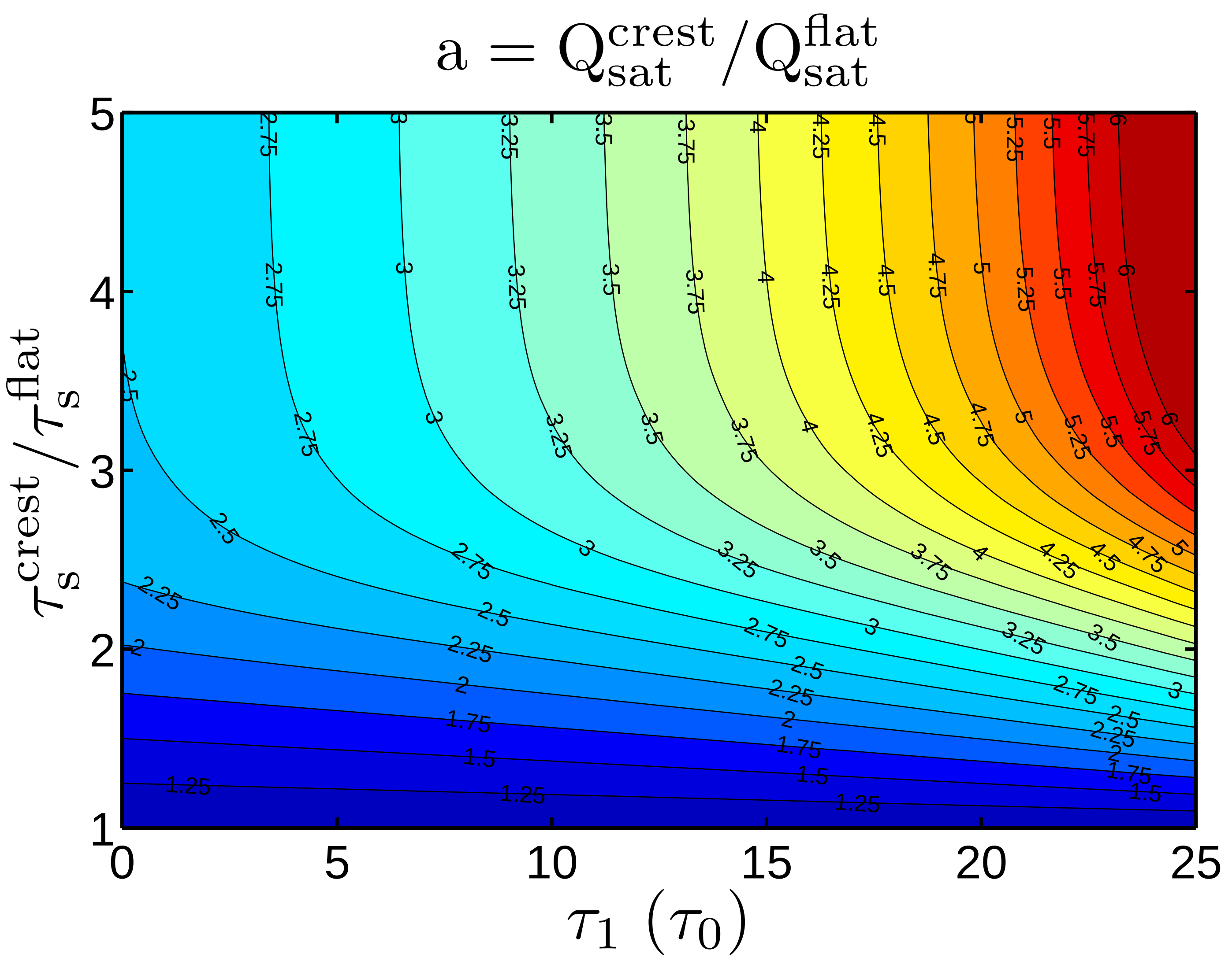
(a)
flow direction
$t/t_0 = 0$
$t/t_0 = 939$
$t/t_0 = 2673$
$t/t_0 = 3414$
(b)
Distance (l_0)
50 100 150 200 250 300 350
x10³
3.4
4.1
4.8
5.5
Time (t_0)
25 30 35 40
Dune height (l_0)
(c)
$t/t_0 = 5163$
$t/t_0 = 5192$
$t/t_0 = 5221$
1
2
3
4
τ_s (τ_0)
120 100 80 60 40 20 0 -20
250 300 350 400 450
Distance (l_0)

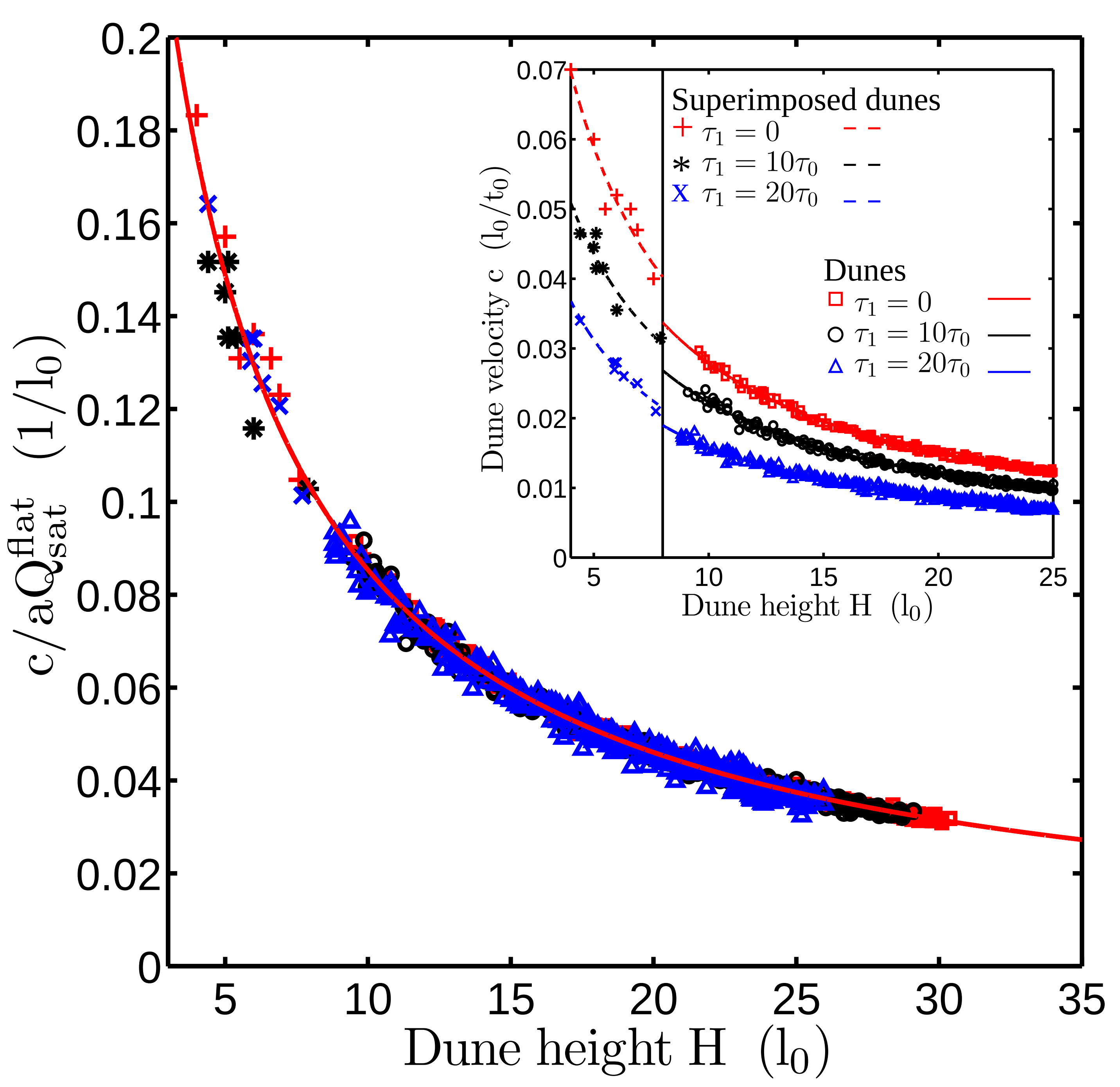


Superimposed dunes
$\tau_1 = 0$
$\tau_1 = 10\tau_0$
$\tau_1 = 20\tau_0$
Dunes
$\tau_1 = 0$
$\tau_1 = 10\tau_0$
$\tau_1 = 20\tau_0$
Dune velocity c (l_0/t_0)
Dune height H (l_0)
c/aQ_{sat}^{flat} $(1/l_0)$
Dune height H (l_0)



