Morphodynamics of superimposed bedforms in a CA dune model

Characteristic length scale for the nucleation of dunes

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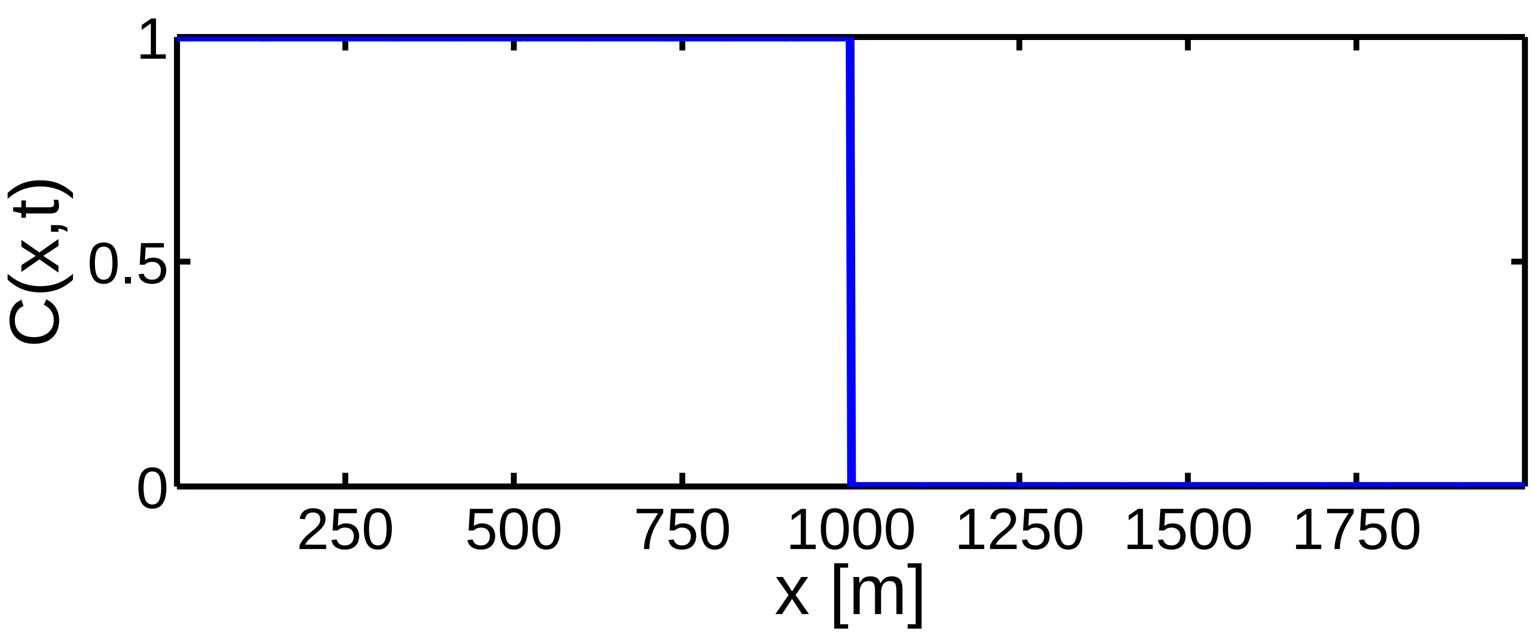
A preamble ...

Deterministic approach for diffusion

$$\frac{\partial C(x,t)}{\partial t} = D\frac{\partial C(x,t)}{\partial x^2}$$

$$\Longrightarrow$$

$$C(x,t) = A\, erf\left(\frac{x}{\sqrt{2Dt}}\right) + B$$



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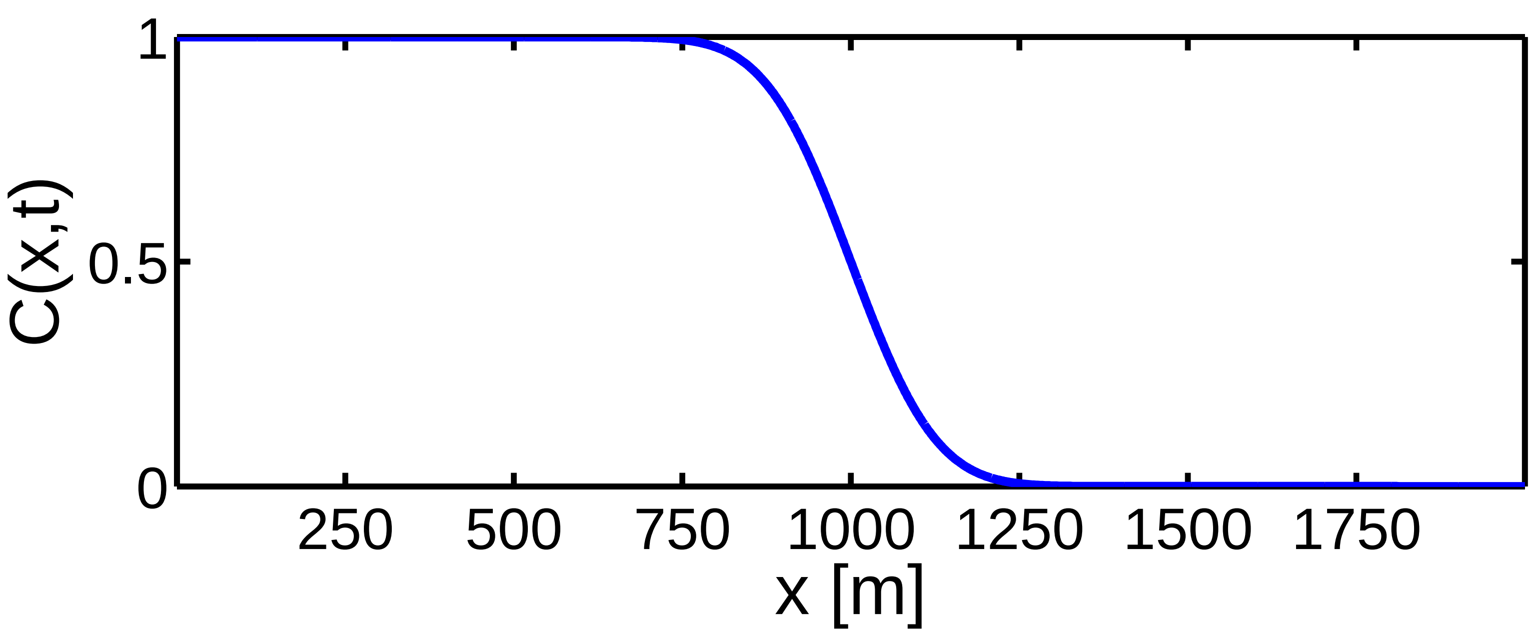

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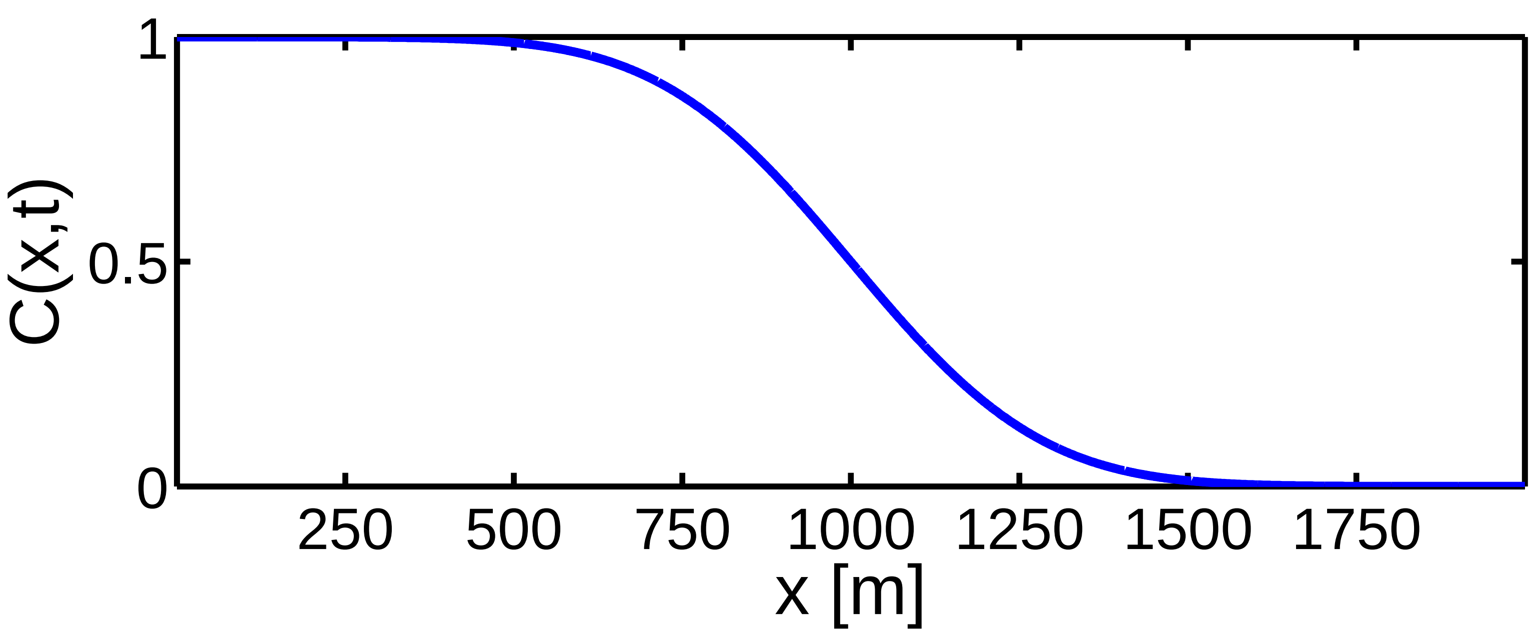

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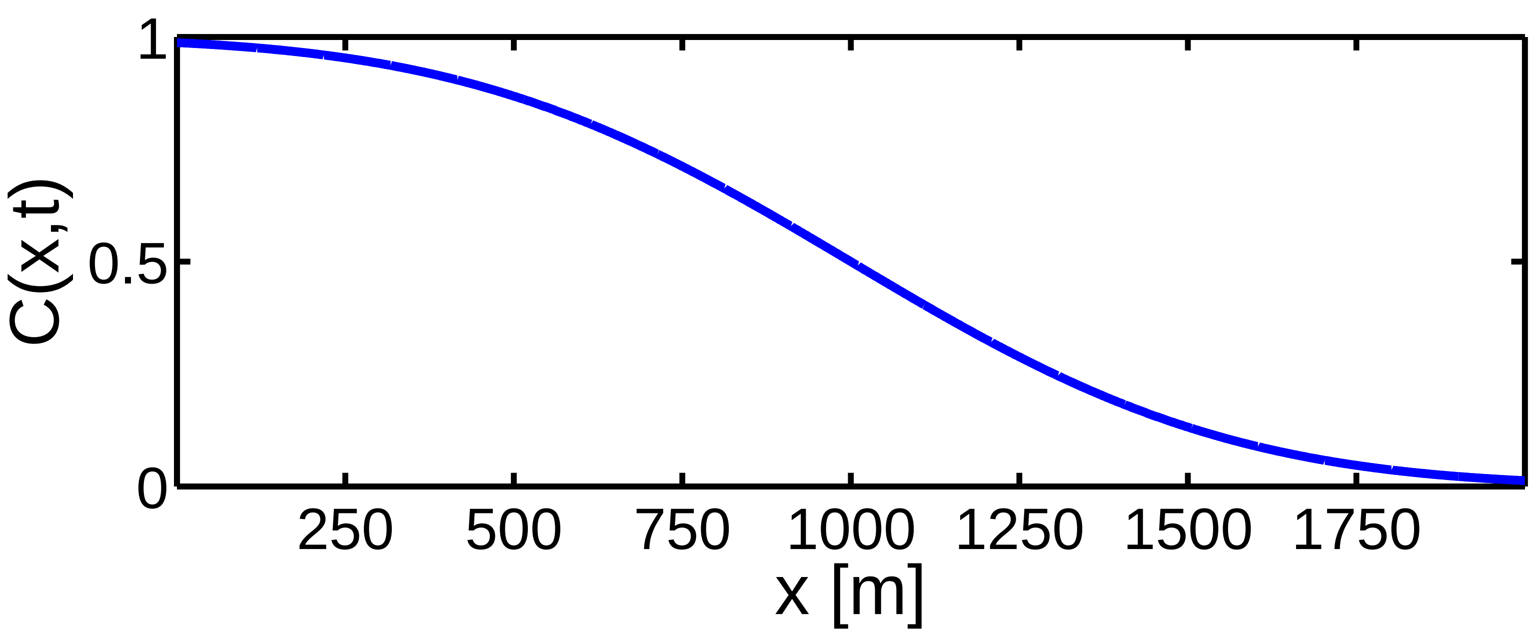

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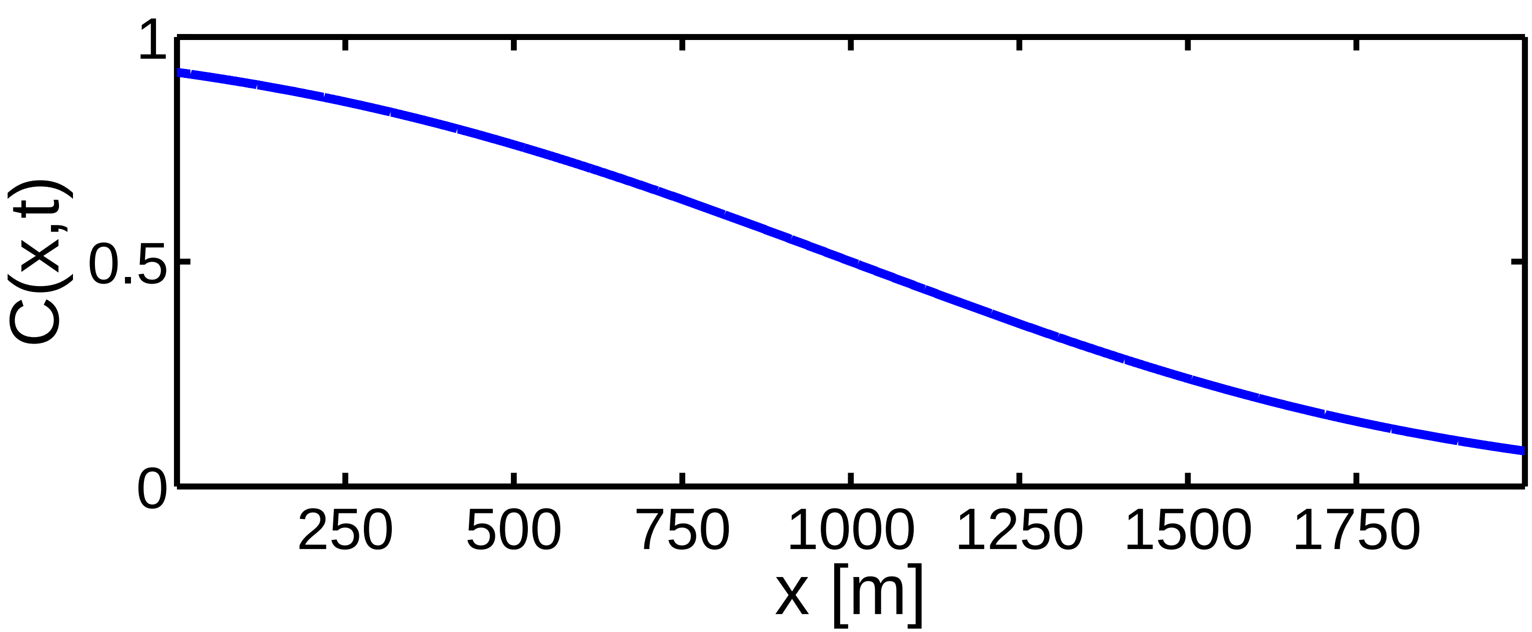
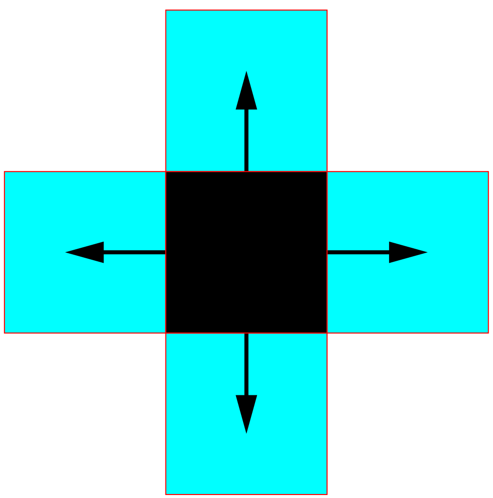
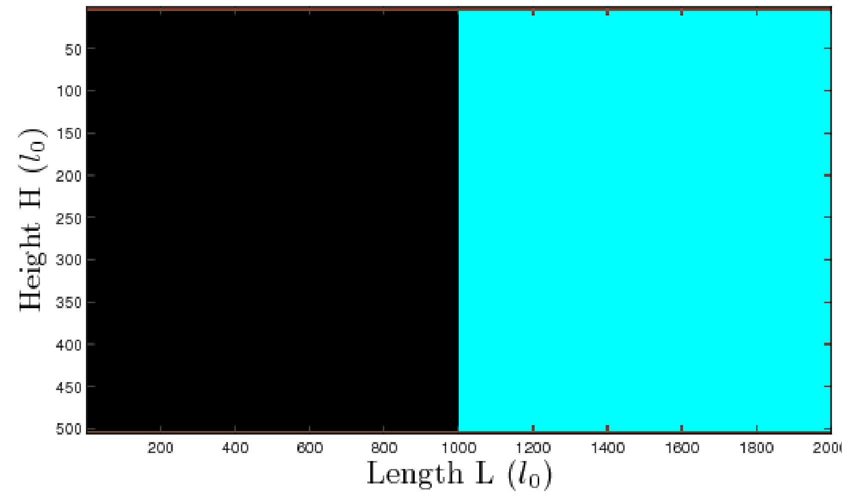
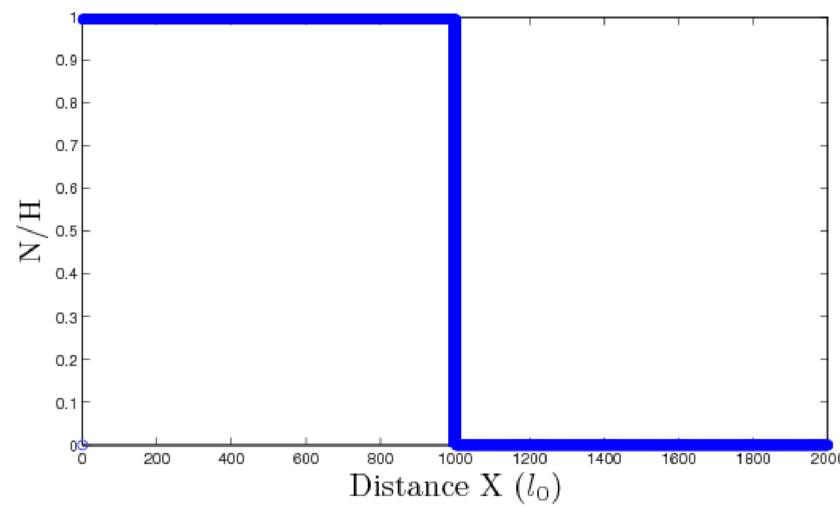