4.5
4
3.5
3
2.5
2
0.14
0.16
0.18
0.2
0.22
0.24
0.26
$\tau_s^{crest}/\tau_s^{flat}$
Aspect ratio H_∞/L_∞


$a = Q_{sat}^{crest} / Q_{sat}^{flat}$
$\tau_s^{crest} / \tau_s^{flat}$
$\tau_1 \ (\tau_0)$
0
5
10
15
20
25
1
2
3
4
5

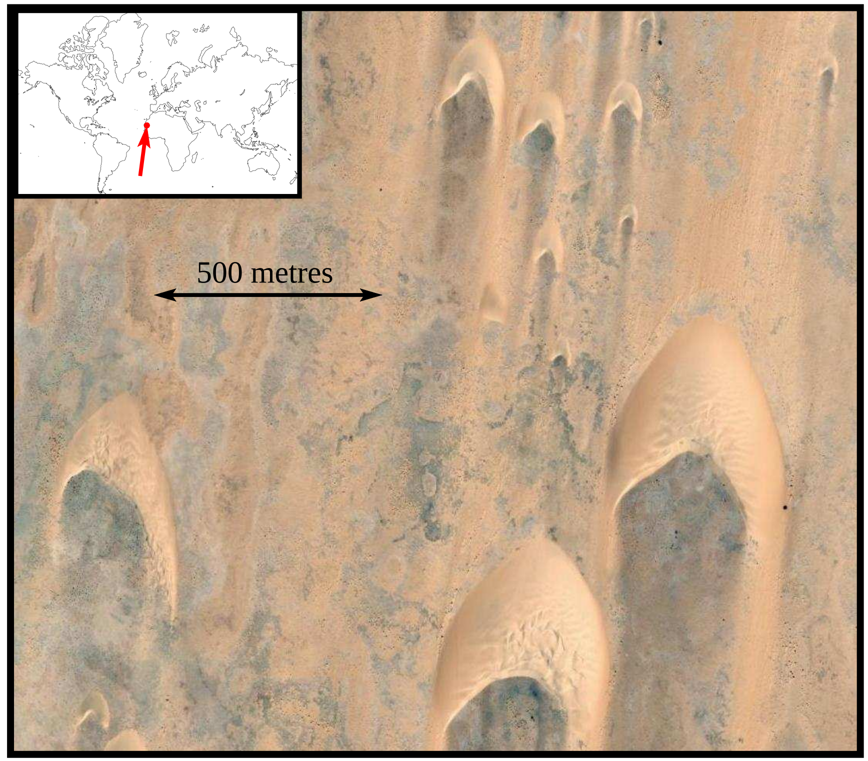


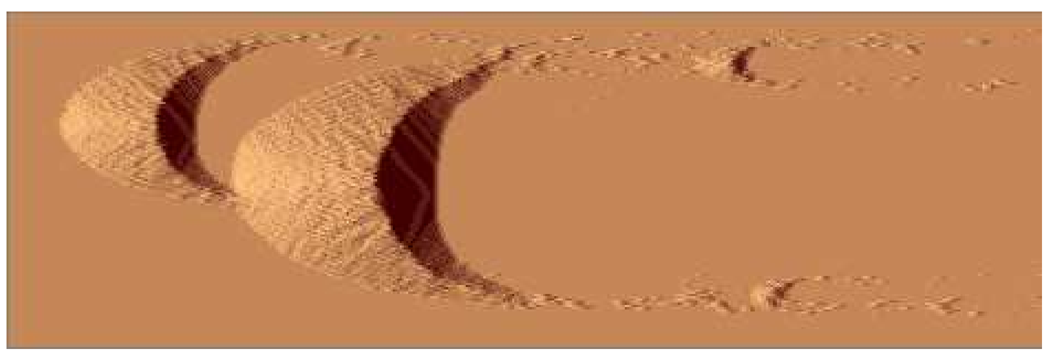
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
c/aQ_{sat}^{flat} $(1/l_0)$
5
10
15
20
25
30
35
Dune height H (l_0)
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
Dune velocity c (l_0/t_0)
Dune height H (l_0)
Superimposed dunes
$\tau_1 = 0$
$\tau_1 = 10\tau_0$
$\tau_1 = 20\tau_0$
Dunes
$\tau_1 = 0$
$\tau_1 = 10\tau_0$
$\tau_1 = 20\tau_0$

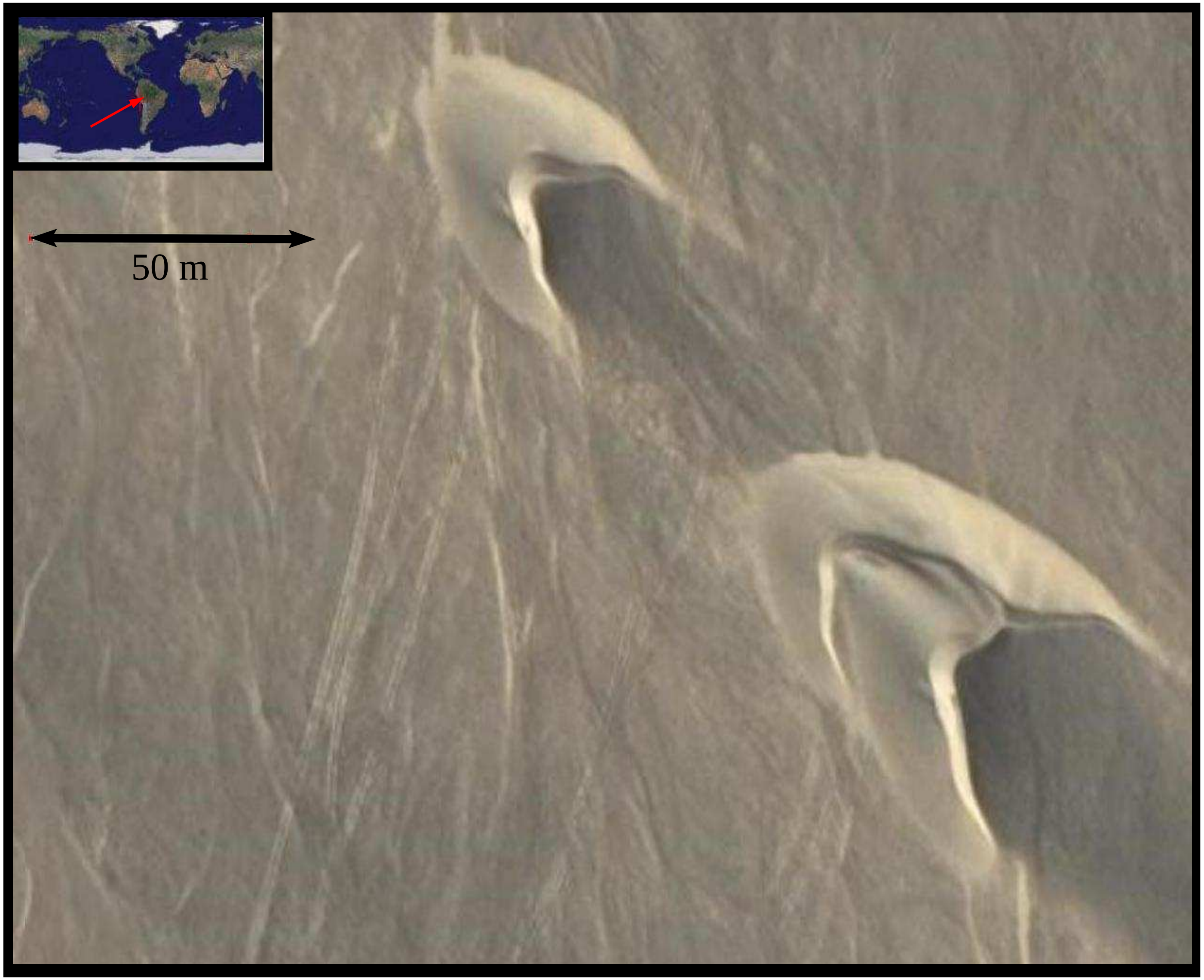
Conclusion

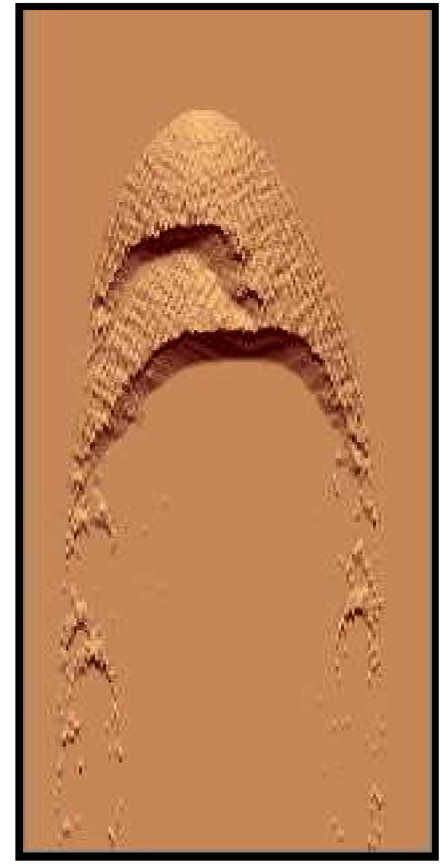
The numerical engine is ready for applications