Stochastic approach for diffusion

Brownian motion

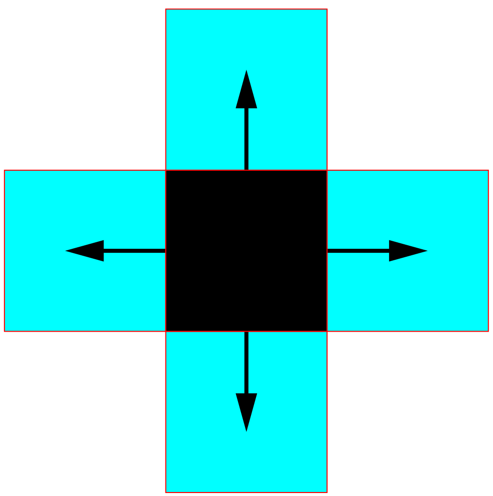


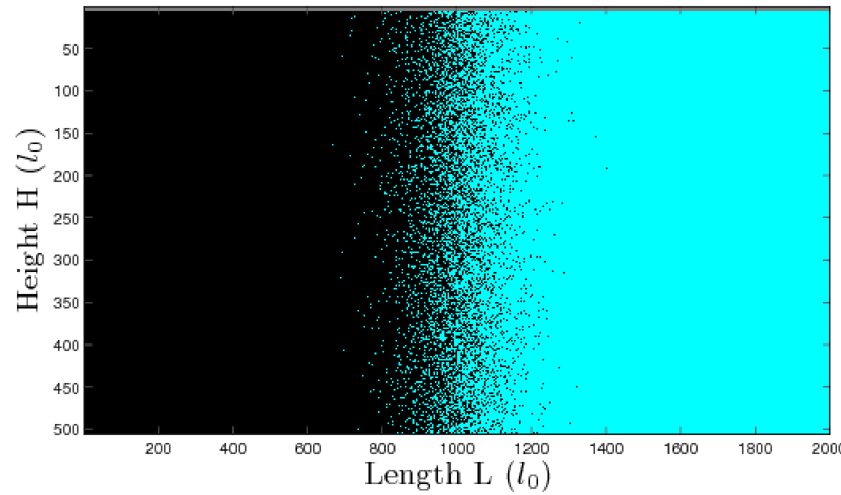


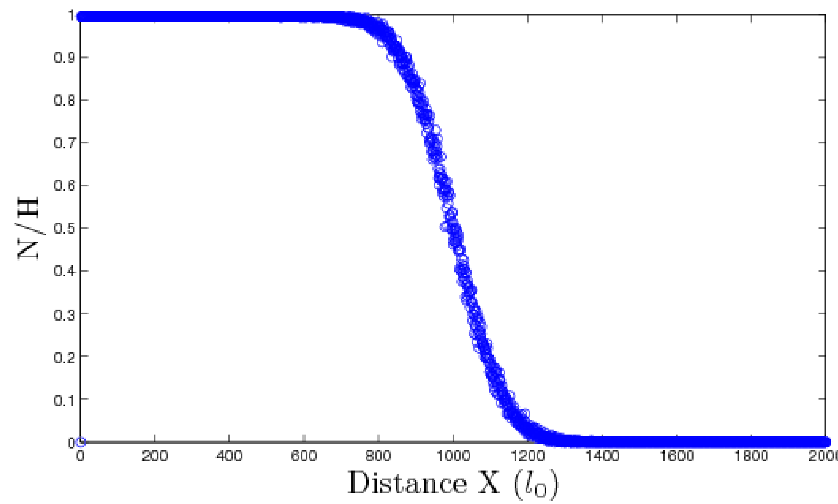


Stochastic approach for diffusion

Brownian motion

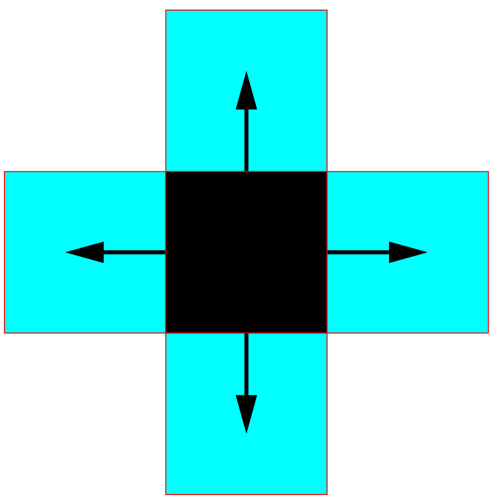


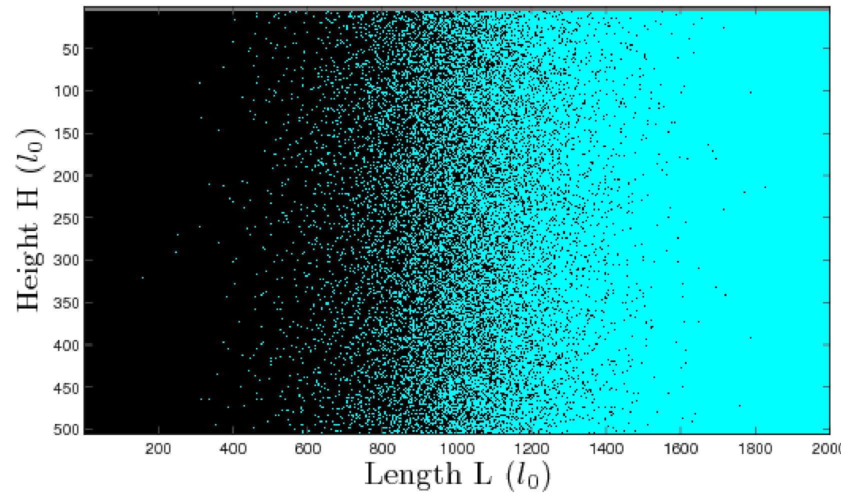


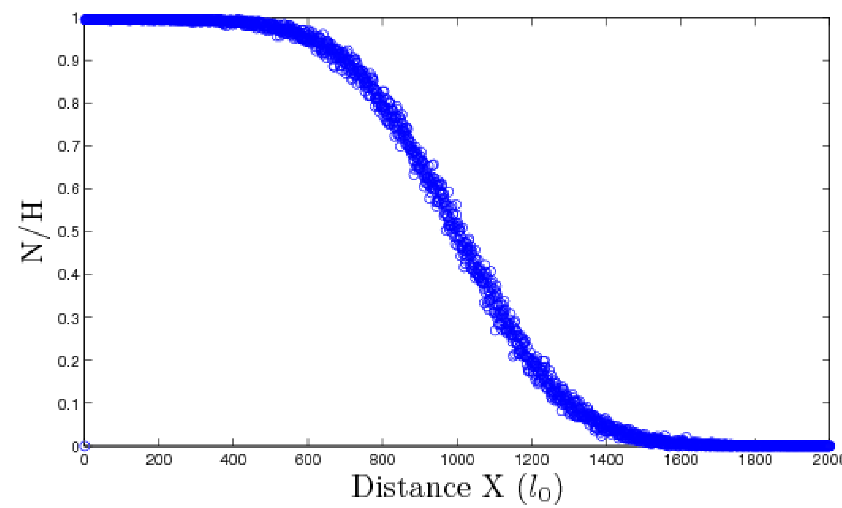


Stochastic approach for diffusion

Brownian motion

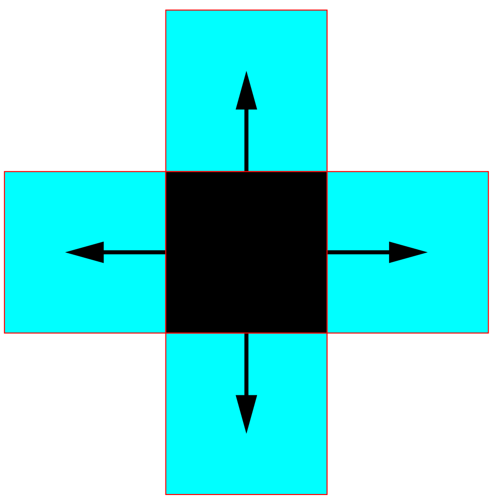


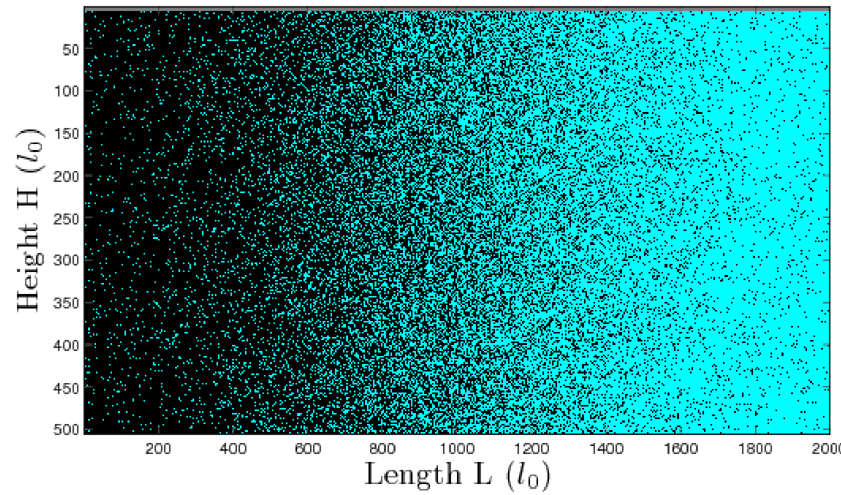


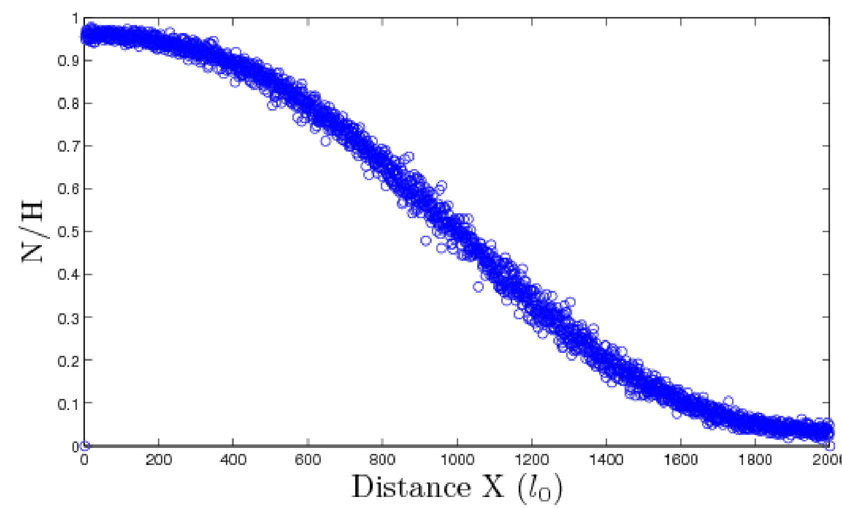


Stochastic approach for diffusion

Brownian motion

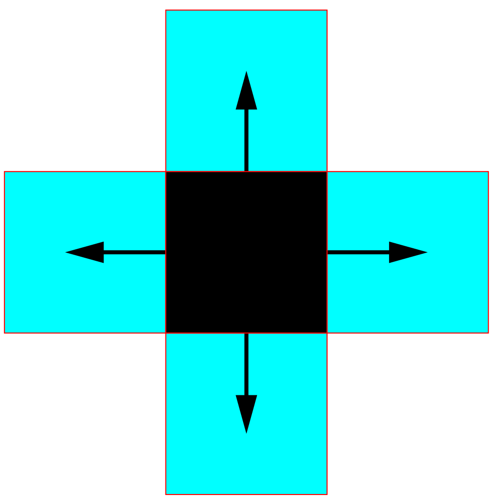


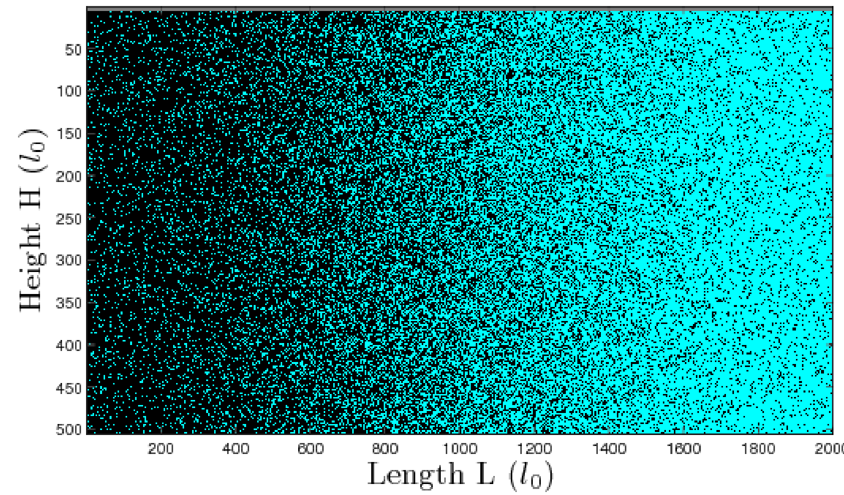


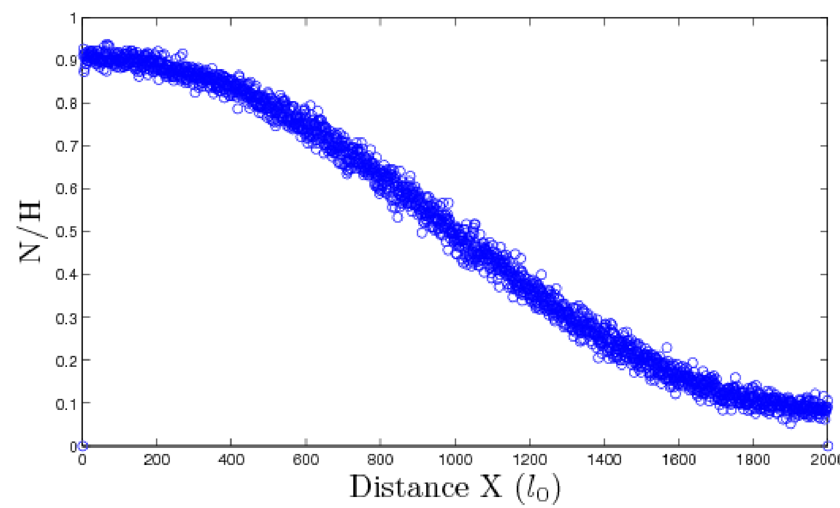


Stochastic approach for diffusion

Brownian motion







Deterministic approach for diffusion

$$\approx$$

Stochastic approach for diffusion

Asymptotic behaviours of different sets of partial differential equations

$$\frac{\partial h}{\partial t} = F(q, S, ...)$$
$$\frac{\partial q}{\partial x} = G(h, S, ...)$$

Scaling

A cellular automaton approach

- A grid of specified shape.
- A collection of cells with different states.
- Rules based on the states of neighboring cells.

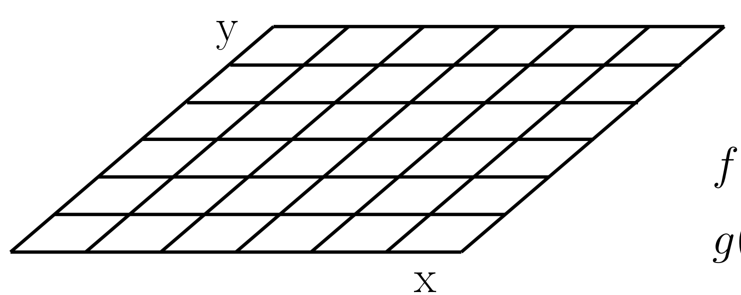
Emergence

Traditional cellular automatons in geophysics

Emergence ?

Scaling ?

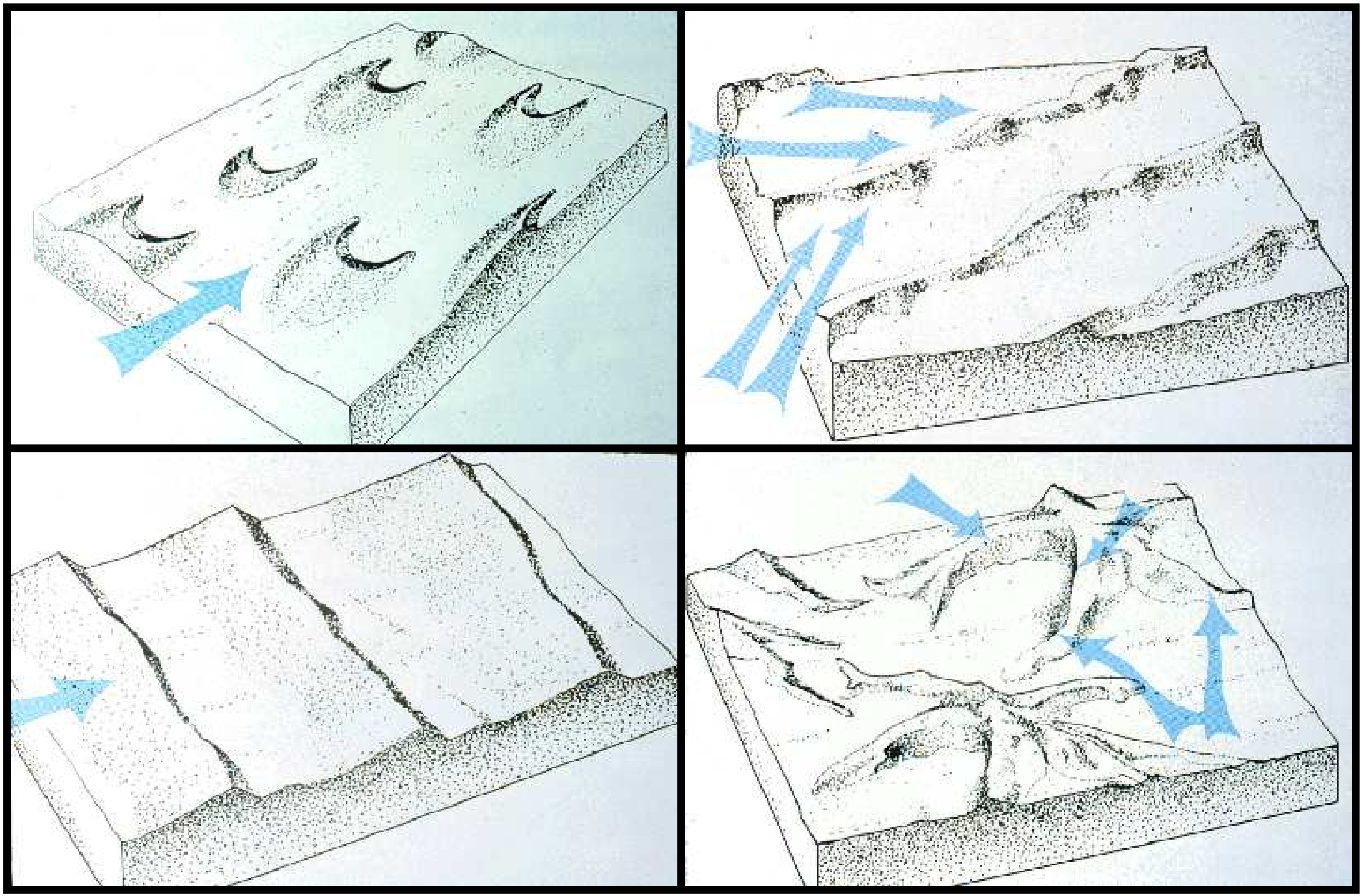
$$x \in \mathbb{N}$$
$$y \in \mathbb{N}$$
$$\Delta t = cte$$