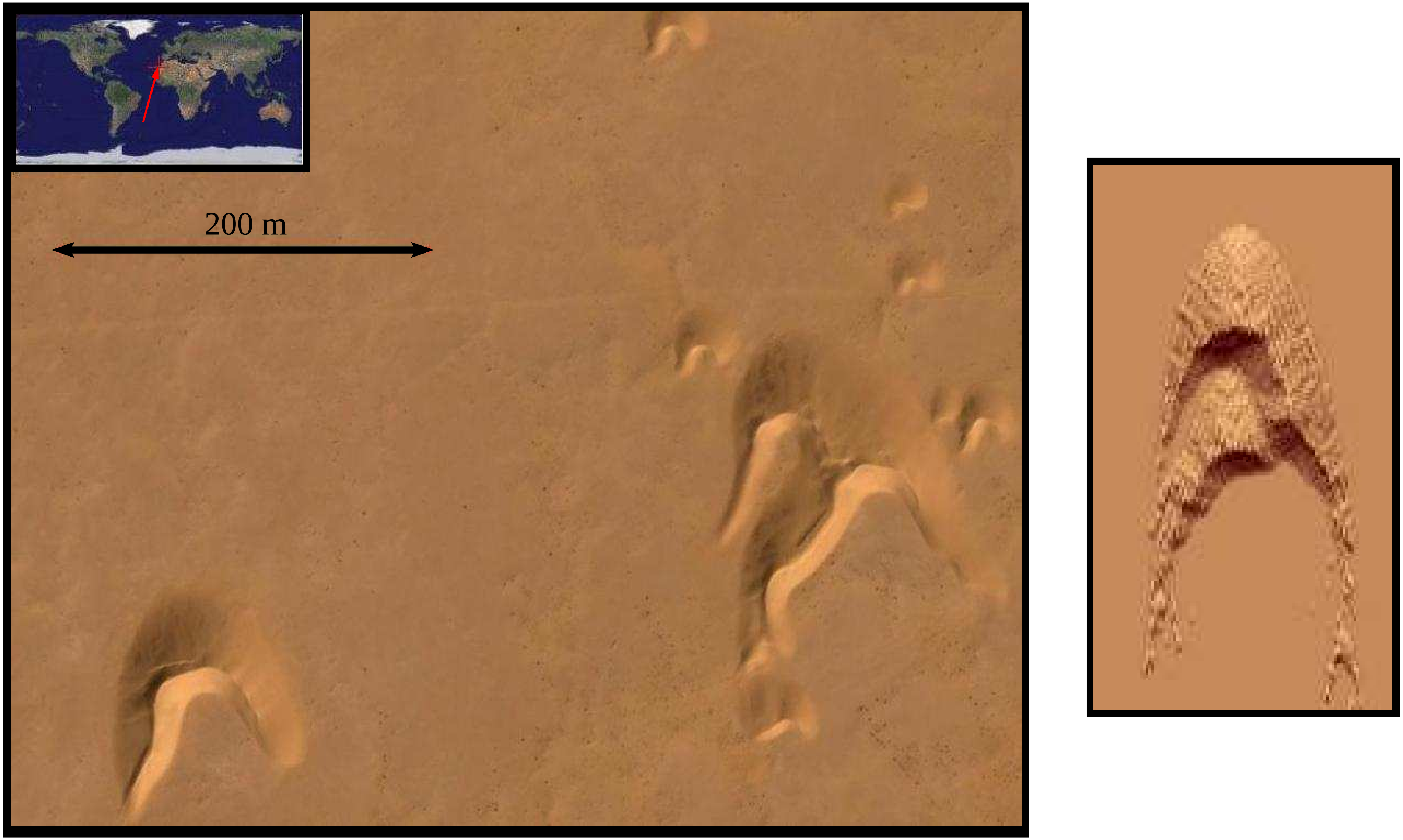
Collision of dunes



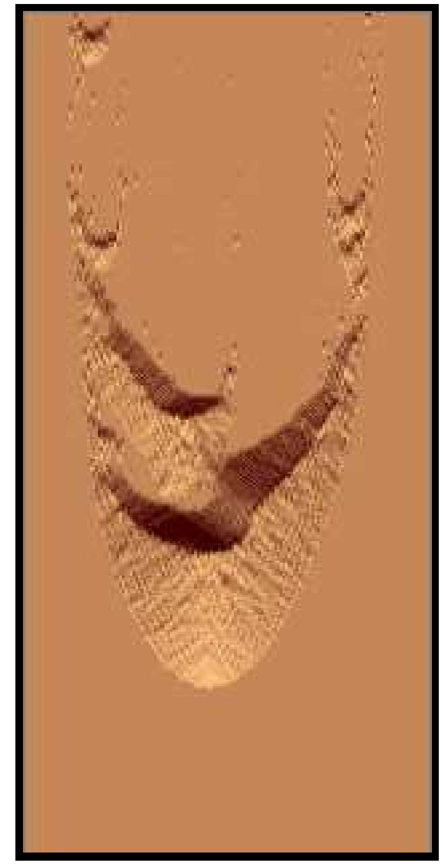



50 m

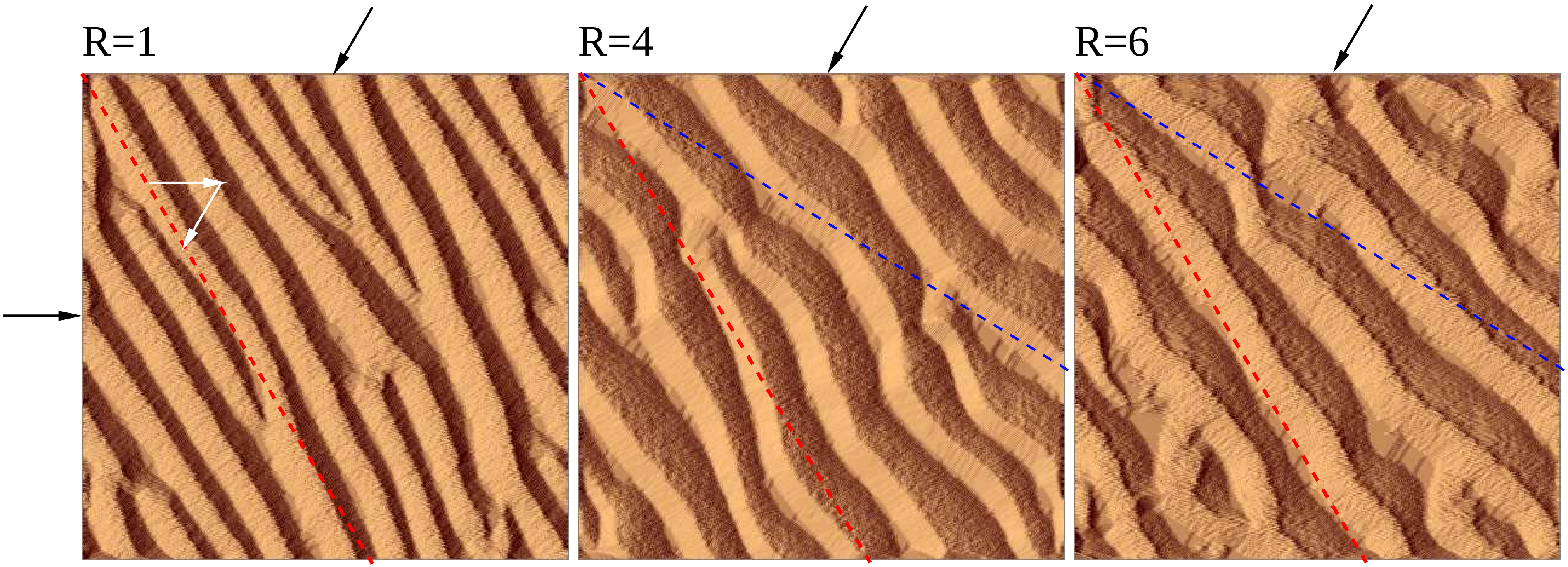



200 m


300 m

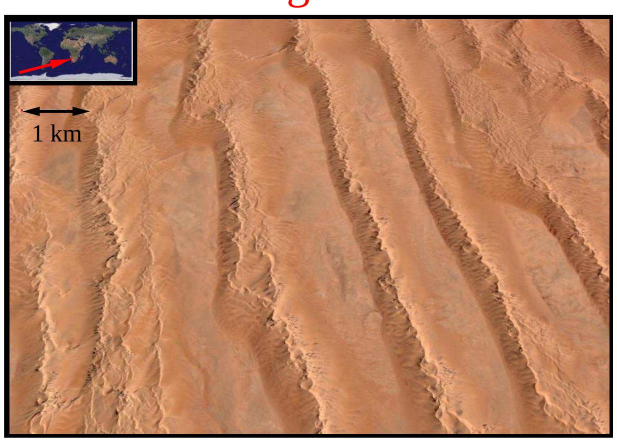


Longitudinal dunes