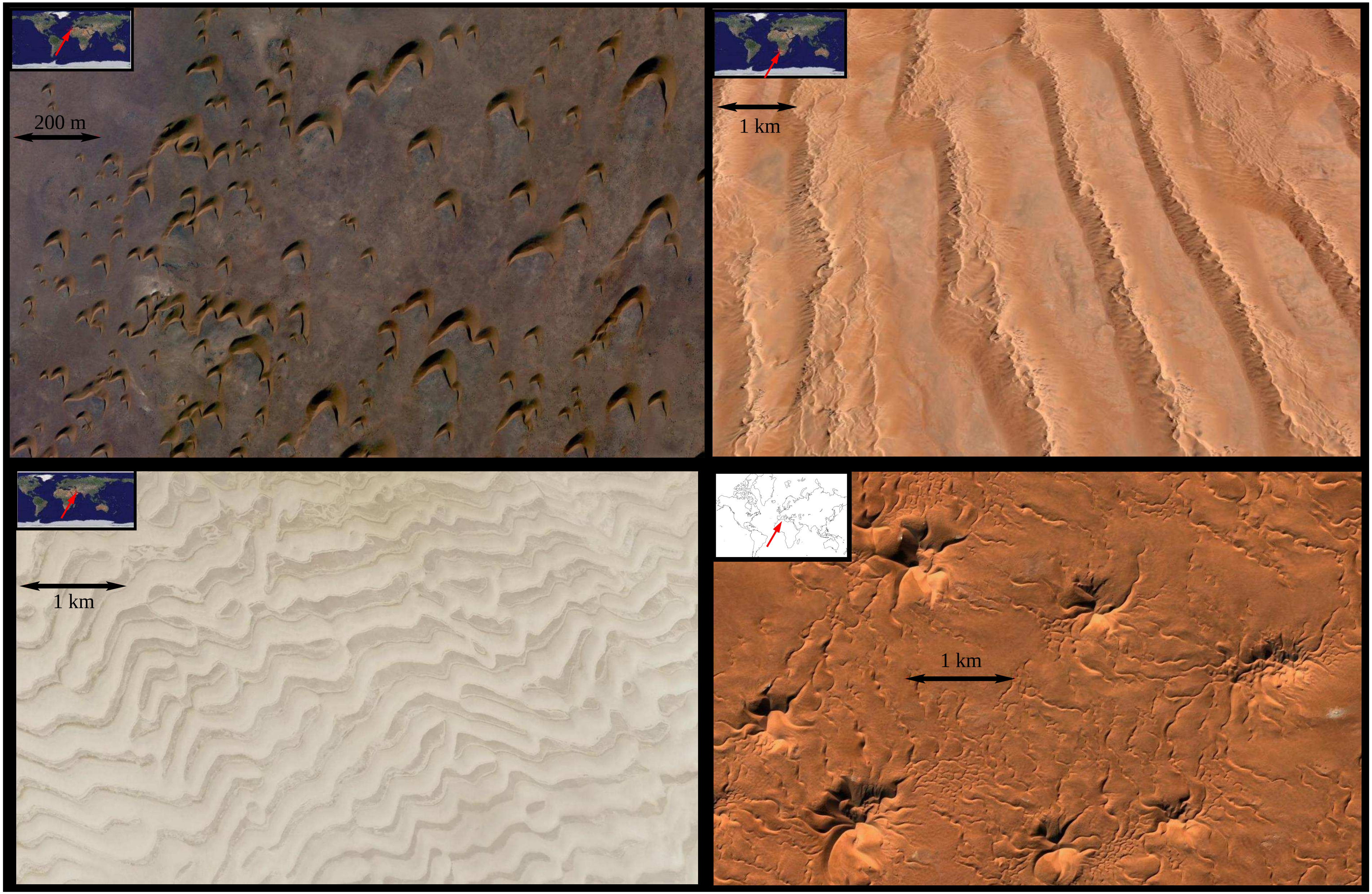


$$f(h(x,y,t), q(x,y,t)) \longrightarrow h(x,y,t+\Delta t)$$
$$g(h(x,y,t), q(x,y,t)) \longrightarrow q(x,y,t+\Delta t)$$

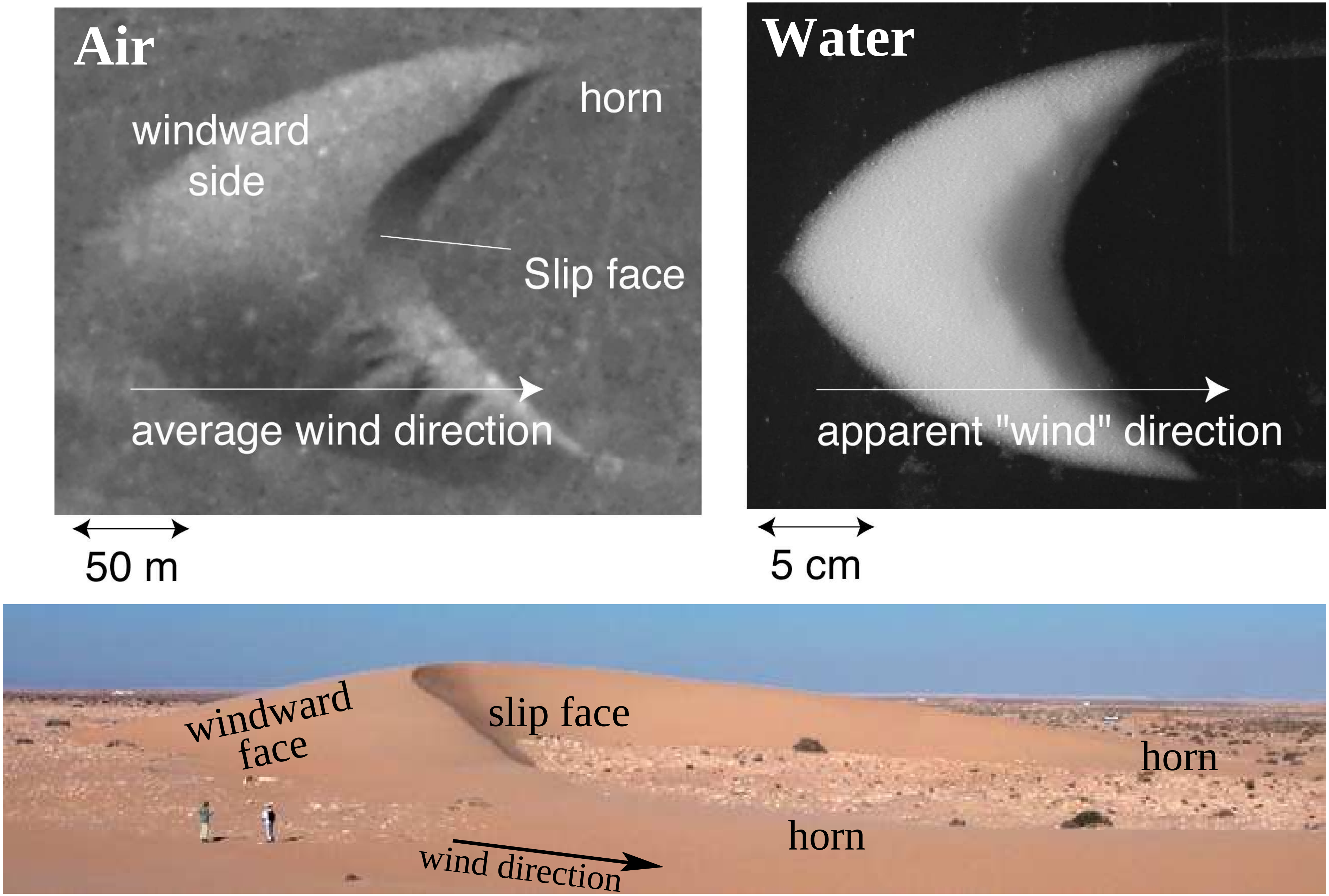
Let us develop new and more theoretical CAs
to combine more efficiently scaling and emergence

Different types of sand dunes





200 m
1 km
1 km
1 km


Air
horn
windward
side
Slip face
average wind direction
50 m
Water
apparent "wind" direction
5 cm
windward
face
slip face
horn
horn
wind direction

A minimum size for dunes

$$l_{drag} = \frac{\rho_s}{\rho_f}\, d_s$$

ρ_f fluid density.

ρ_s sediment density.

d_s grain diameter.

The physics of aeolian dunes

h the height profile

q the sand flux per unit width

$$\frac{\partial h}{\partial t} = - \frac{\partial q}{\partial x}$$

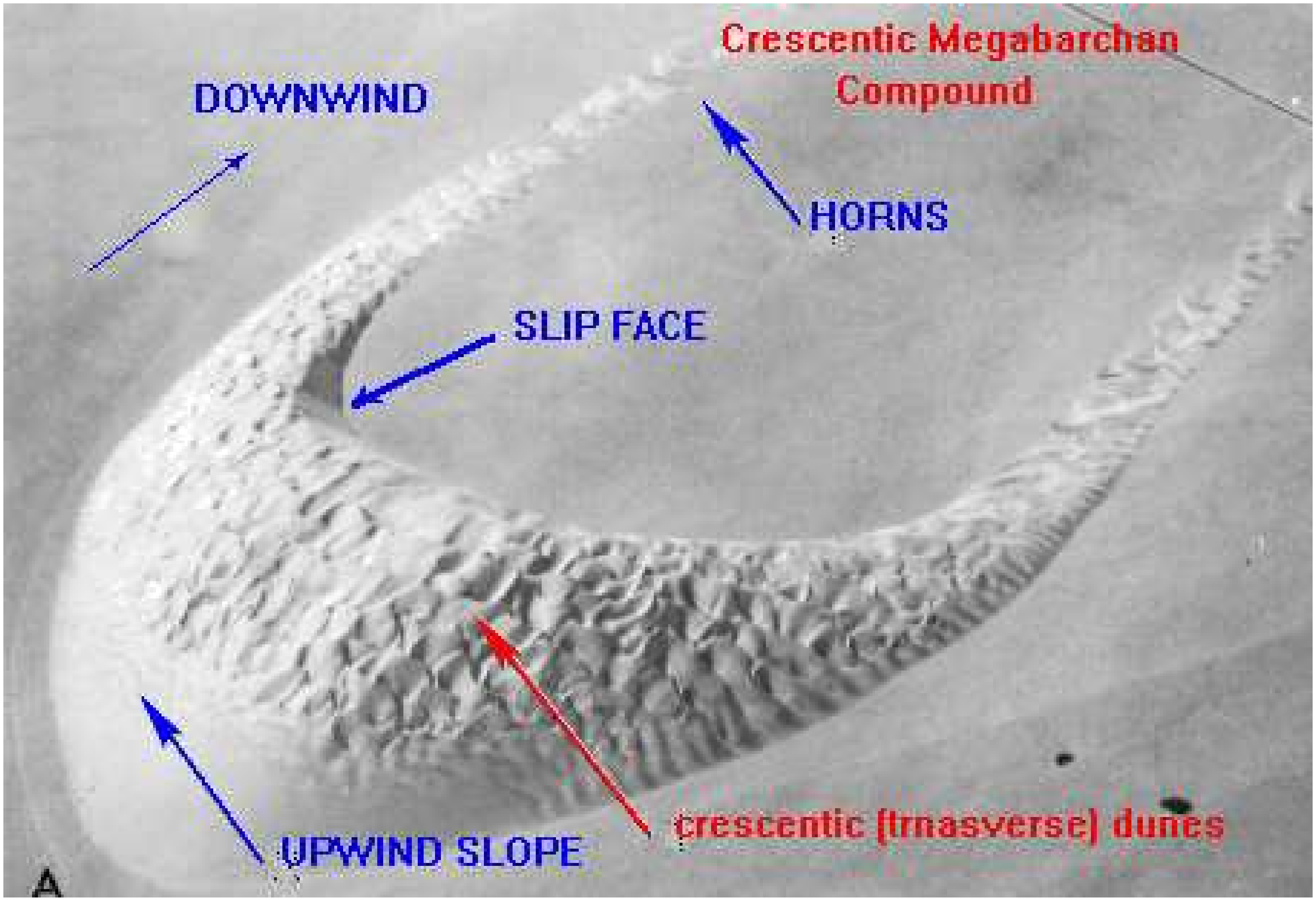
Q_{sat} the saturated sand flux

l_{sat} the saturation length.

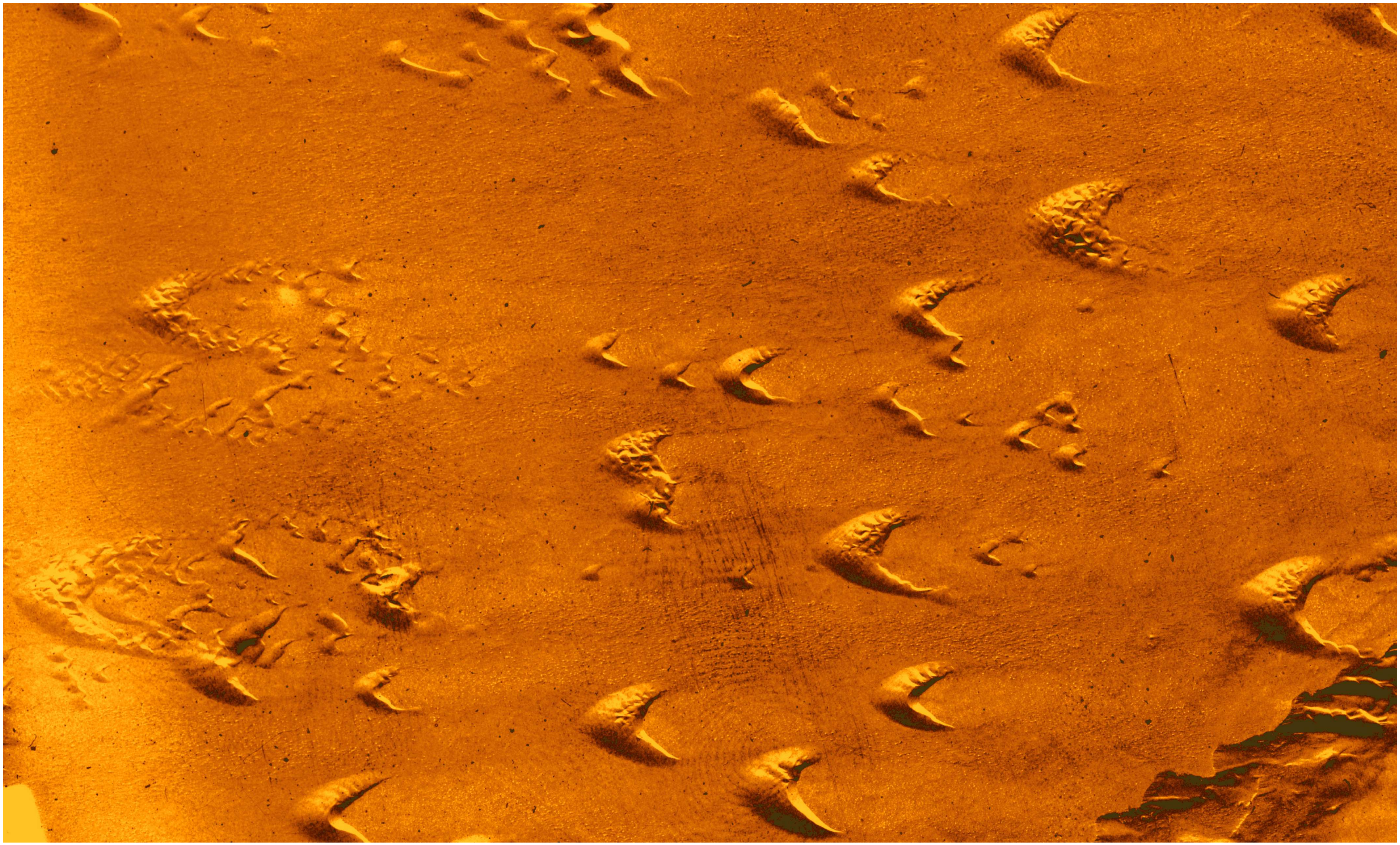
$$\frac{\partial q}{\partial x} \sim \frac{Q_{sat} - q}{l_{sat}}$$

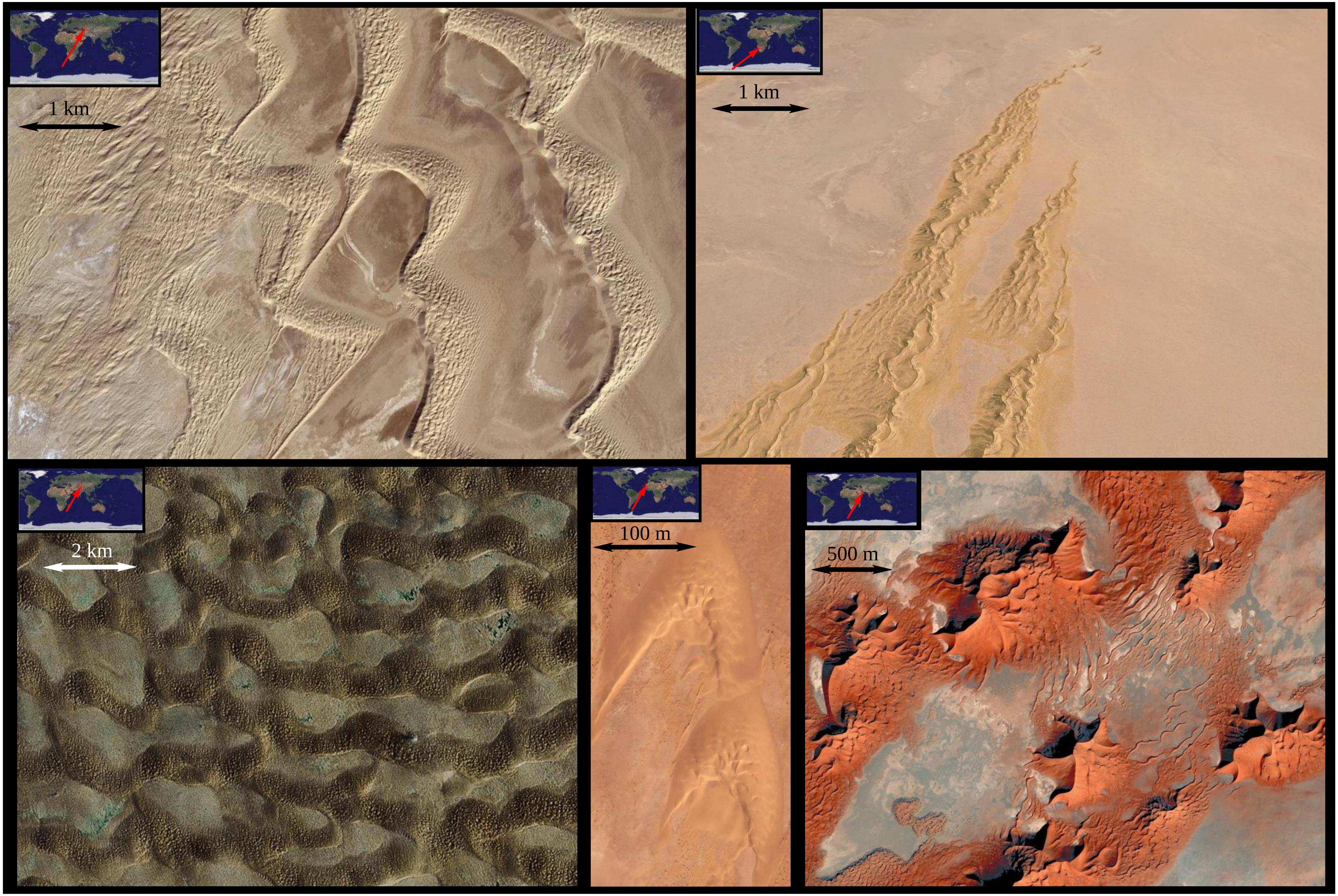
An elegant formalism which gives solutions that can be used as a benchmark for numerical codes.

Superimposed dune patterns



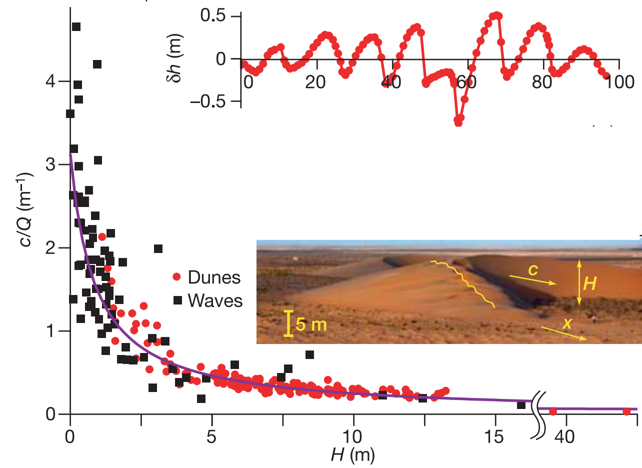
Superimposed dune patterns



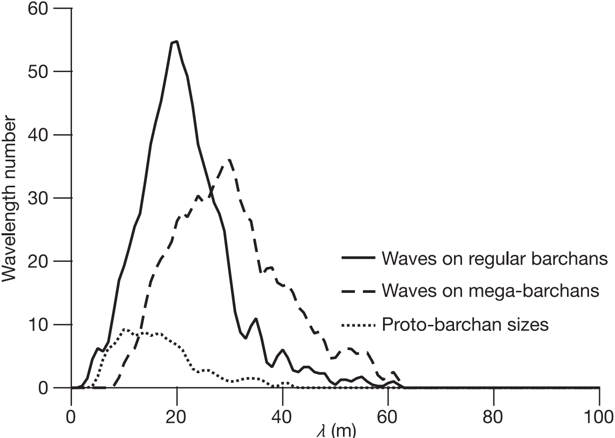

1 km
1 km
2 km
100 m
500 m

Superimposed dune patterns

Elberithi et al. (2005)

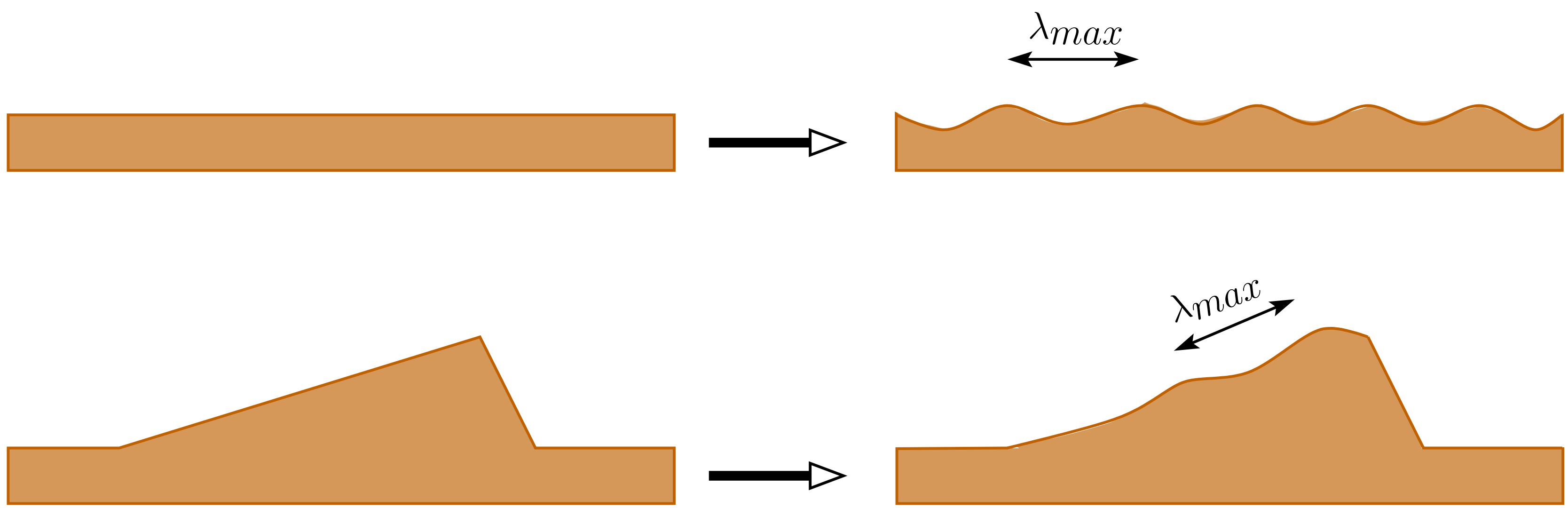


δh (m)
0.5
0
−0.5
0
20
40
60
80
100
c/Q (m−1)
4
3
2
1
0
Dunes
Waves
c
H
5 m
x
0
5
10
15
40
H (m)



Wavelength number
60
50
40
30
20
10
0
0
20
40
60
80
100
λ (m)
Waves on regular barchans
Waves on mega-barchans
Proto-barchan sizes

A sand wave instability



Characteristic length scale for the nucleation of dunes

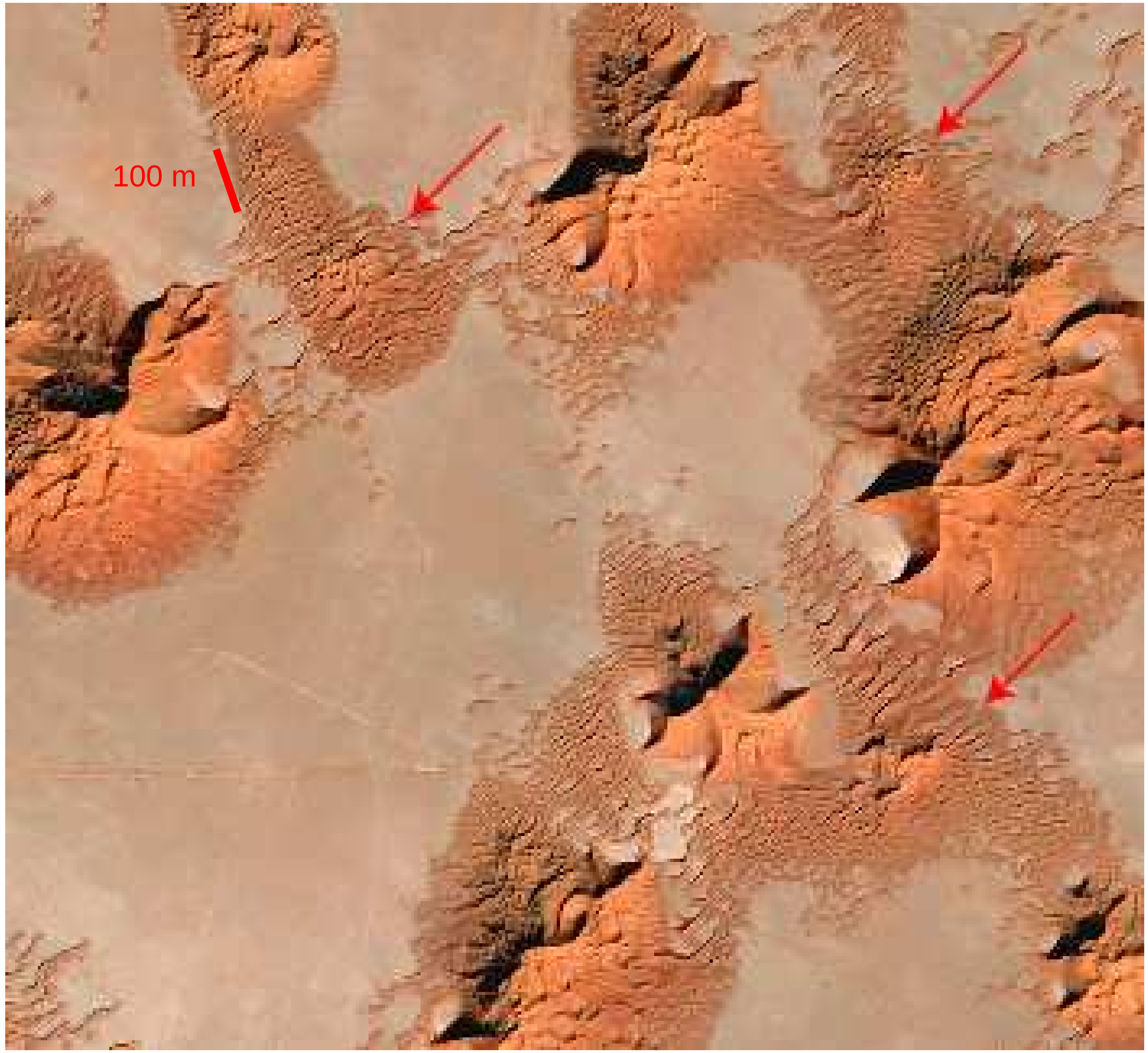
$$\lambda_{max} = 20 \text{ m}$$

on a flat sand bed and dunes (if they are big enough).

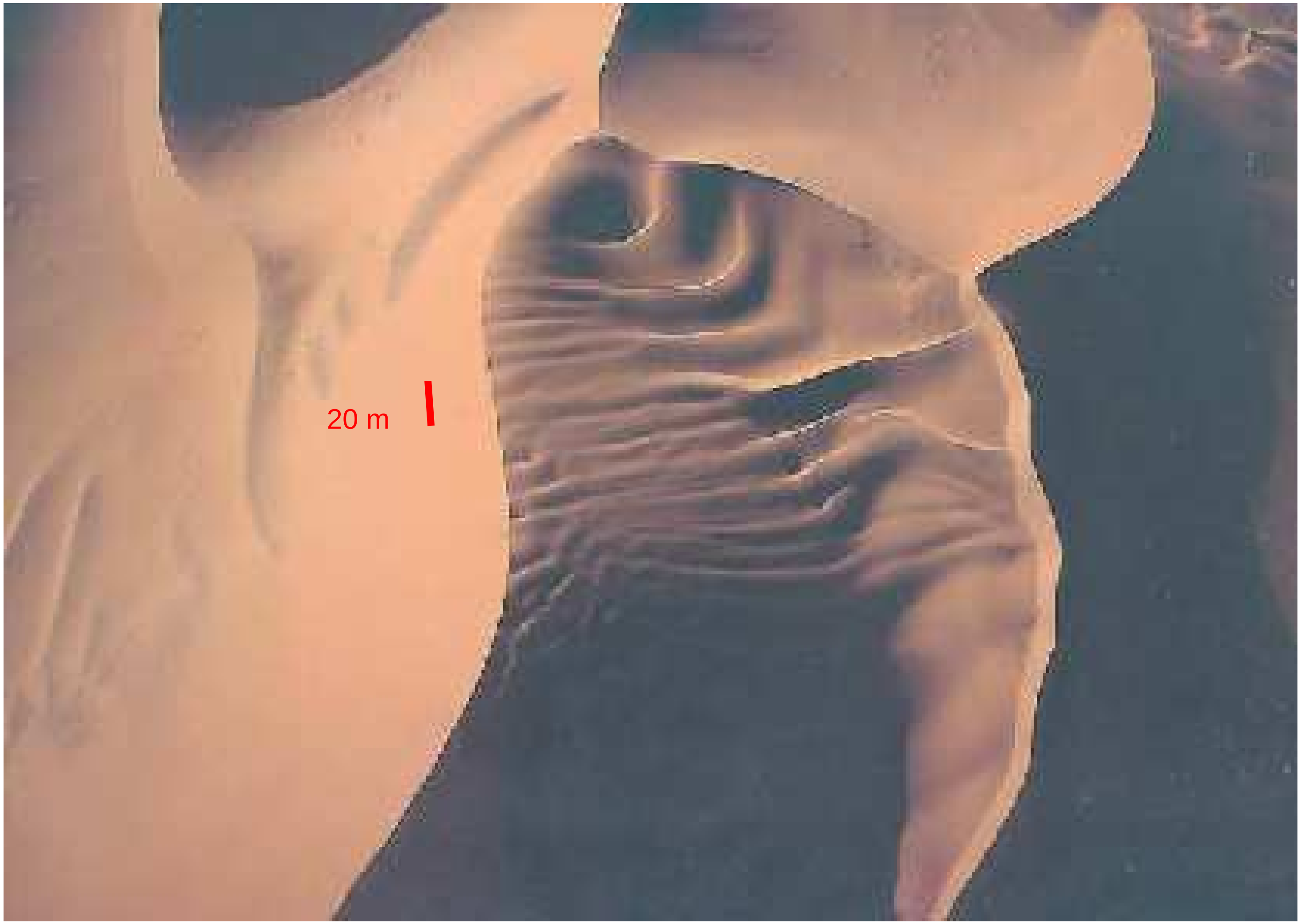
Superimposed dune patterns

Superimposed dune patterns

Superimposed dune patterns



Superimposed dune patterns



A large set of observations ☑

Qualitative descriptions of dune shapes ☑

Elegant physical formalisms ☑

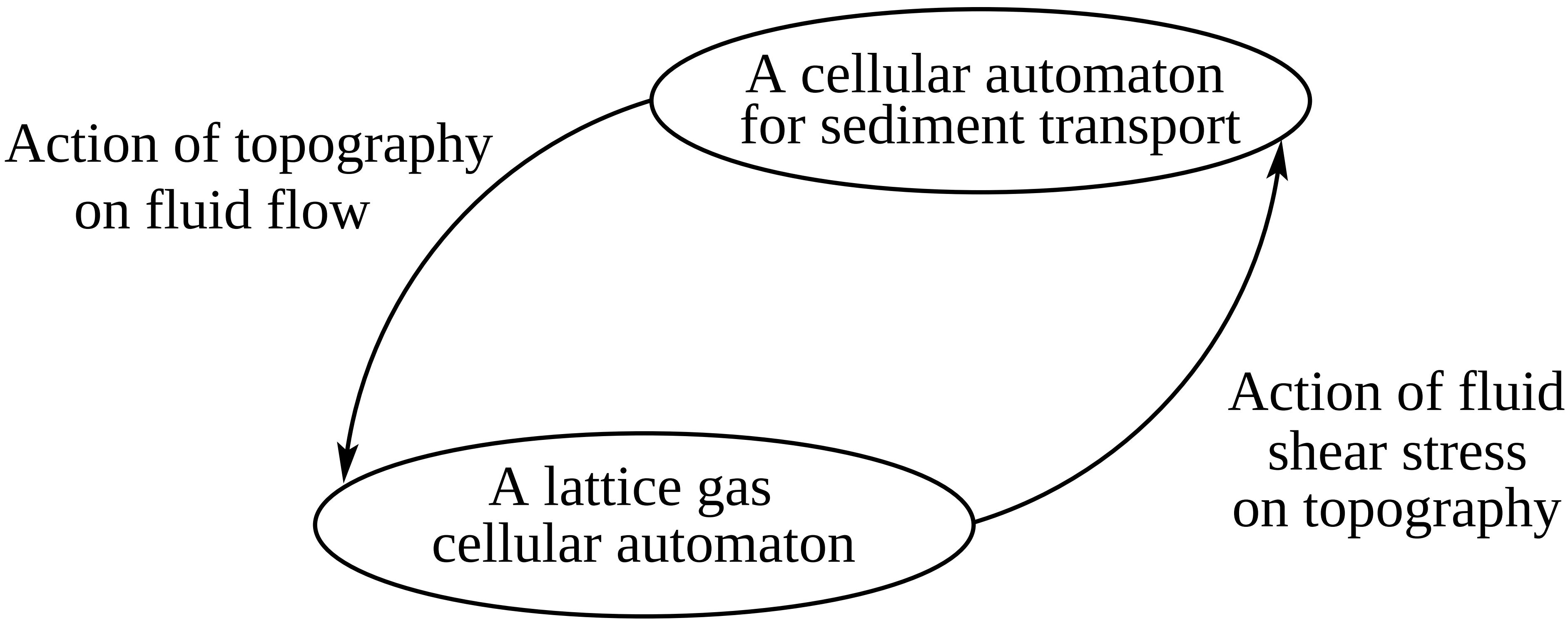
Pattern selection ☐

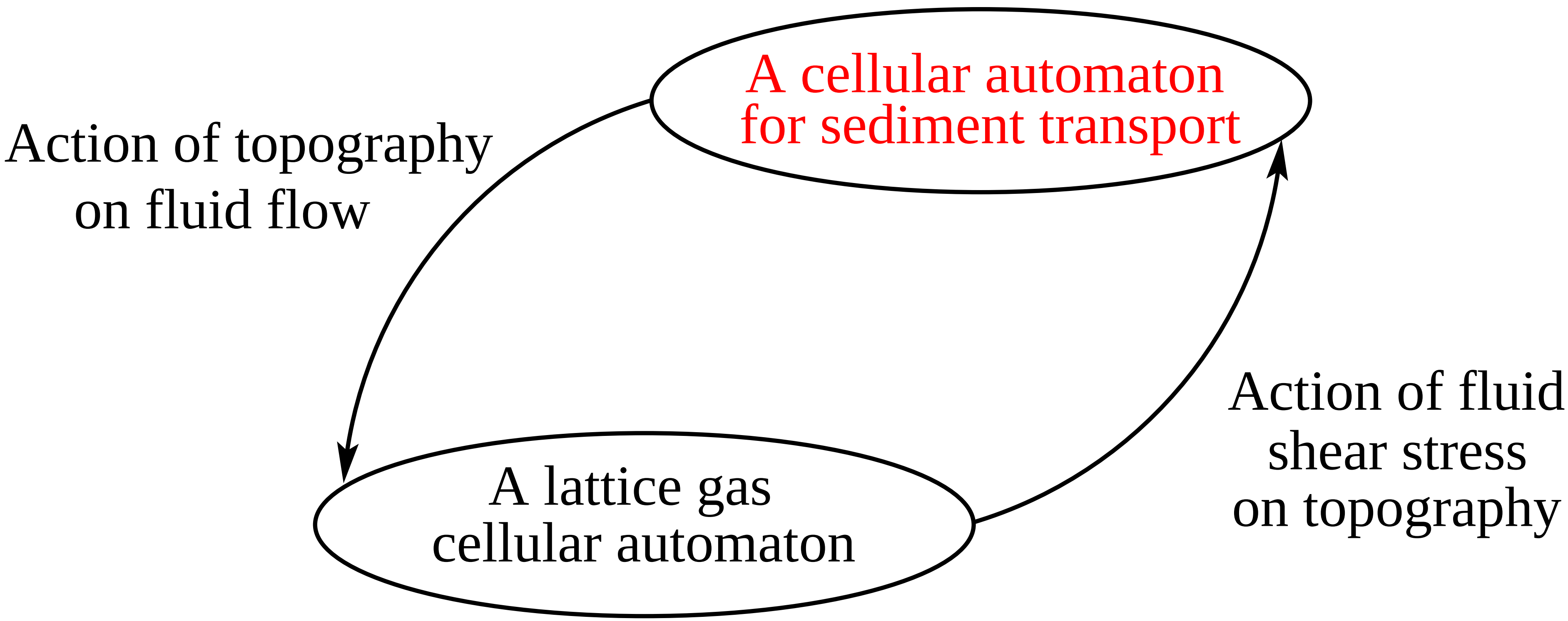
Population dynamics ☐

A quantitative analysis of sediment transport ☐

$\Downarrow$

The physics of sand dunes
as a complex system


A cellular automaton
for sediment transport
Action of topography
on fluid flow
A lattice gas
cellular automaton
Action of fluid
shear stress
on topography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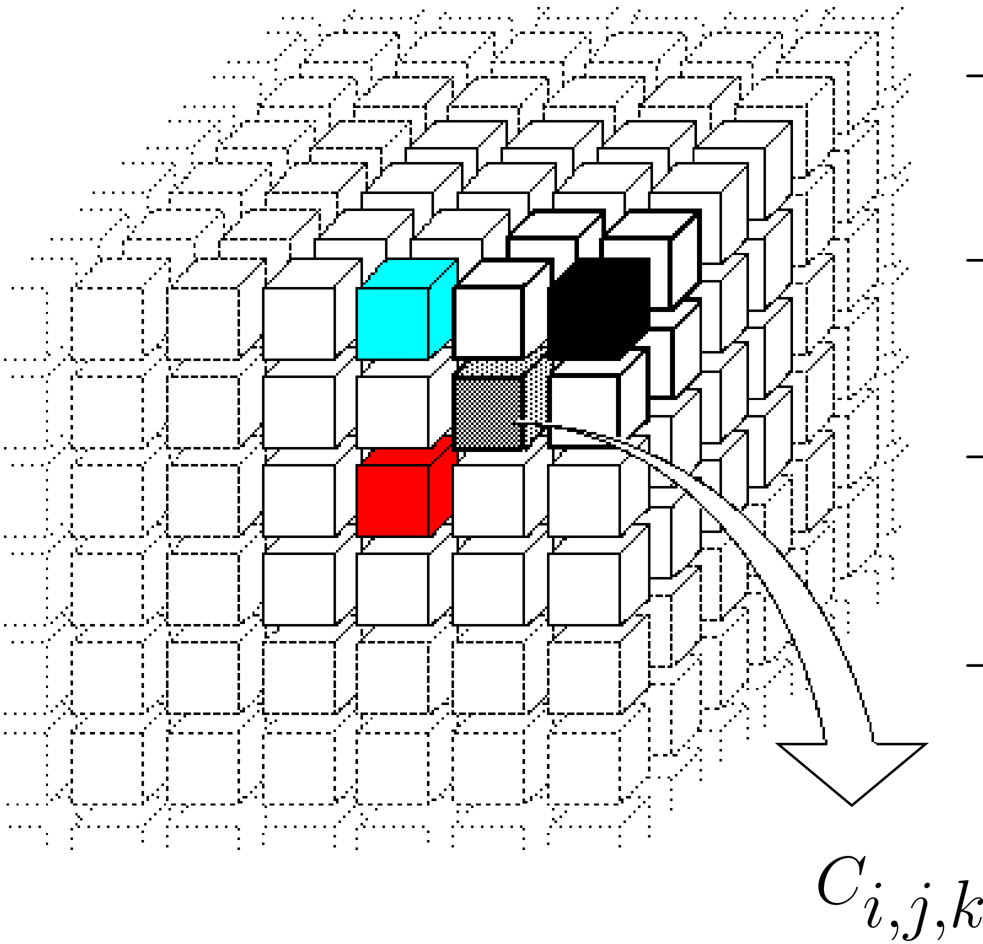
on topography

Cellular automaton for sediment transport



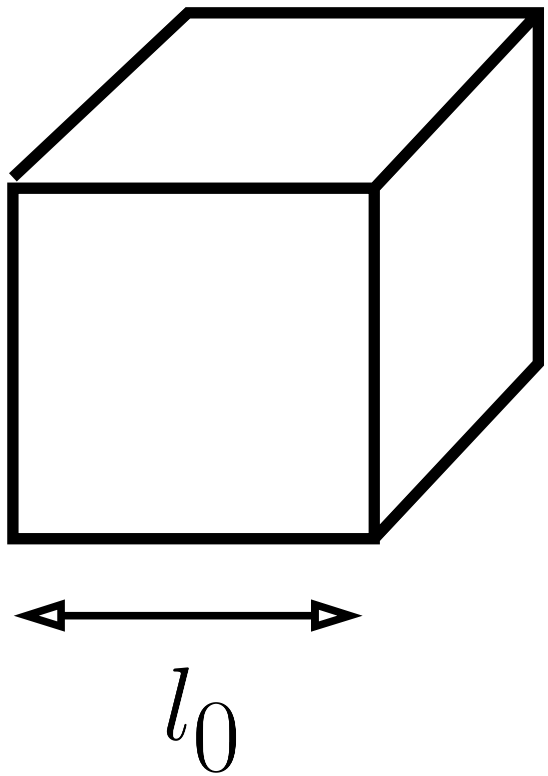
→ A 3 dimensional model.