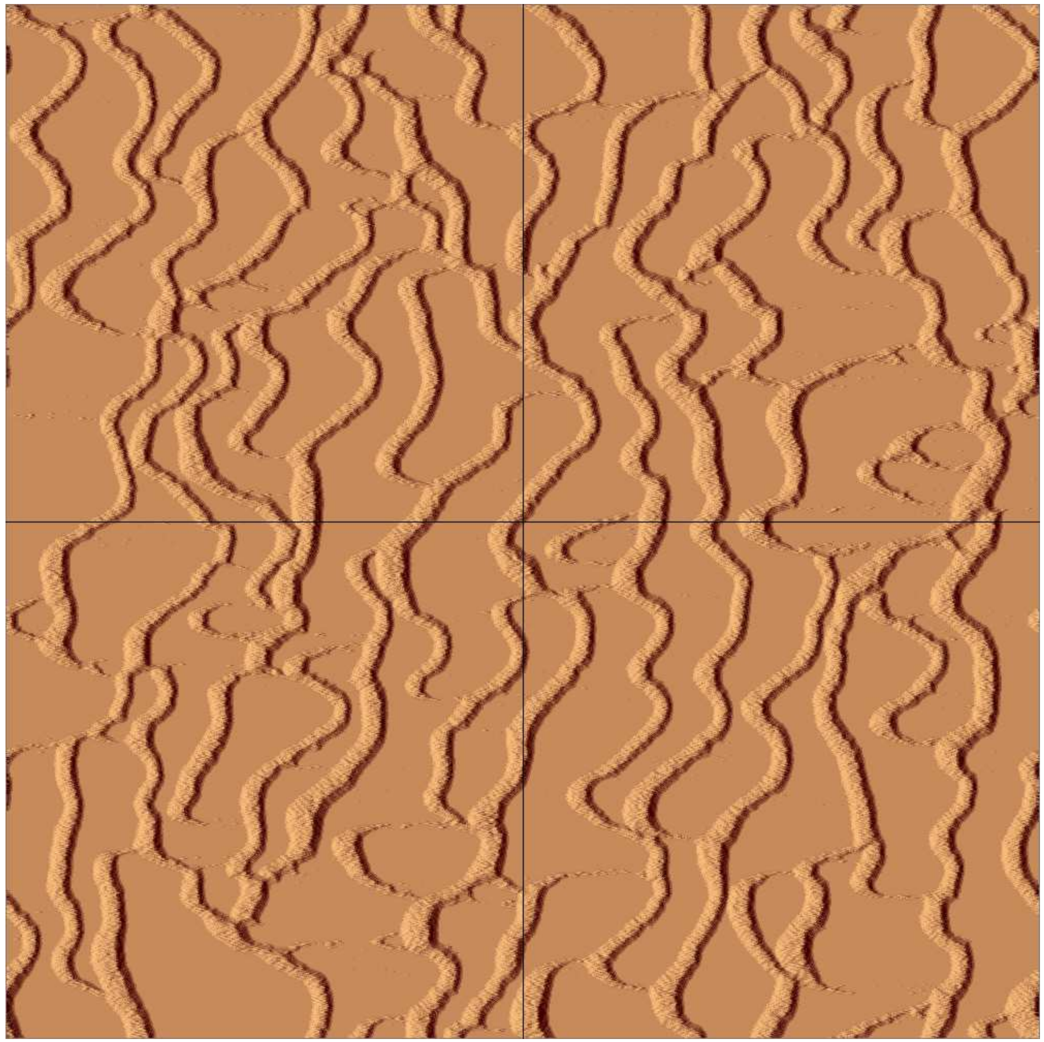


.... Longitudinal

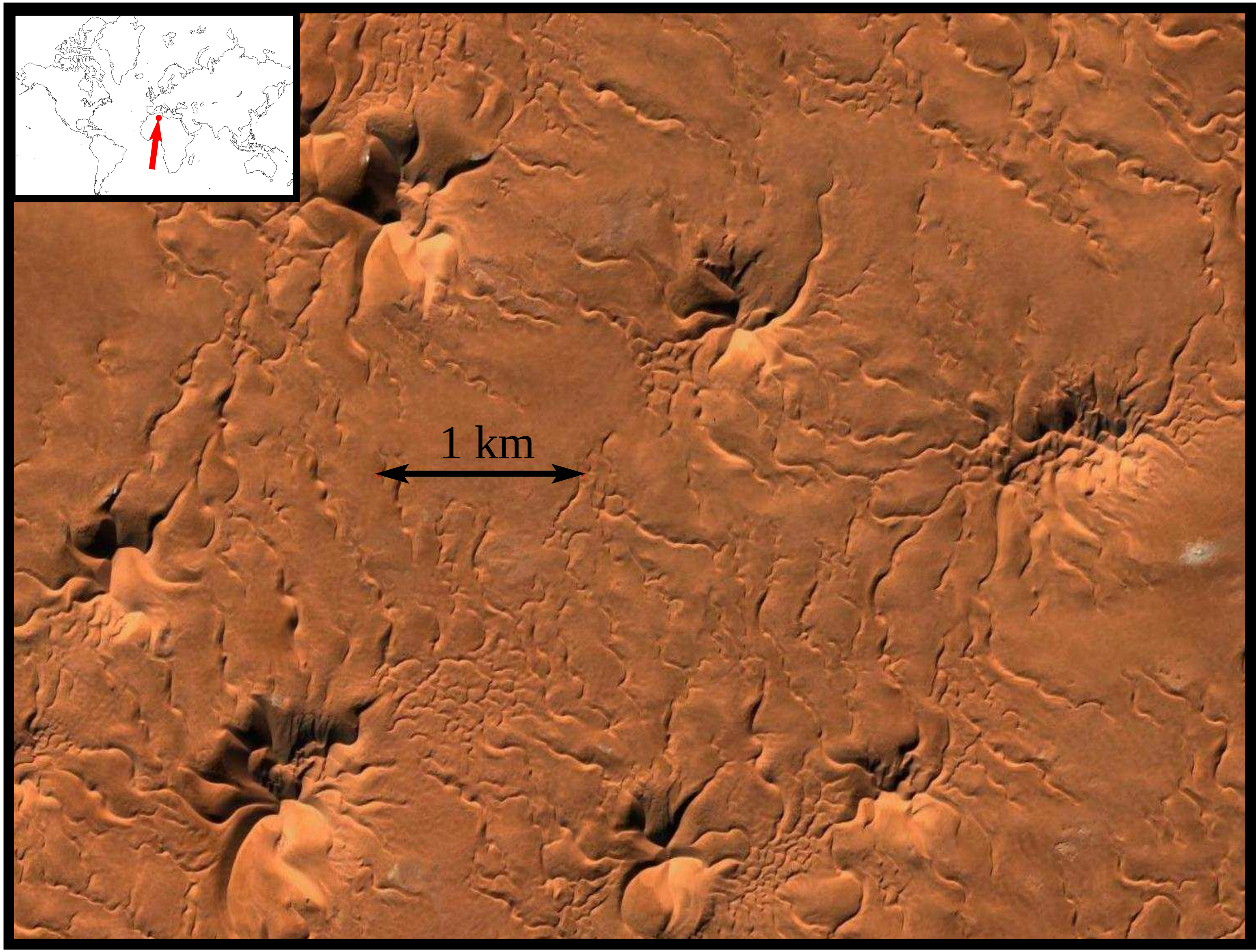
.... Transverse

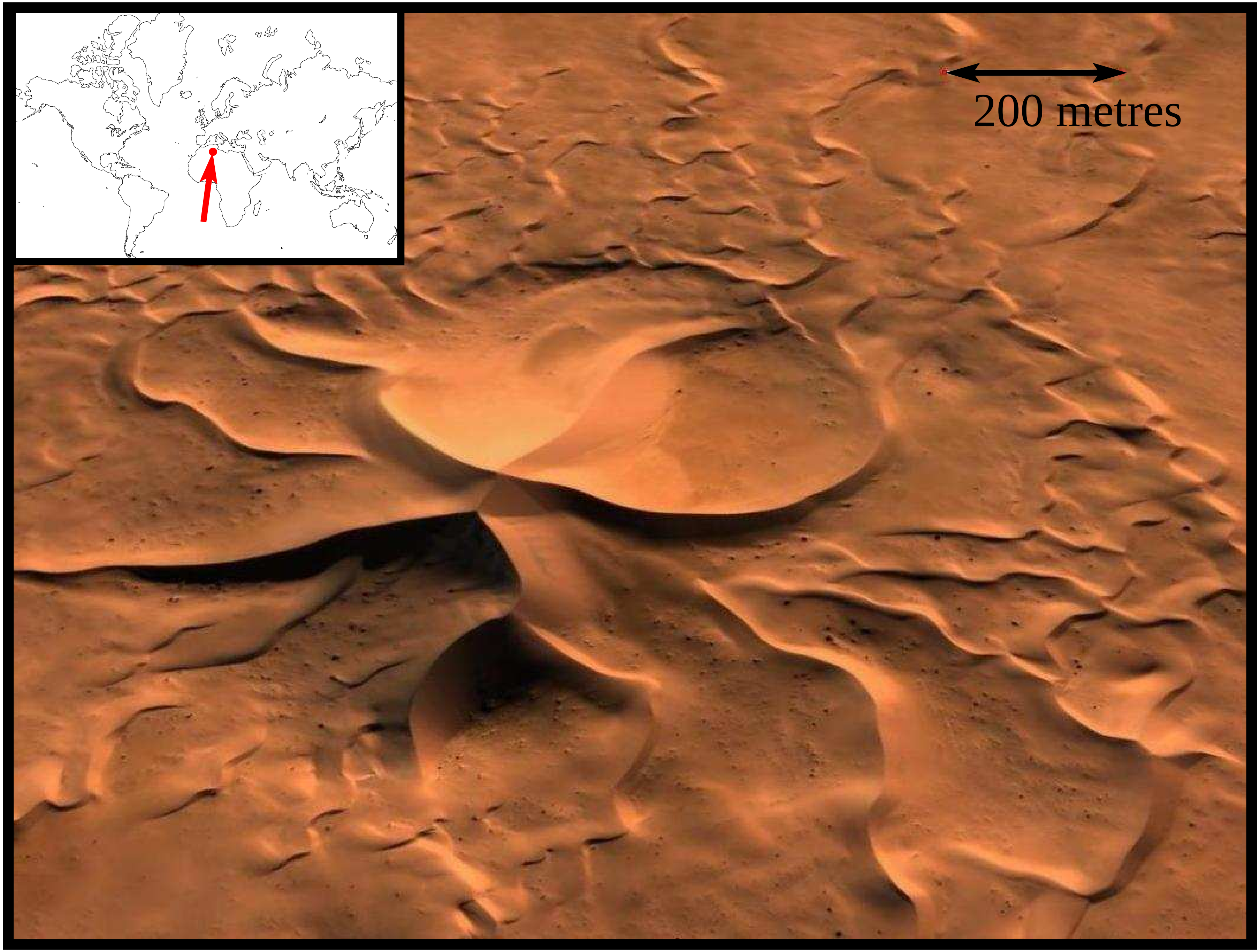


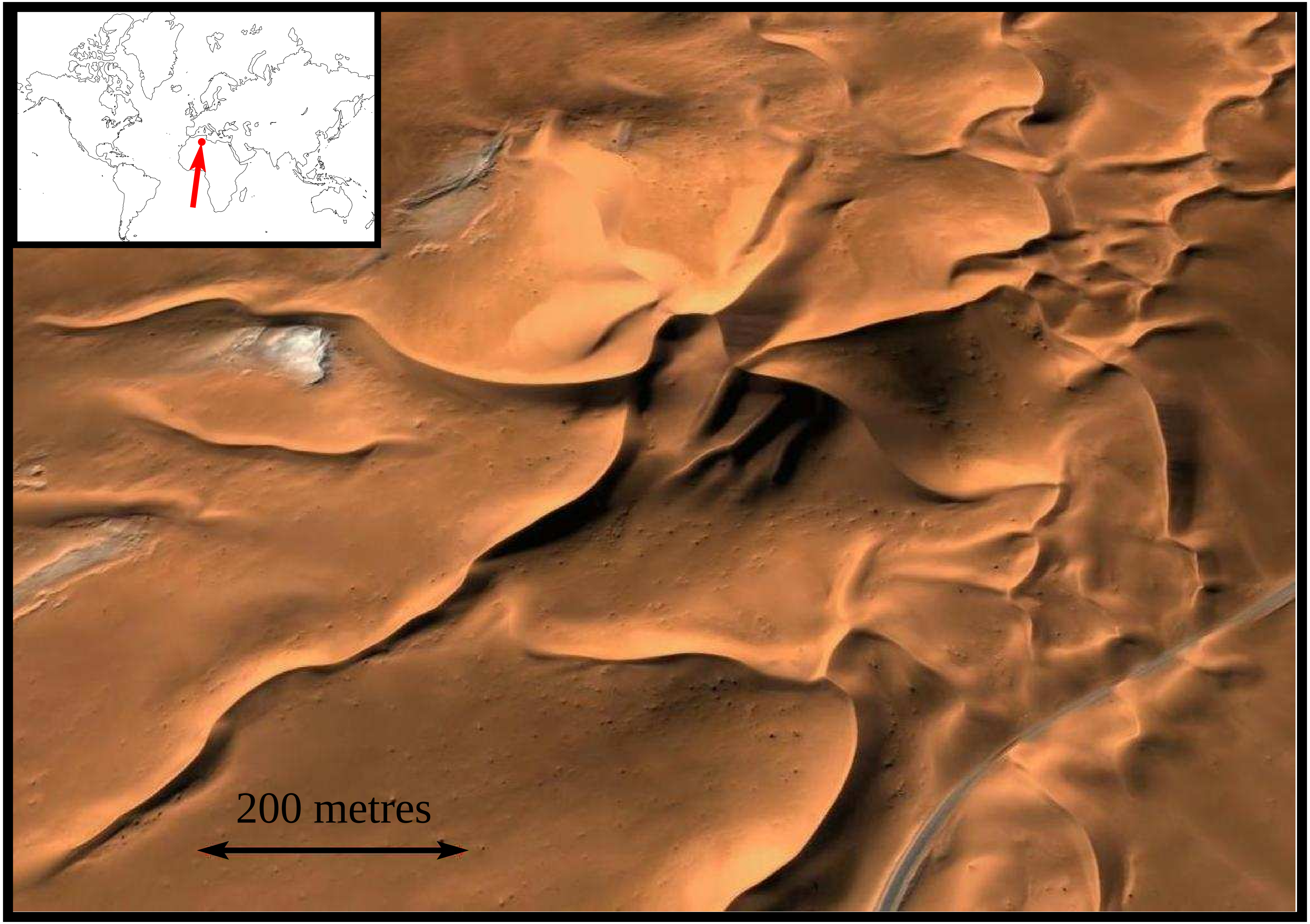
From transverse to barchan dunes