→ State variables.

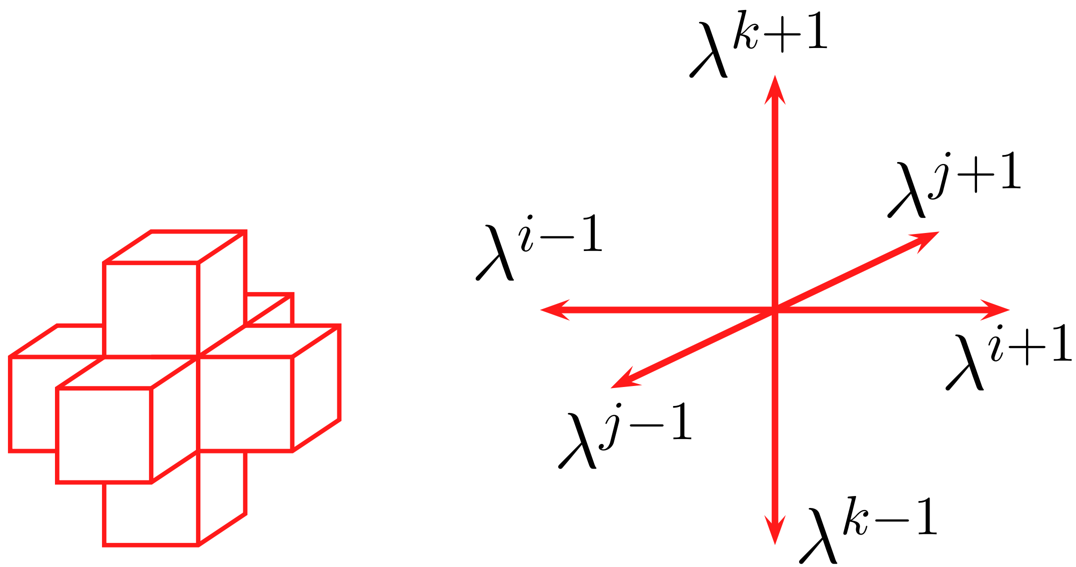
→ Nearest neighbour interactions.

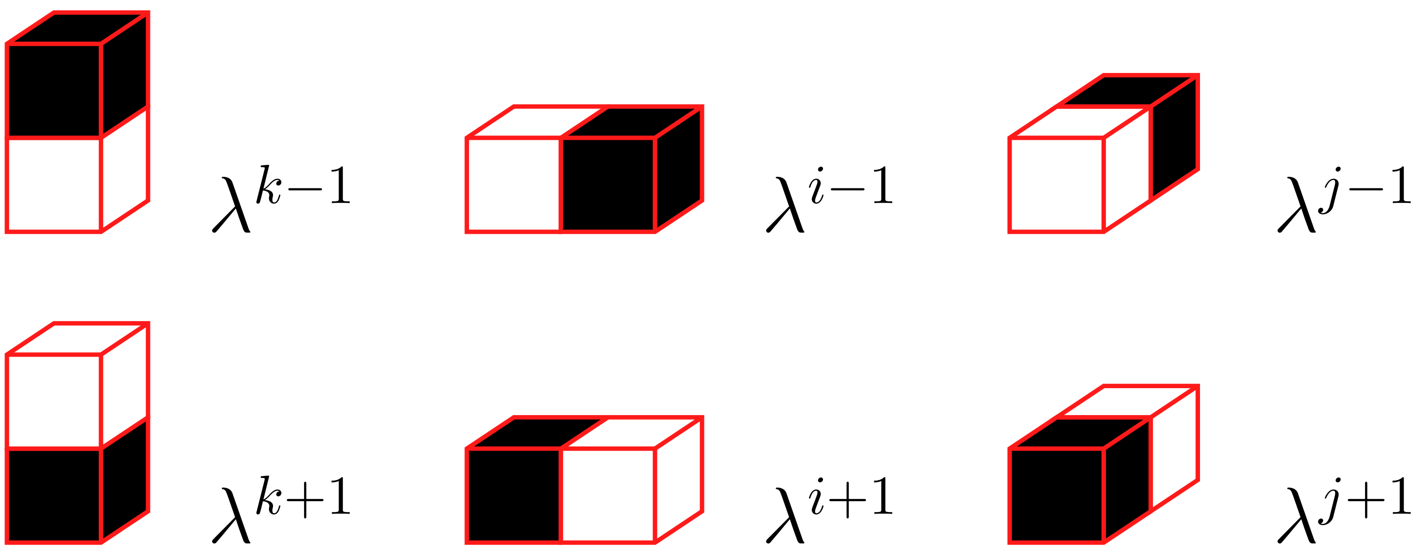
→ Individual physical processes.

An elementary length scale

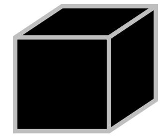
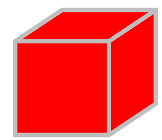
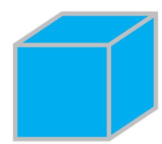


Nearest neighbour interactions

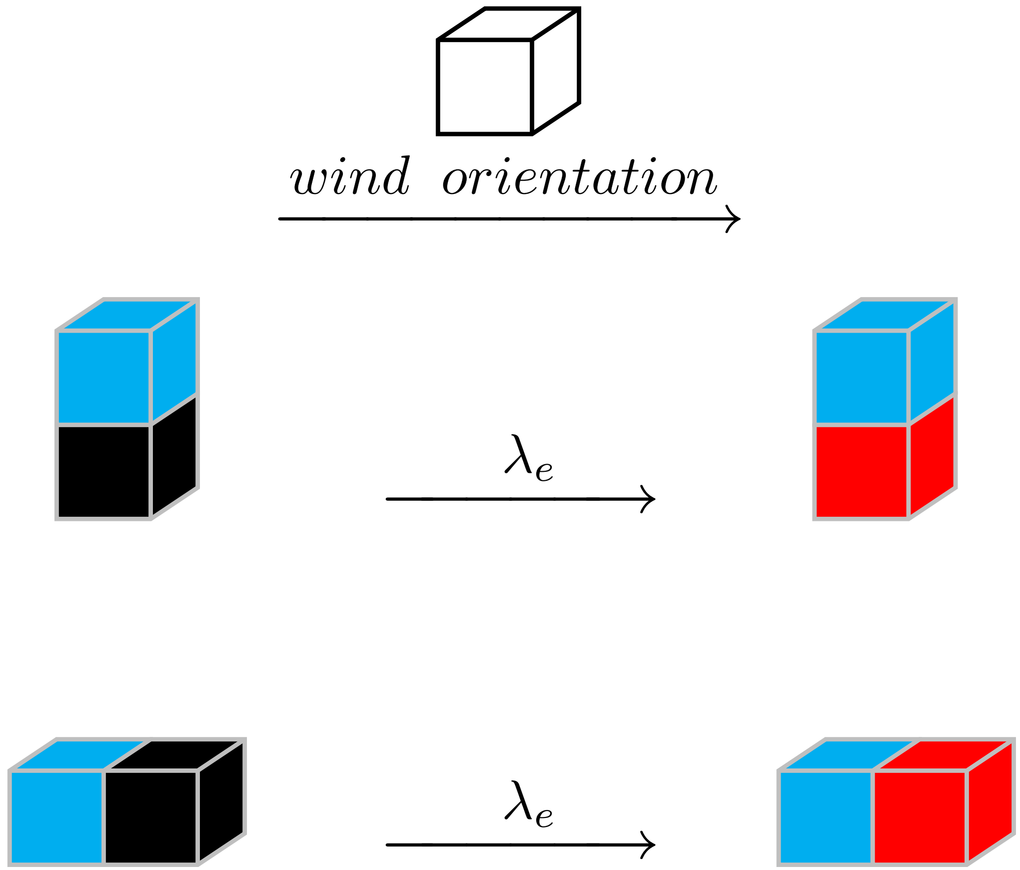




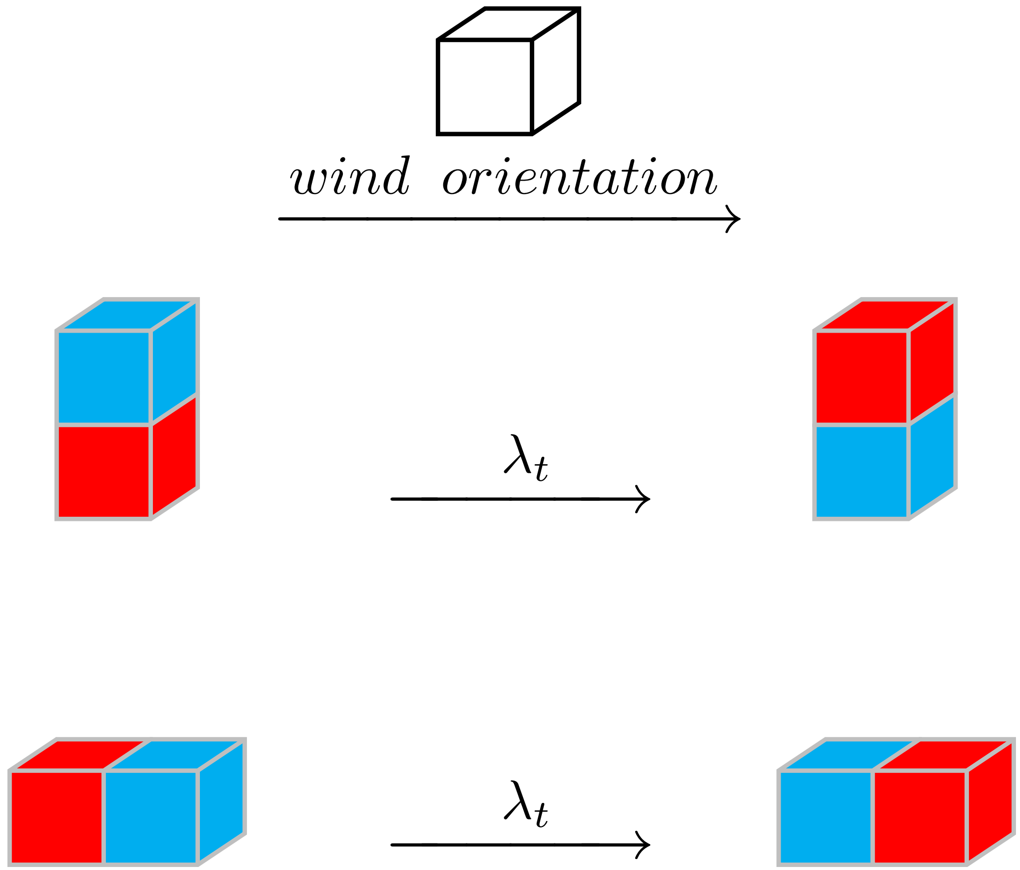
Physical states

Granular material	 Grain
Mobilized granular material	 Mobilized grain
Fluid flow	 Fluid

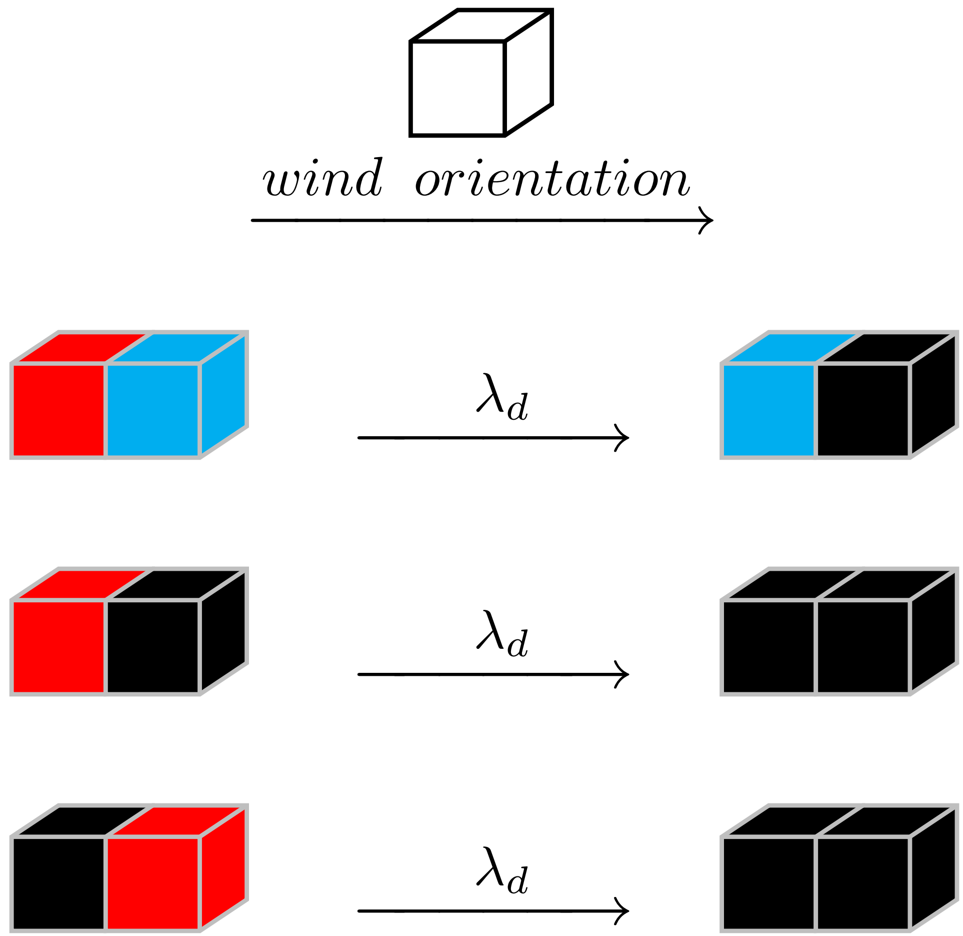
Erosion



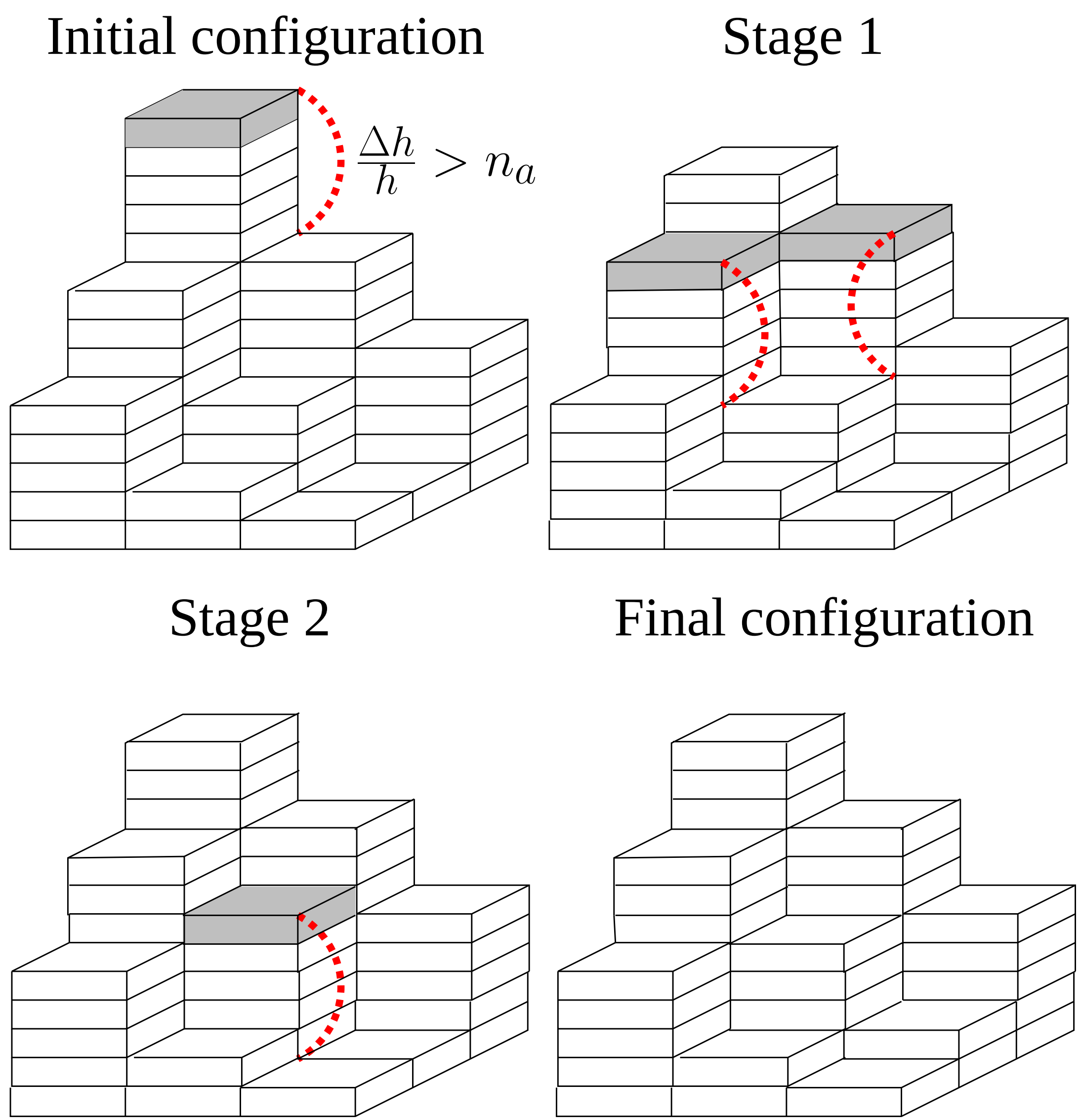
Transport



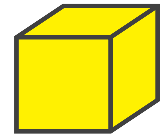
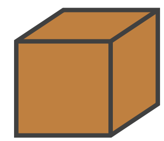
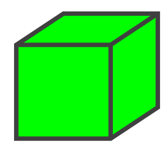
Deposition

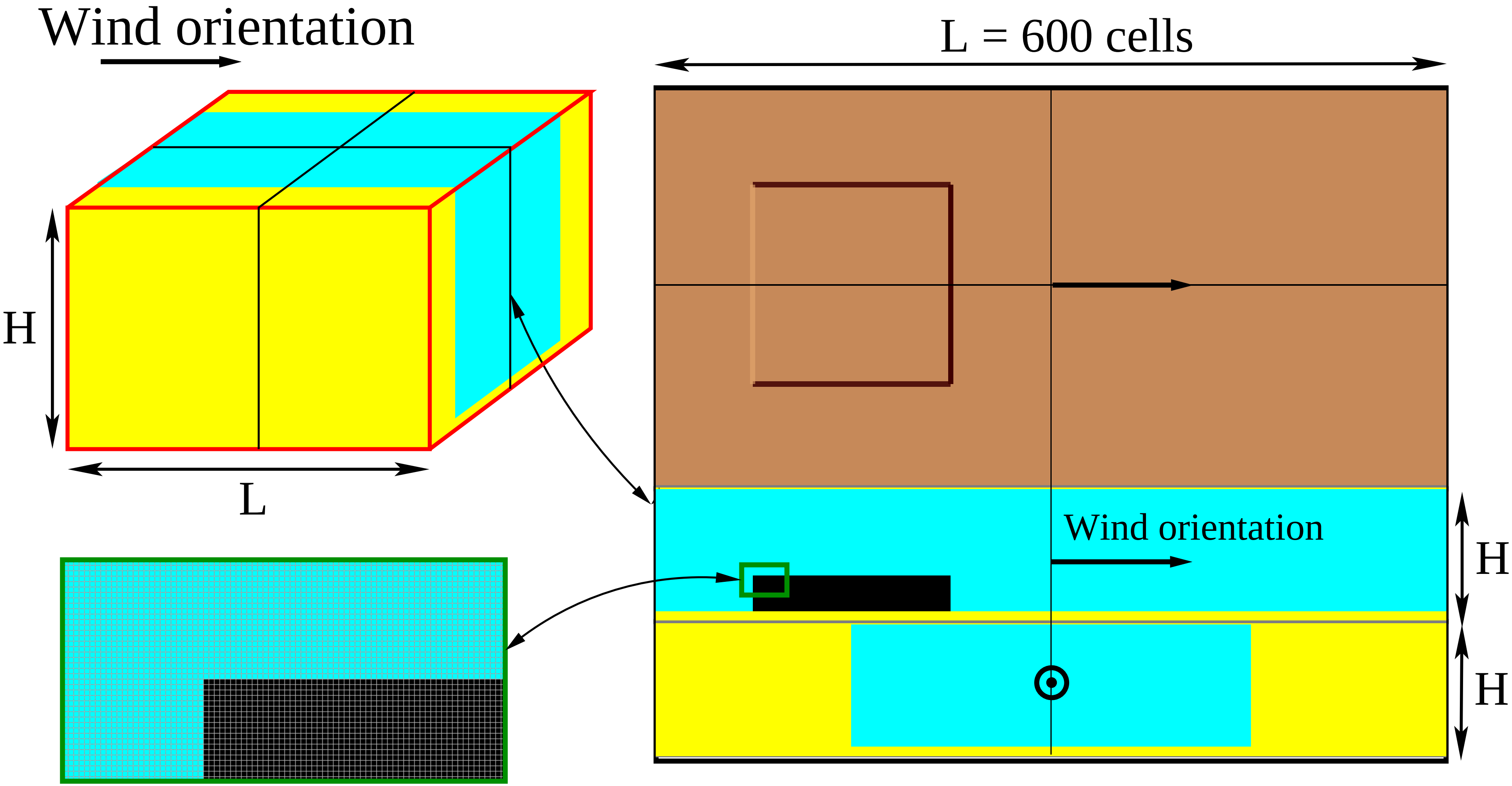


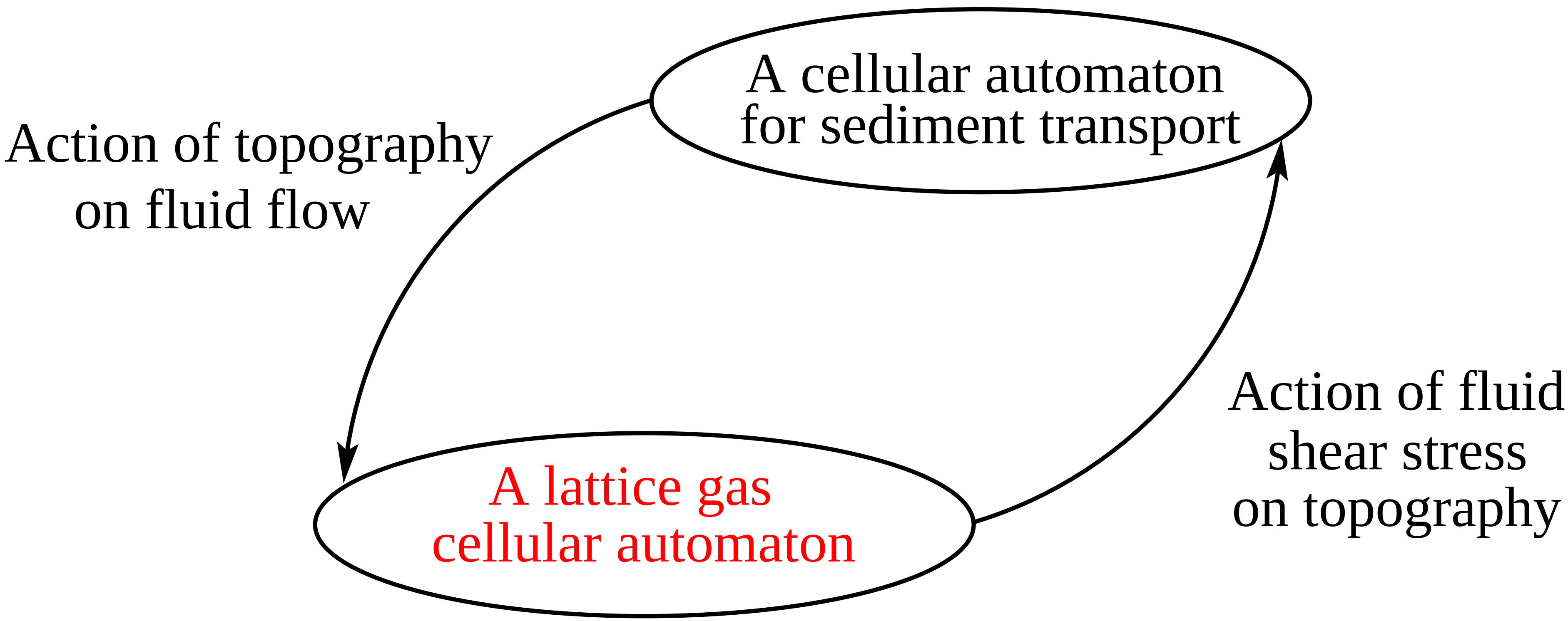
Avalanches


Initial configuration
Stage 1
$\frac{\Delta h}{h} > n_a$
Stage 2
Final configuration

Boundary conditions

Neutral boundaries	 Solid
Removal boundaries	 Out
Injection boundaries	 In


Wind orientation
H
L
L = 600 cells
Wind orientation
H
H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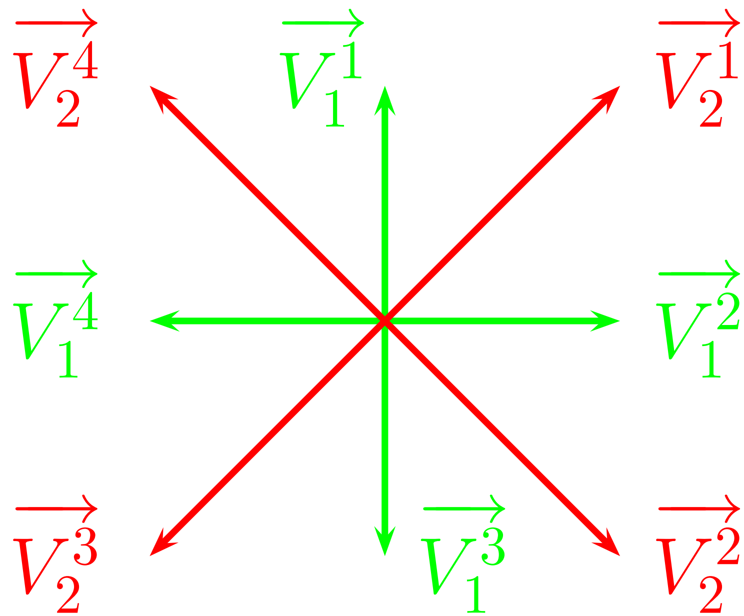
A lattice gas model

A lattice gas model is composed of a set of fluid particles flying from one lattice node to its neighbour in one unit of time. Possible velocity vectors are



with

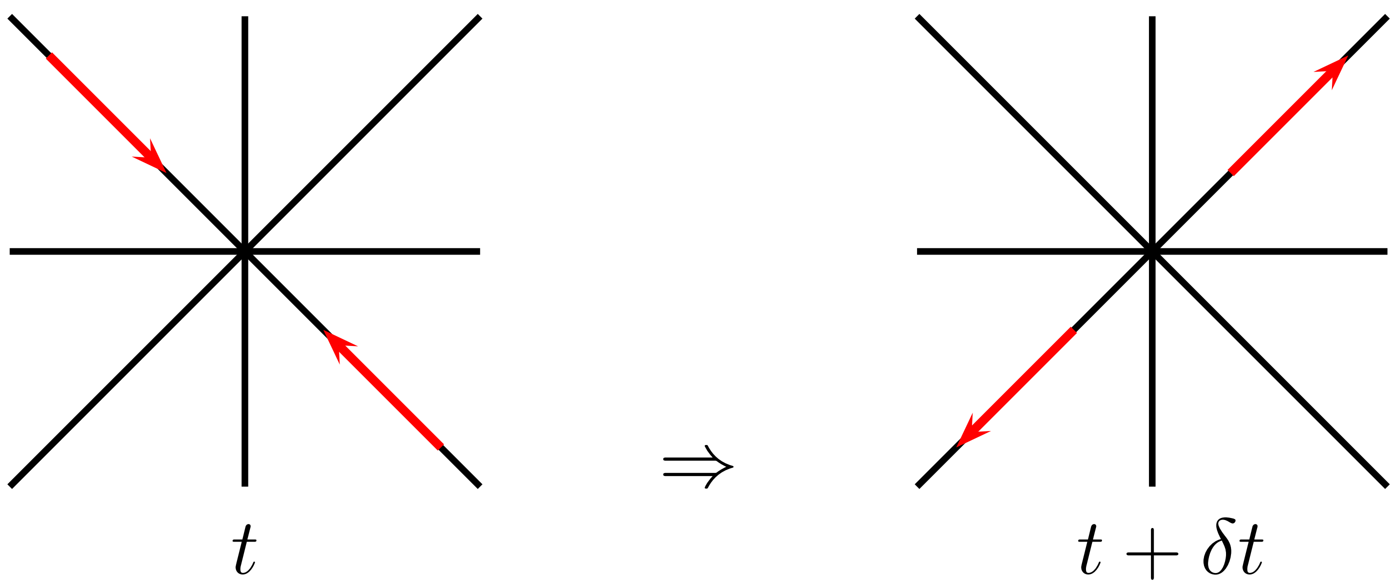
$$\|\overrightarrow{V_2}\| > \|\overrightarrow{V_1}\|.$$

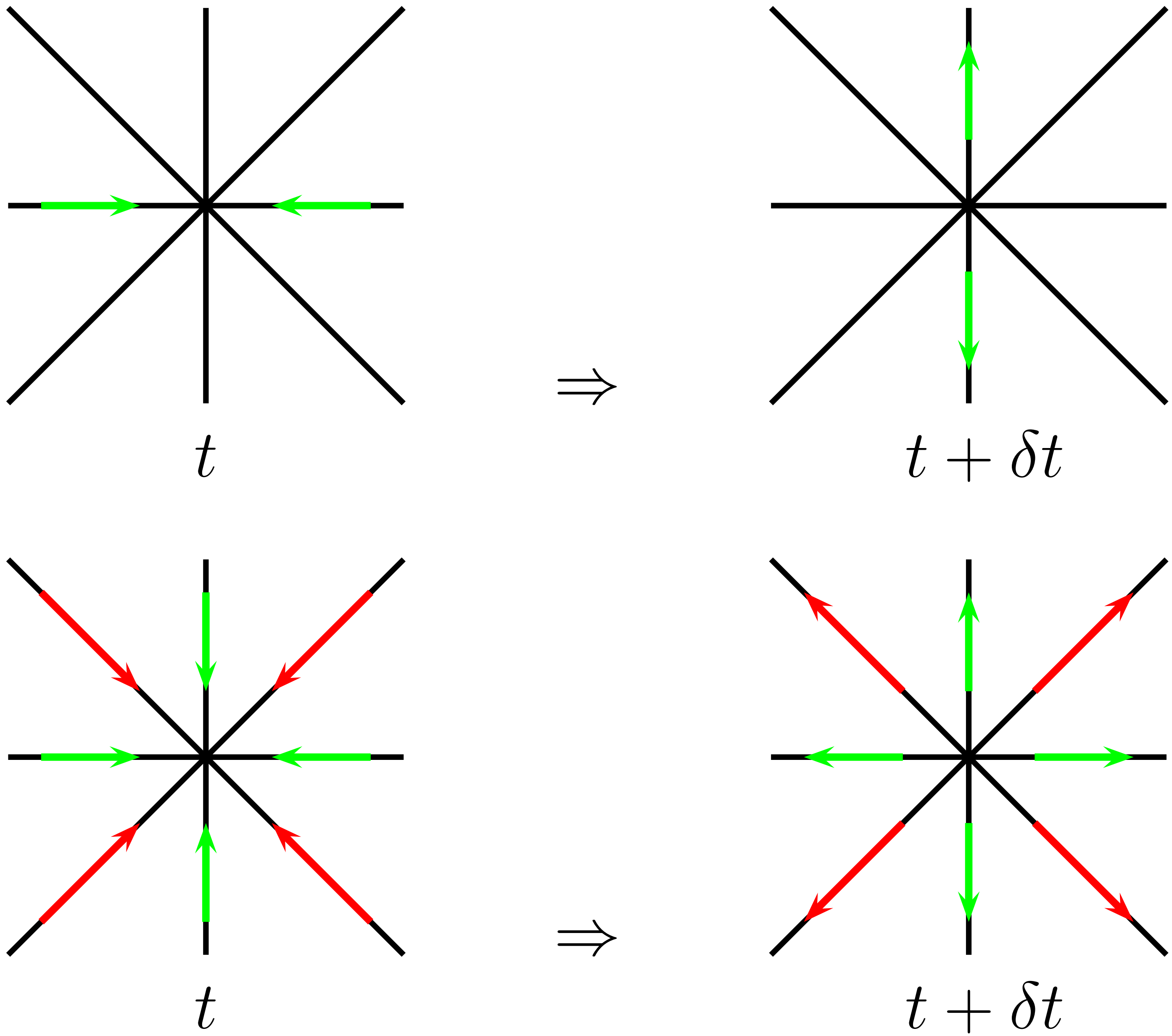
A lattice gas model

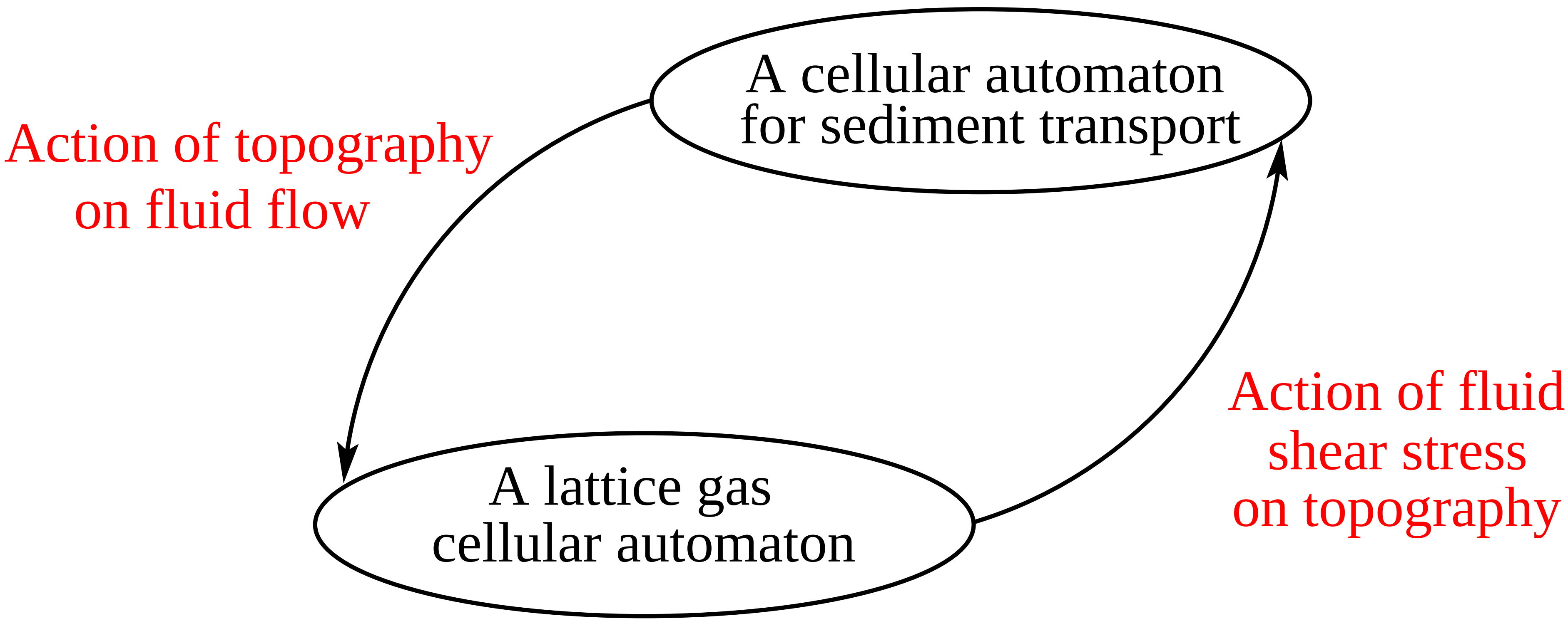
Two particles cannot sit simultaneously on the same node.

• Propagation: the particles move from their node to the nearest neighbour in the direction of their velocity vector.

• Collision: particles on the same node may exchange momentum according to the imposed collision rules.