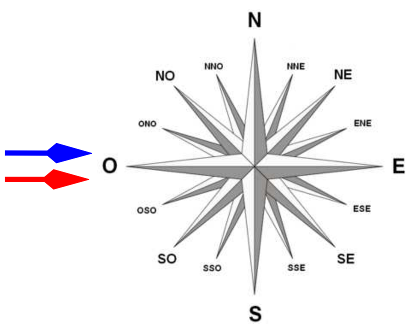
Star dunes



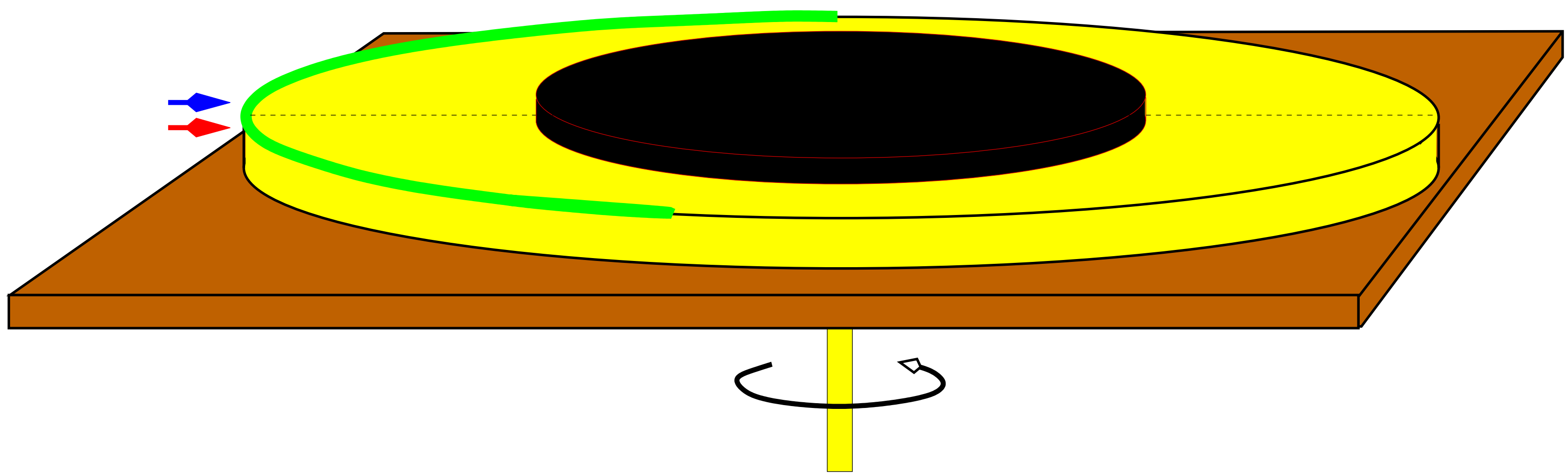

200 metres


200 metres

Rotating table for variable wind orientations

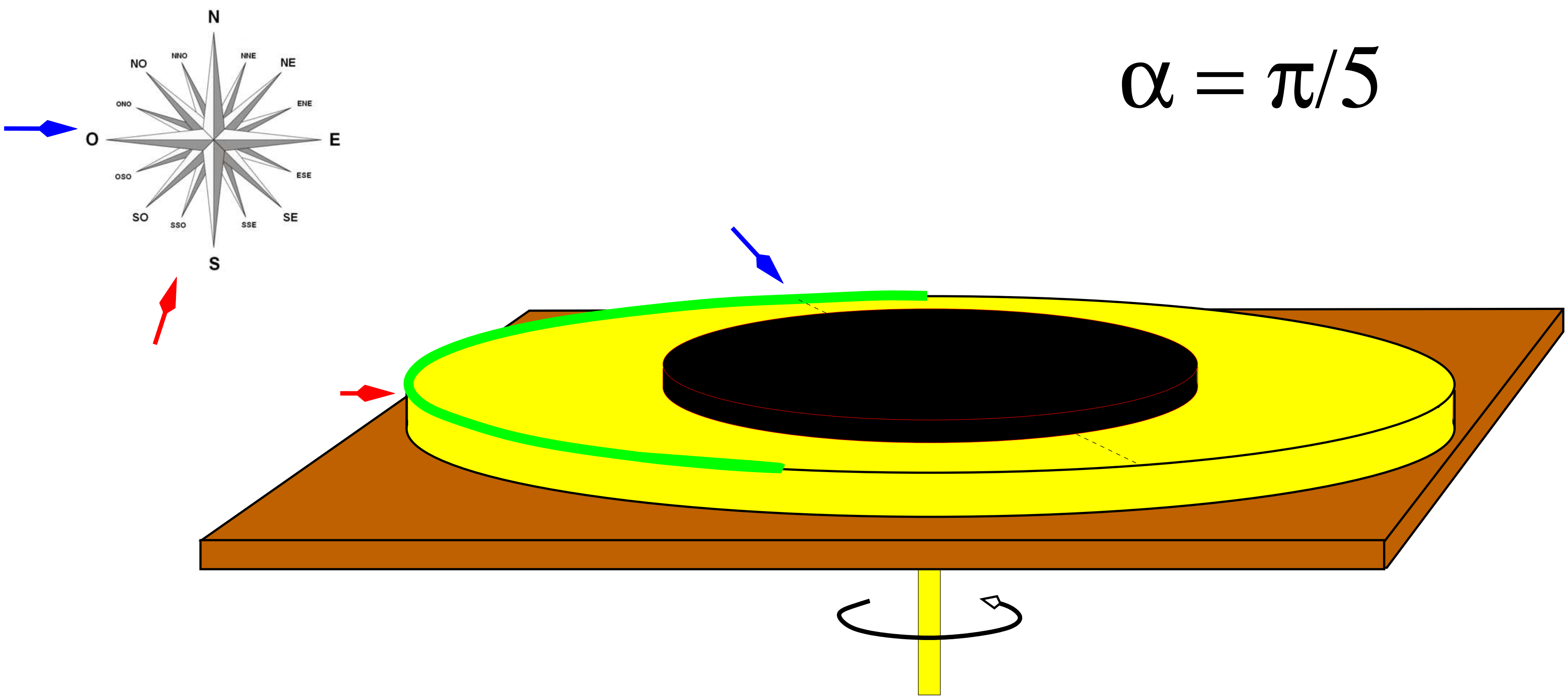


$$\alpha = 0^{\circ}$$