t
⇒
t + δt
t
⇒
t + δt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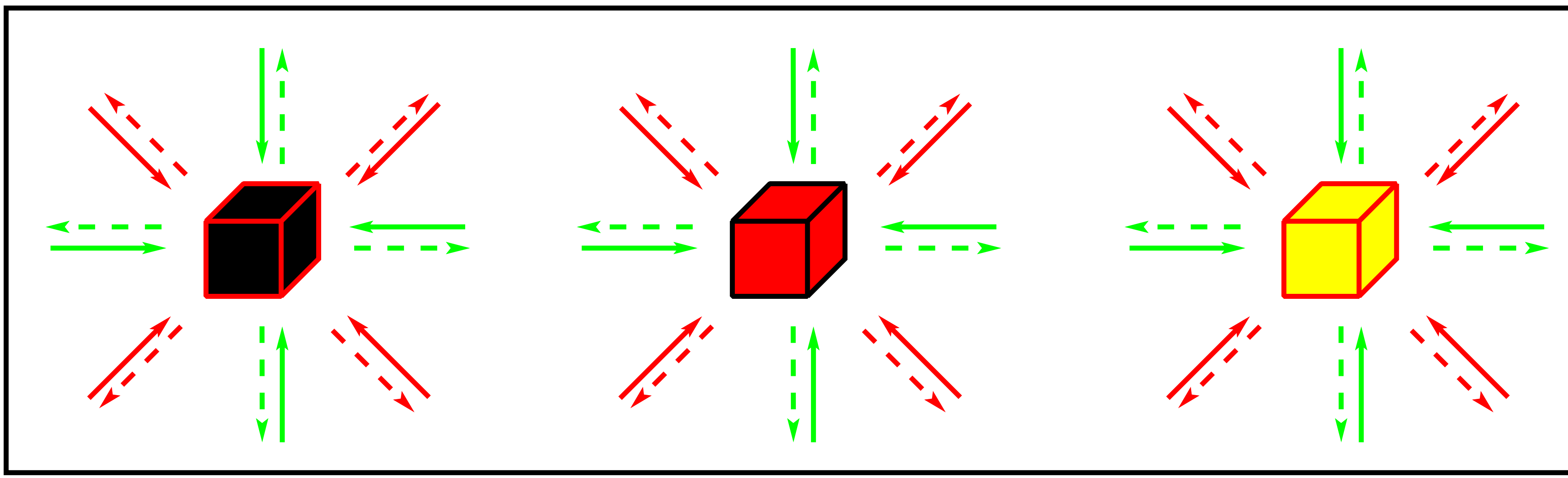
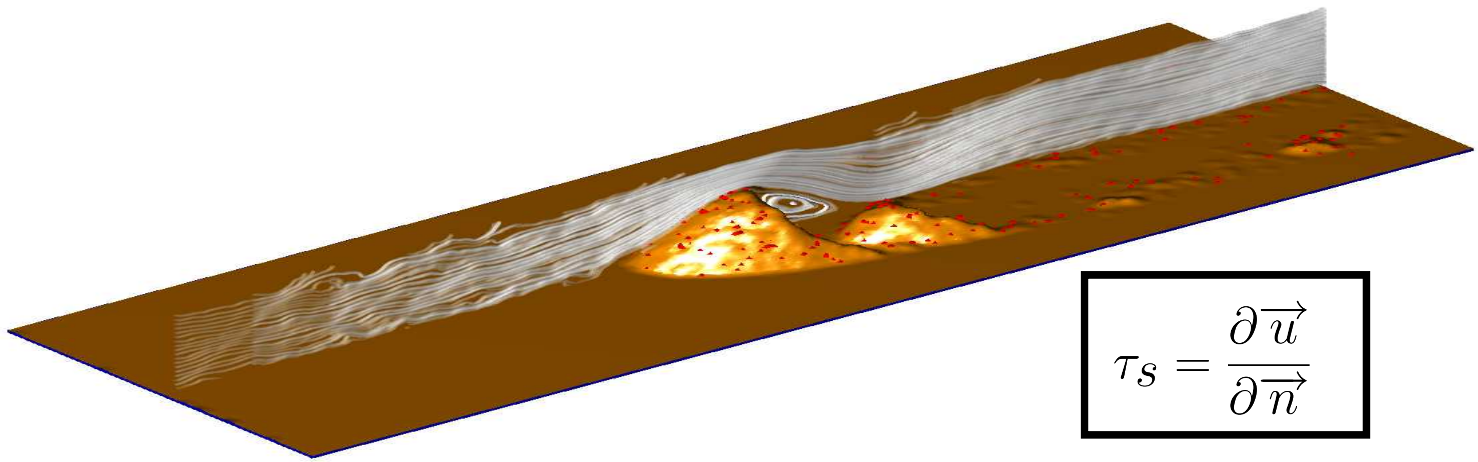
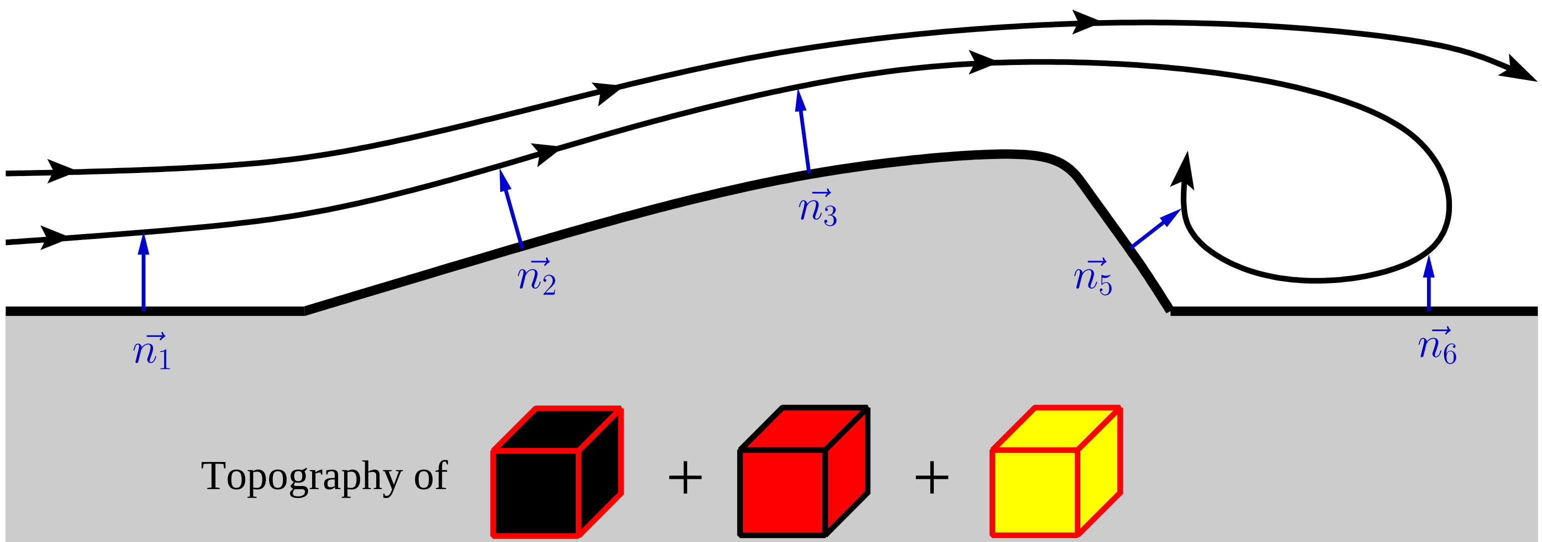
Boundary conditions



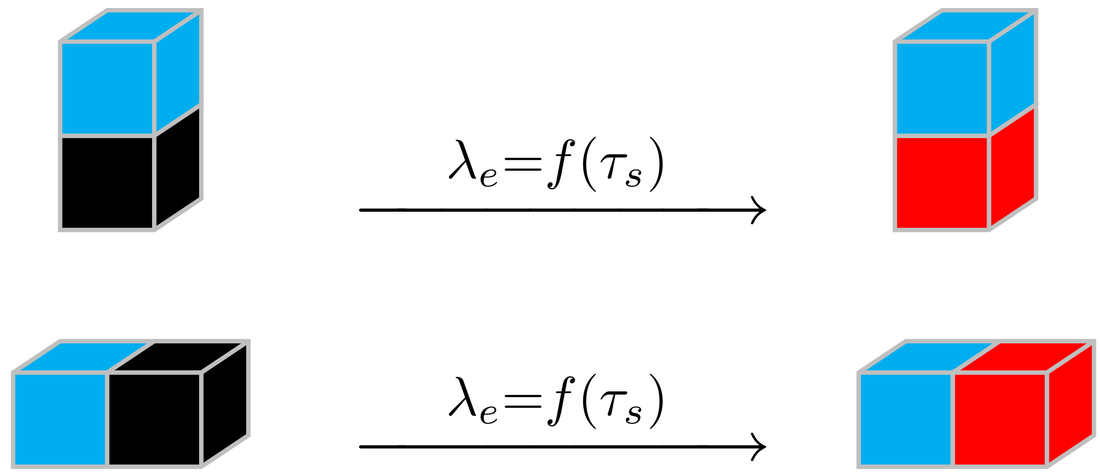
The bed shear stress



$$\tau_s = \frac{\partial \overrightarrow{u}}{\partial \overrightarrow{n}}$$

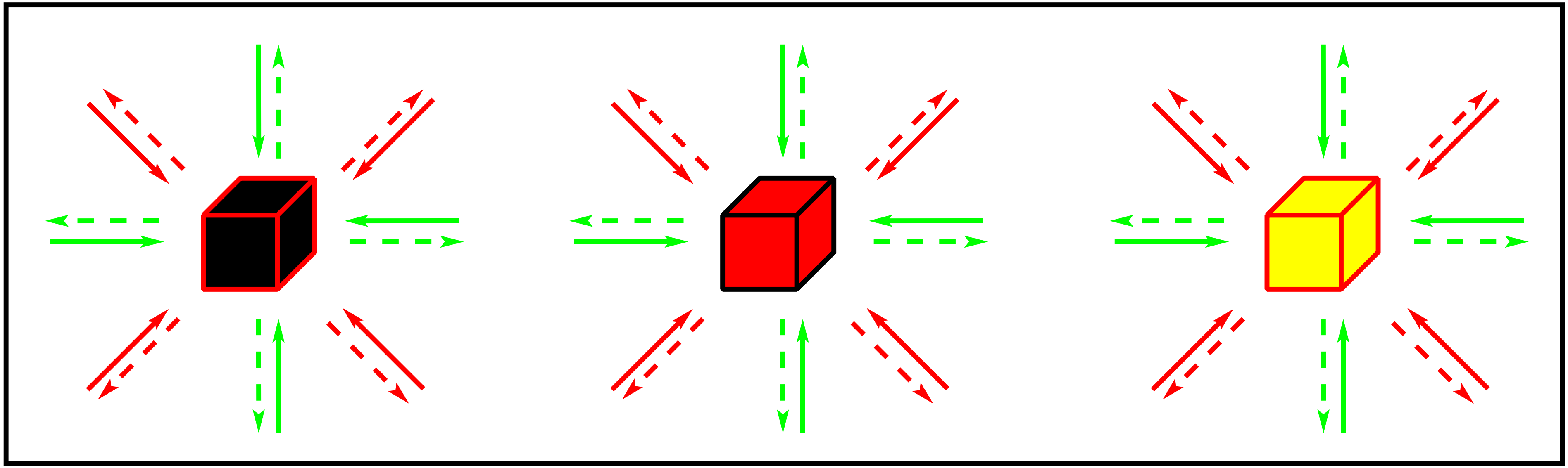


Erosion



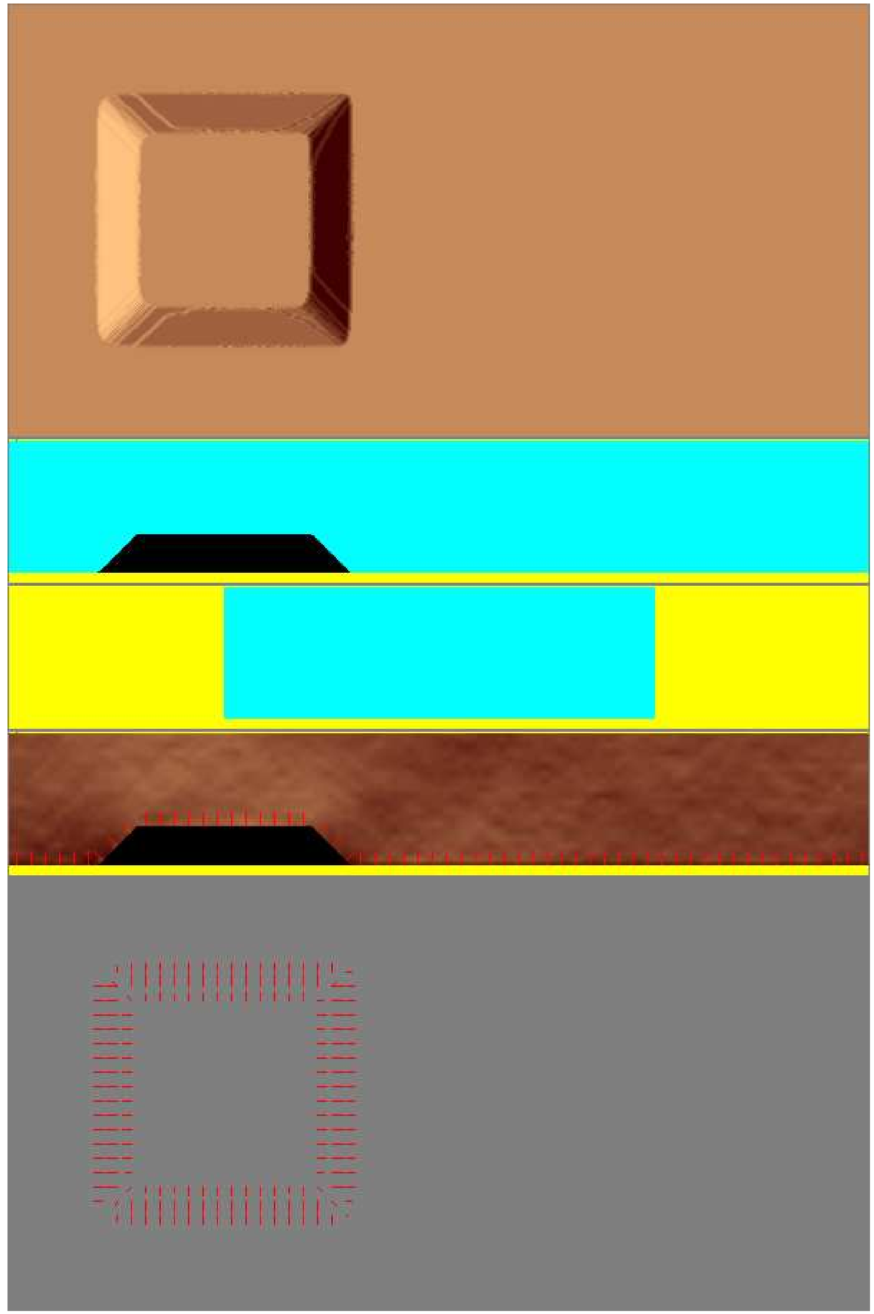
$$\lambda_e = \begin{cases} 0 & \text{for } \tau_s \le \tau_{min}, \\ \lambda_0 \dfrac{\tau_s - \tau_{min}}{\tau_{max} - \tau_{min}} & \text{for } \tau_{min} \le \tau_s \le \tau_{max}, \\ \lambda_0 & \text{else.} \end{cases}$$

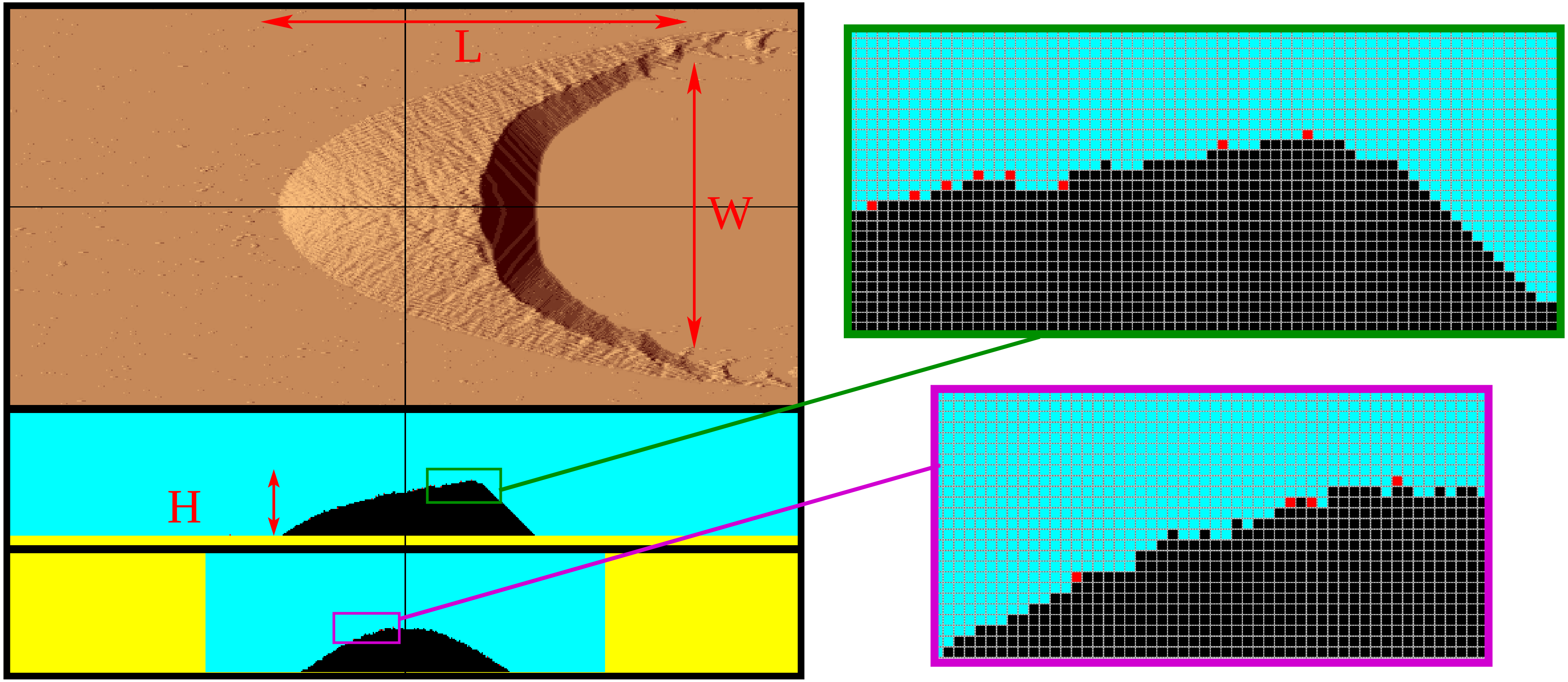
A fully coupled system



$$\lambda_e = \begin{cases} 0 & \text{for } \tau_s \le \tau_{min}, \\ \lambda_0 \dfrac{\tau_s - \tau_{min}}{\tau_{max} - \tau_{min}} & \text{for } \tau_{min} \le \tau_s \le \tau_{max}, \\ \lambda_0 & \text{else.} \end{cases}$$

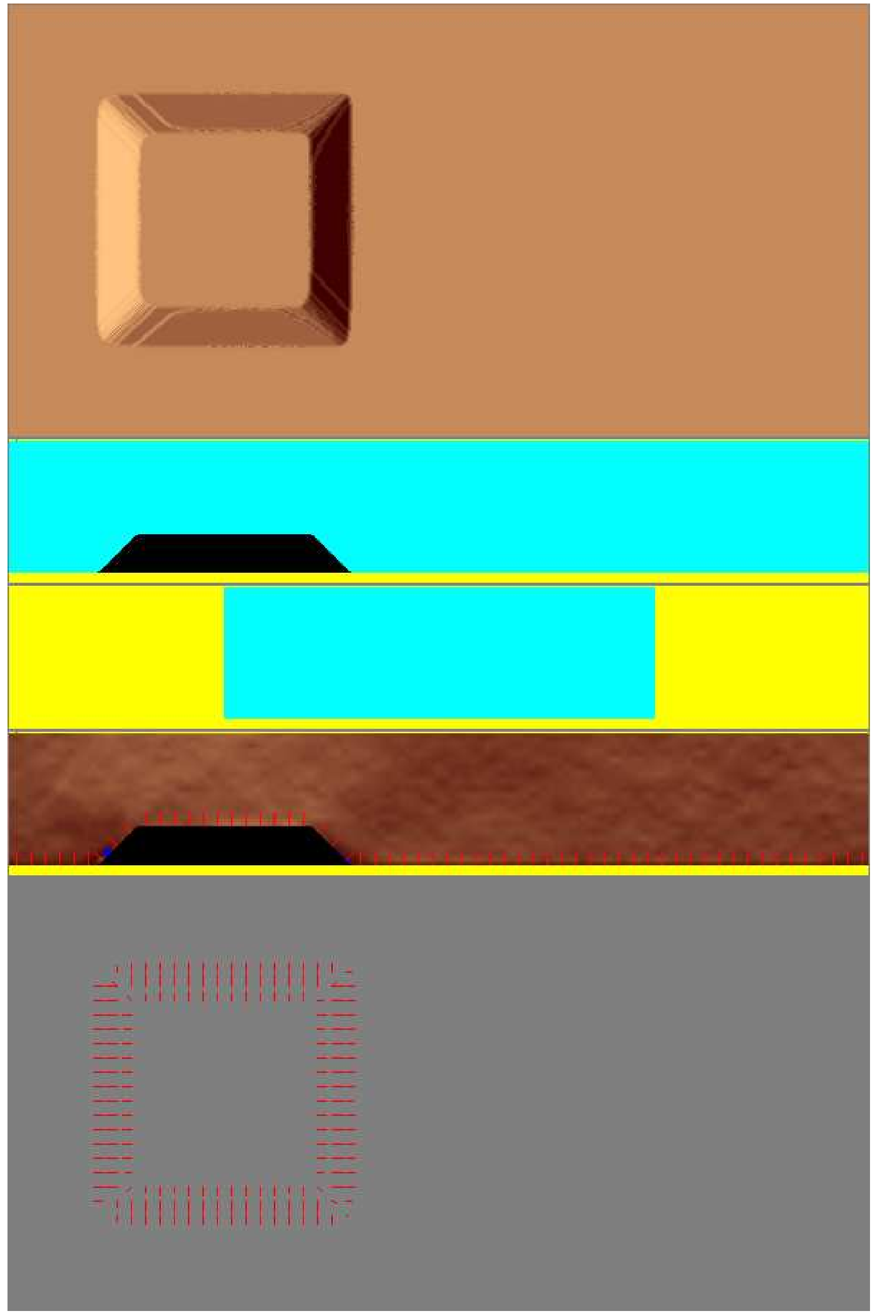
High wind speed $\Longleftrightarrow$ low τ_{min}-value