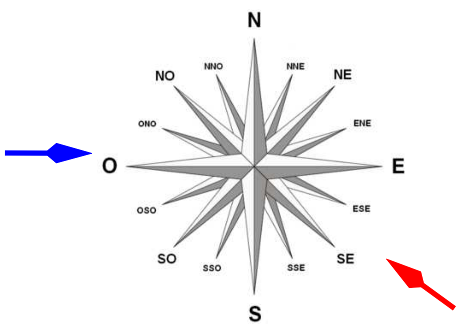


Rotating table for variable wind orientations

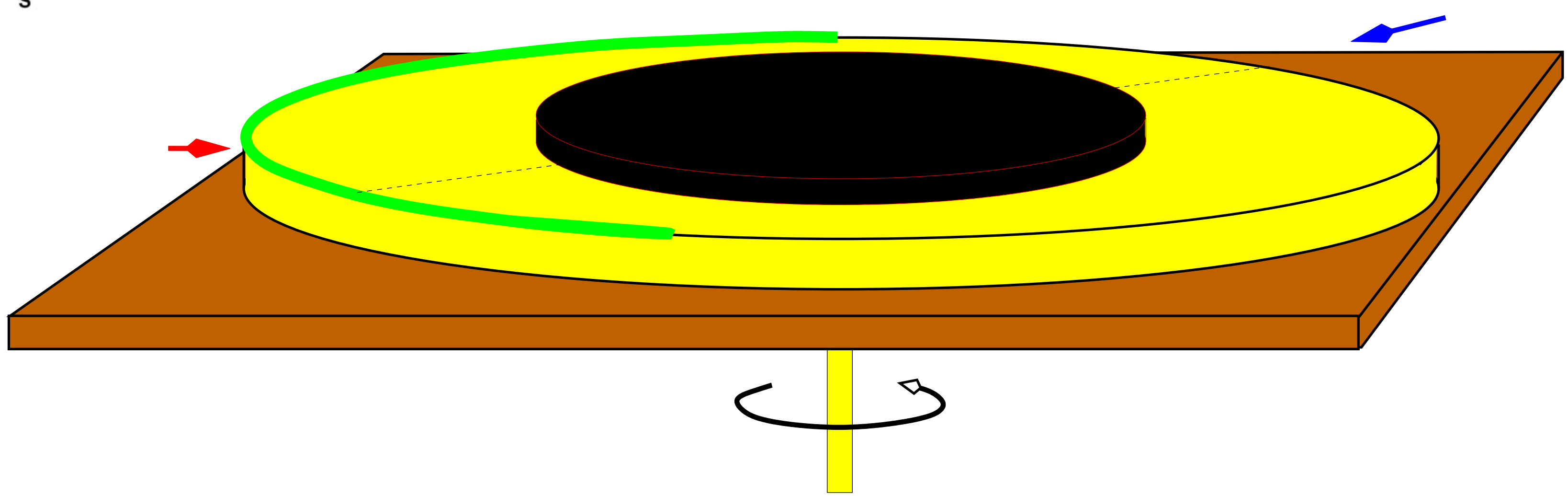
$$\alpha = \pi/5$$



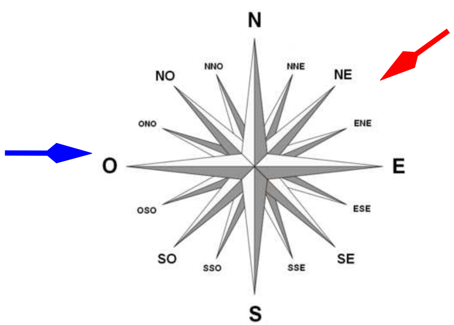
Rotating table for variable wind orientations



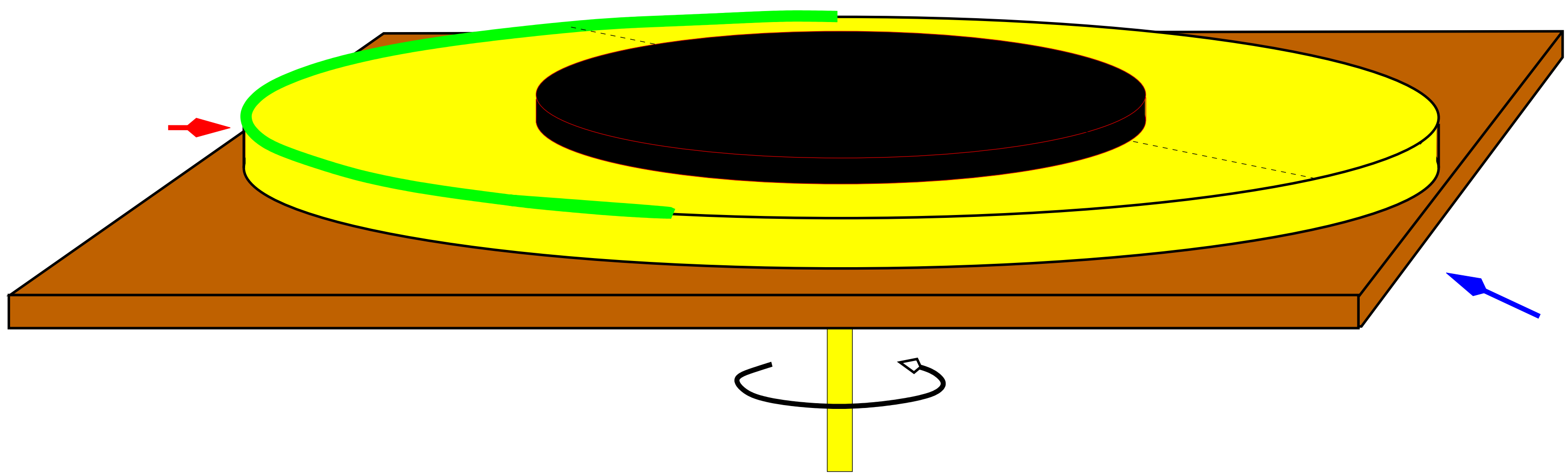
$$\alpha = 2\pi/5$$



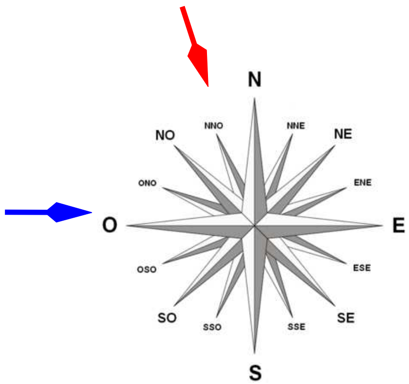
Rotating table for variable wind orientations



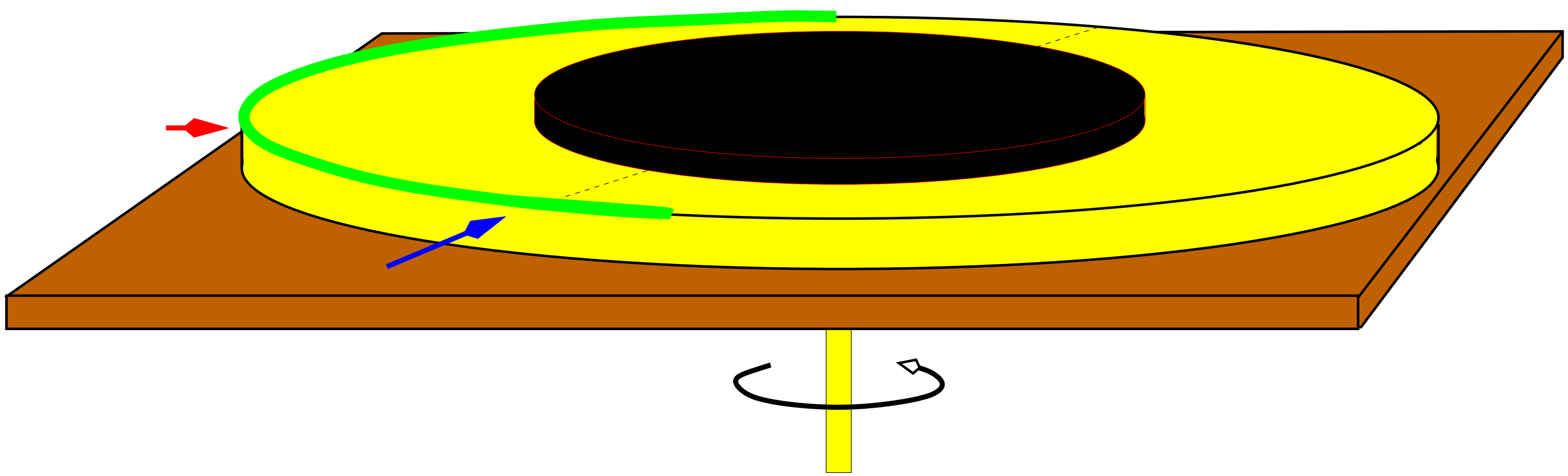
$$\alpha = 3\pi/5$$



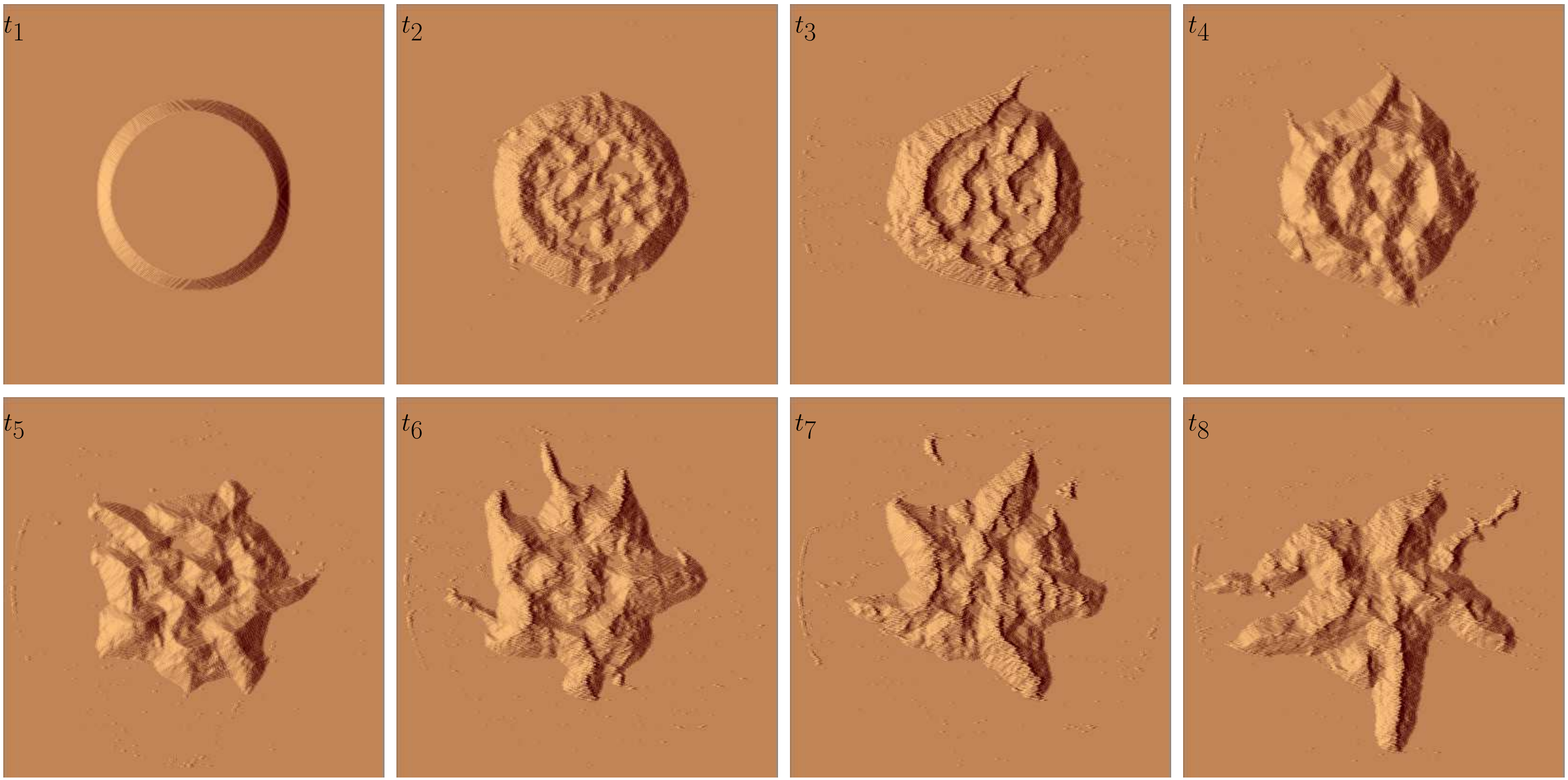
Rotating table for variable wind orientations



$$\alpha = 4\pi/5$$



Star dunes



Star dunes