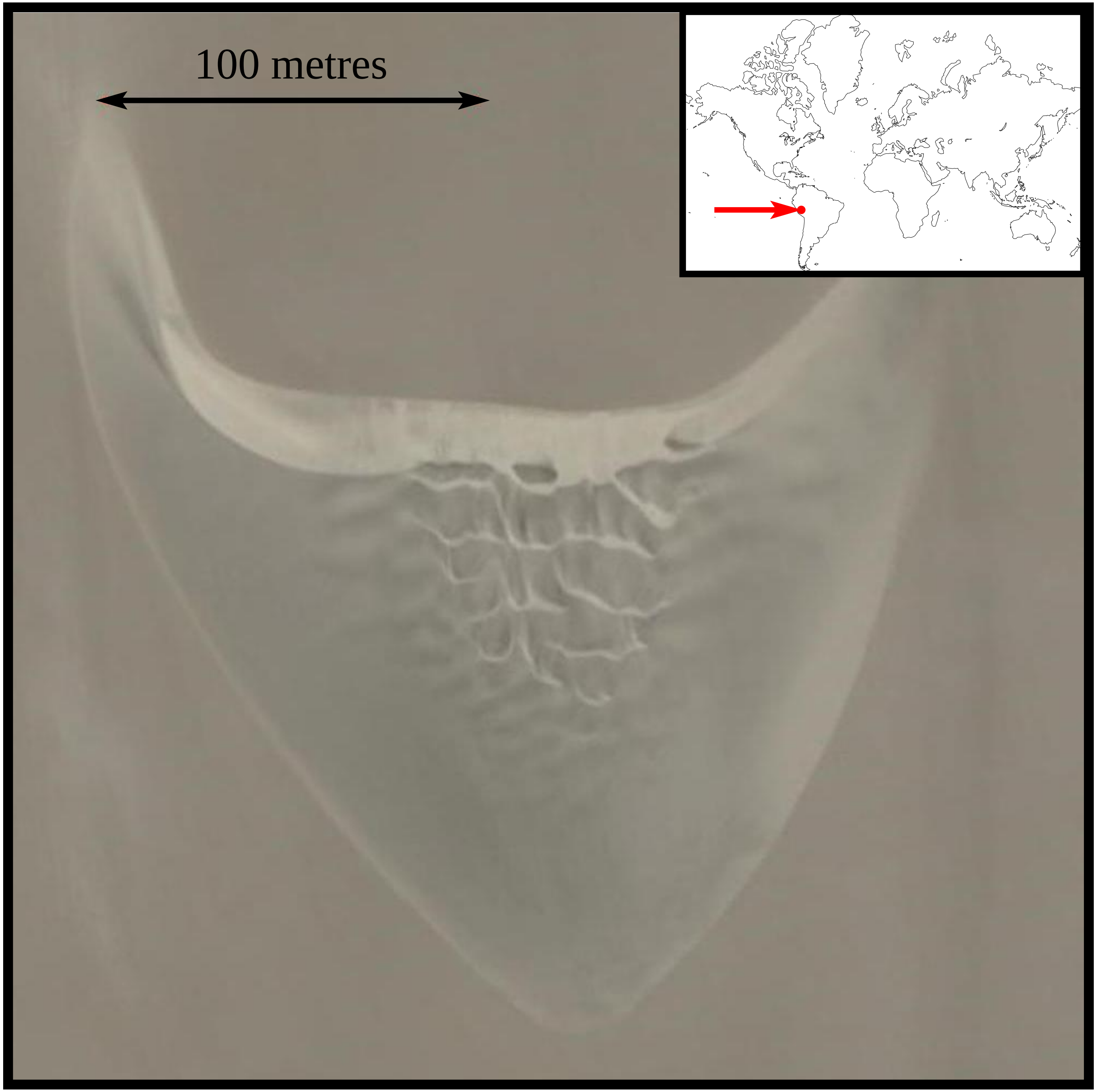


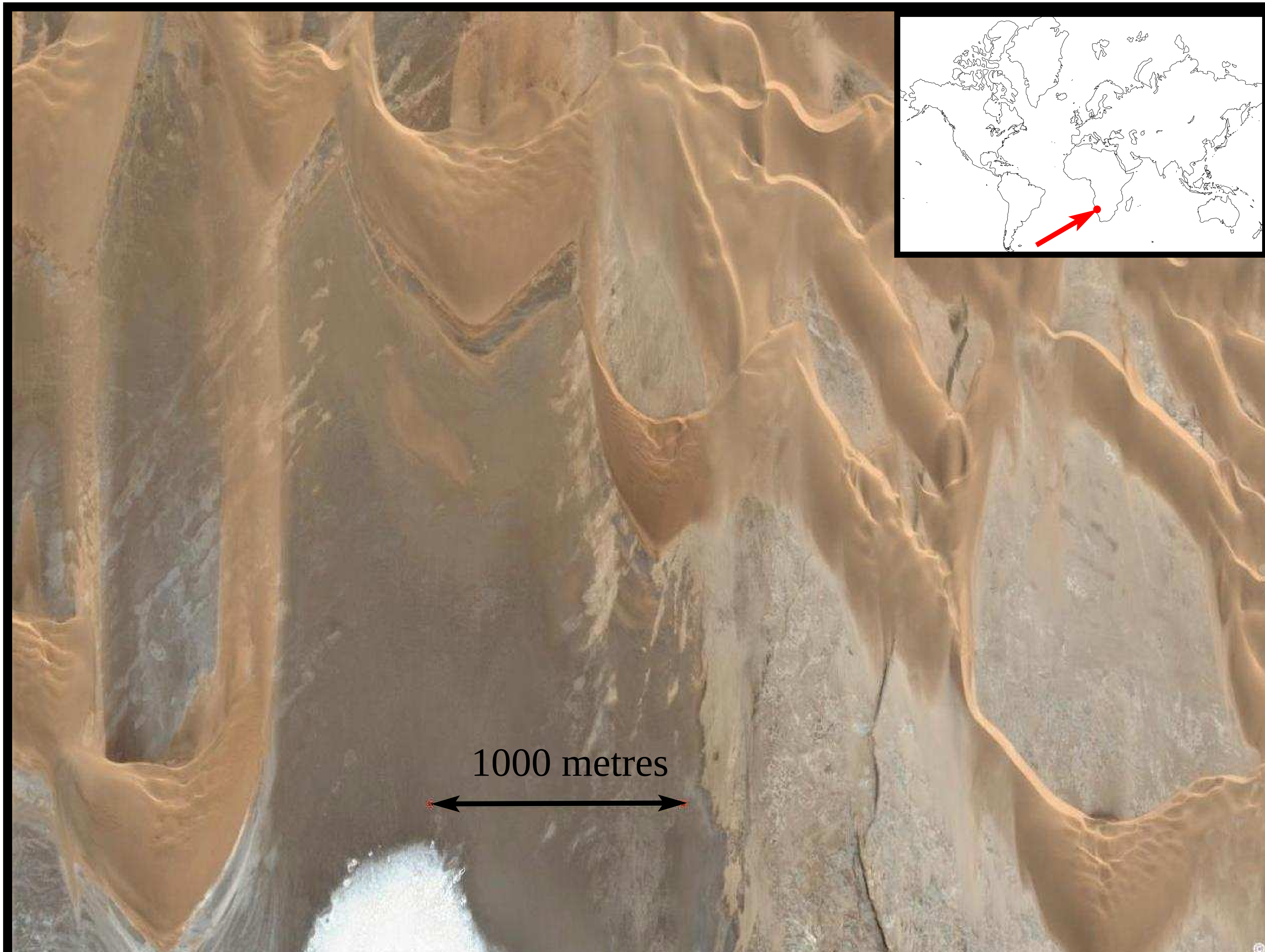


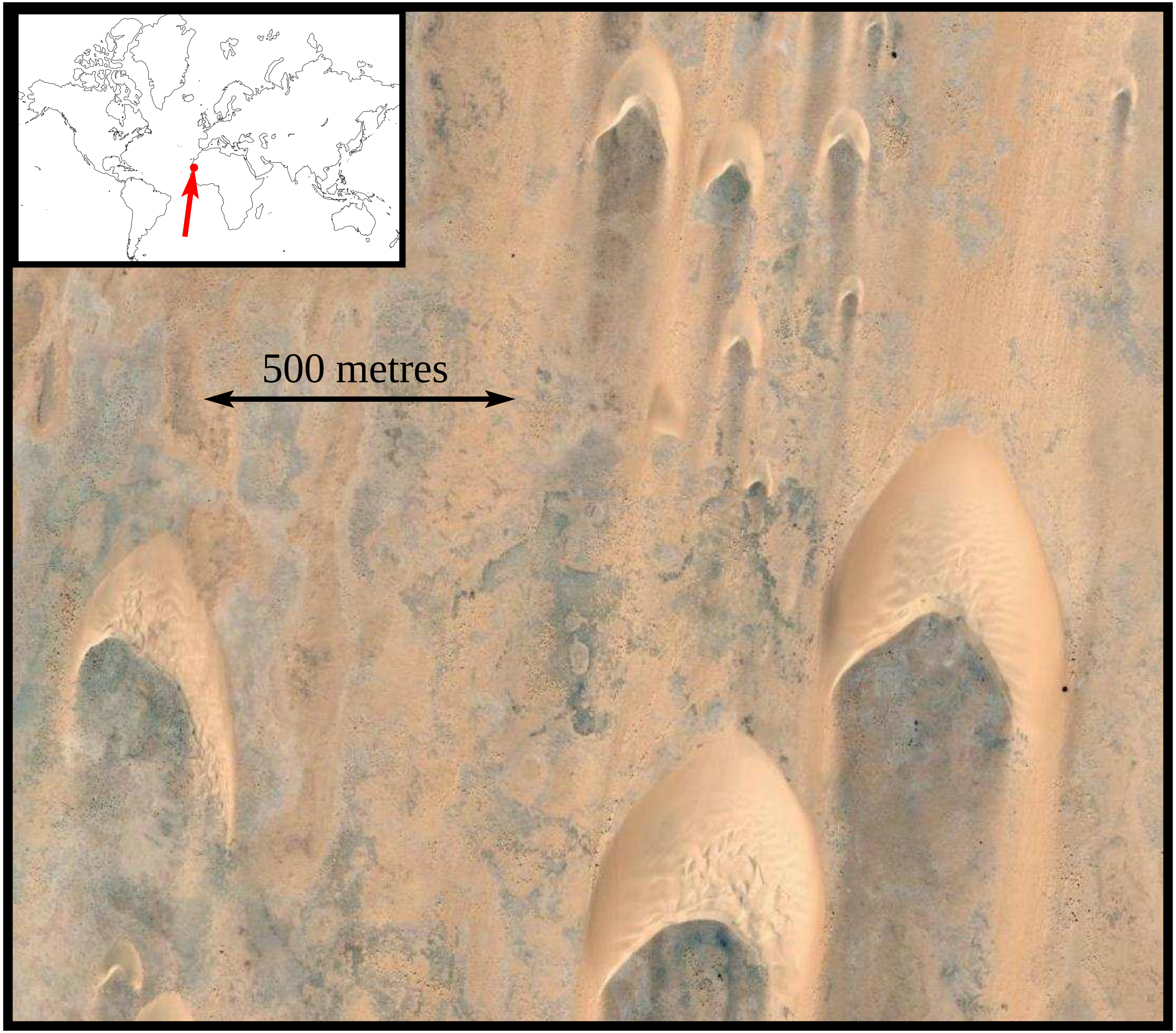
L
W
H

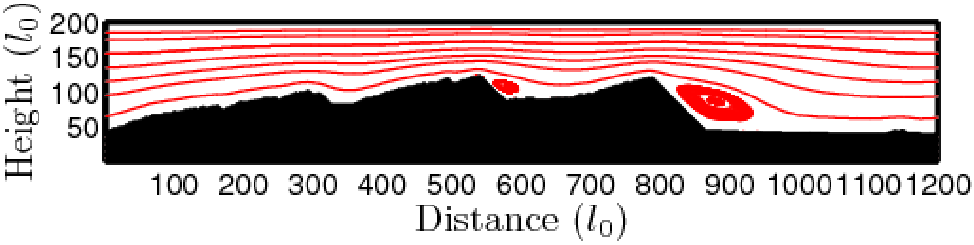
Low wind speed $\Longleftrightarrow$ high τ_{min}-value

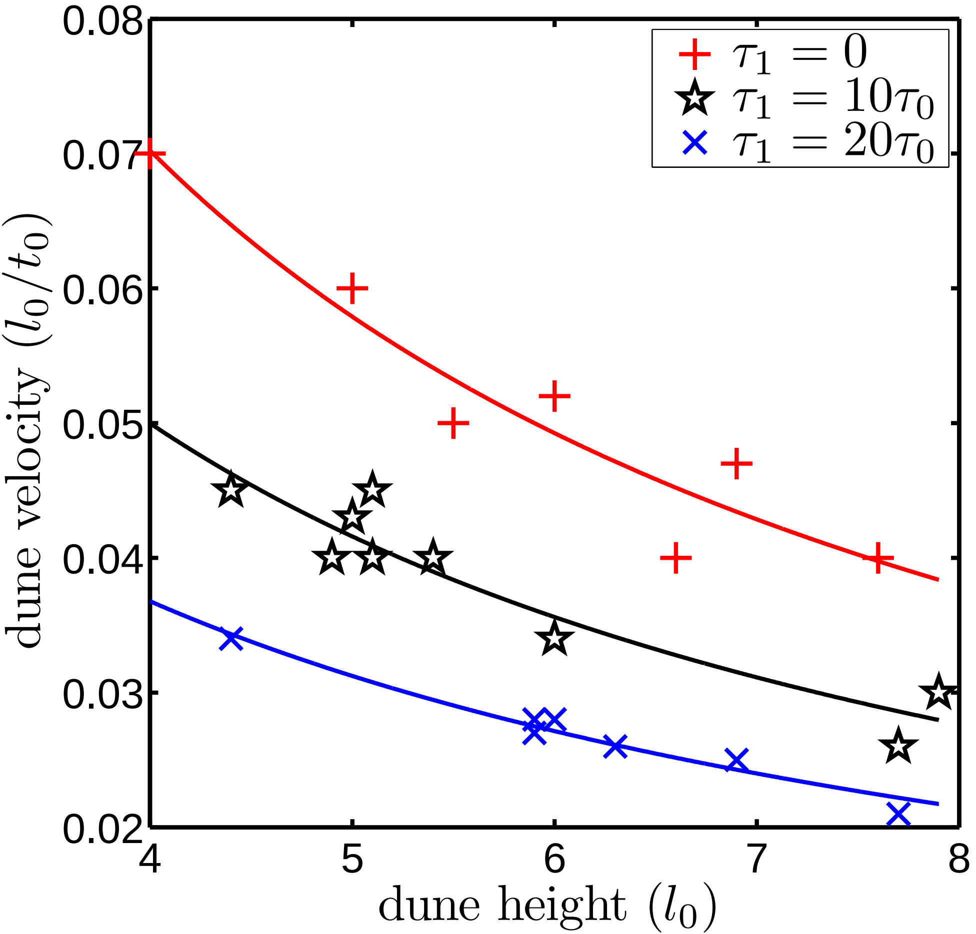


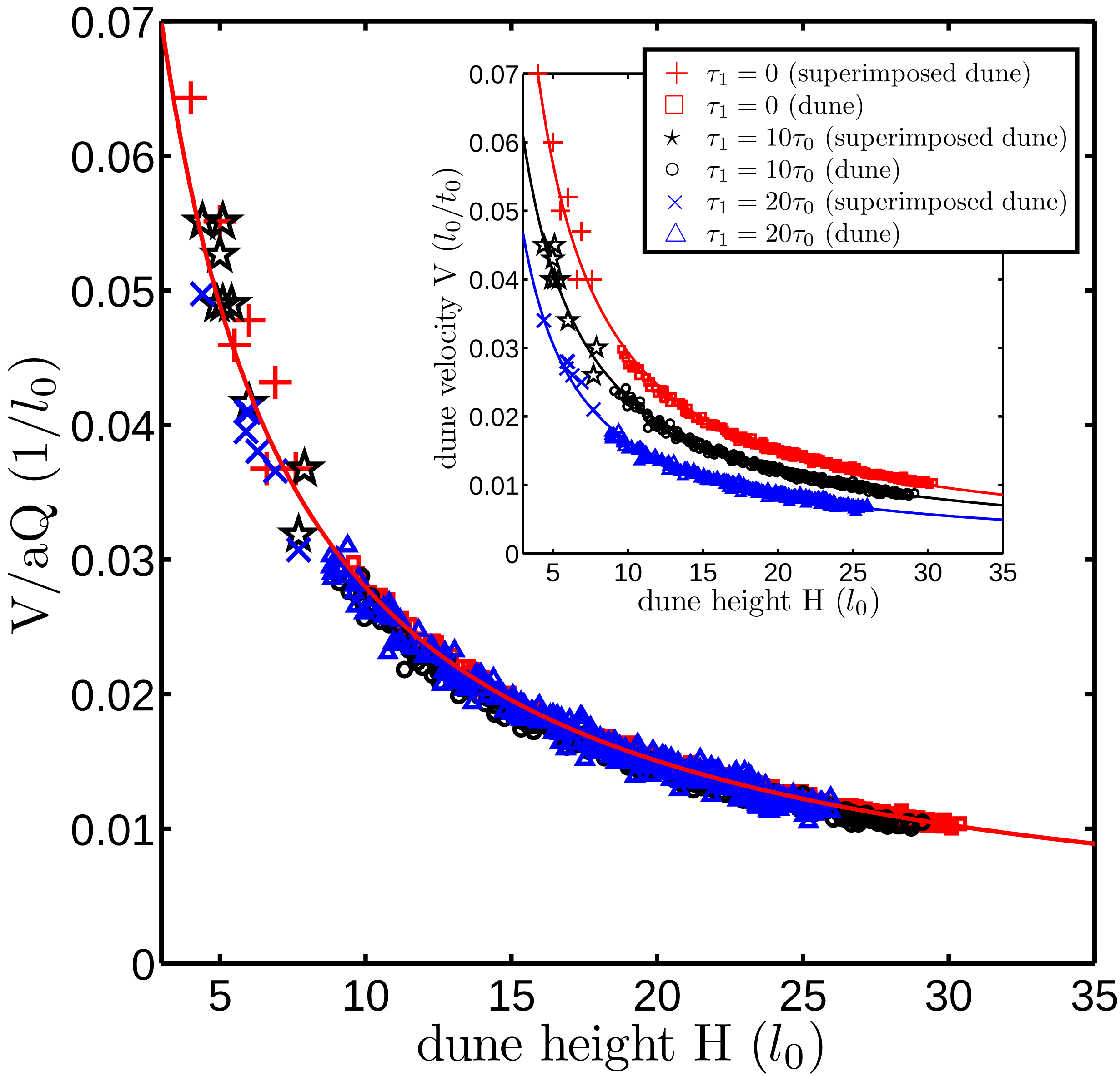

100 metres


1000 metres


500 metres


Height (l_0)
200
150
100
50
100 200 300 400 500 600 700 800 900 1000 1100 1200
Distance (l_0)

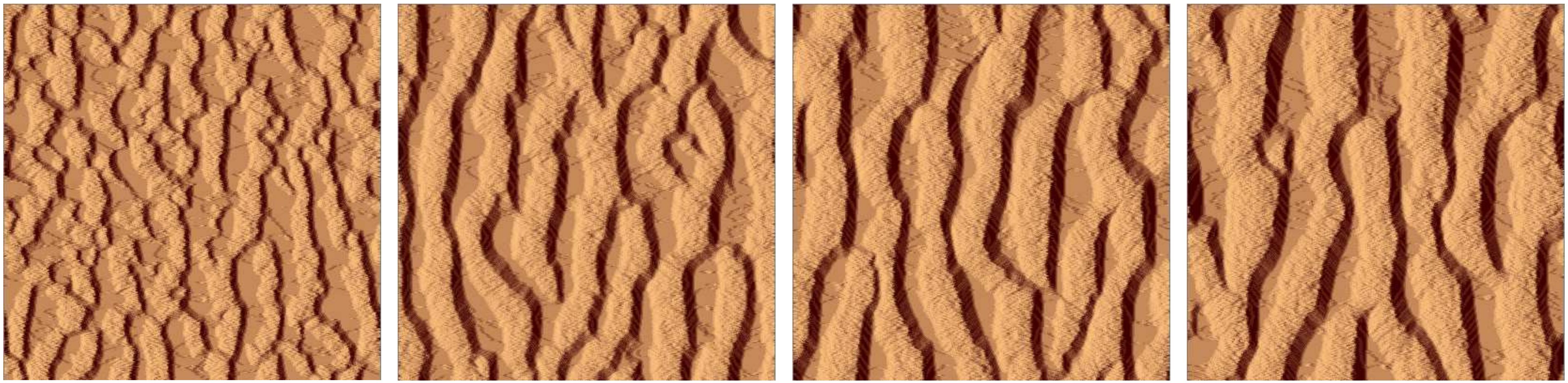

$\tau_1 = 0$
$\tau_1 = 10\tau_0$
$\tau_1 = 20\tau_0$
0.08
0.07
0.06
0.05
0.04
0.03
0.02
4
5
6
7
8
dune velocity (l_0/t_0)
dune height (l_0)



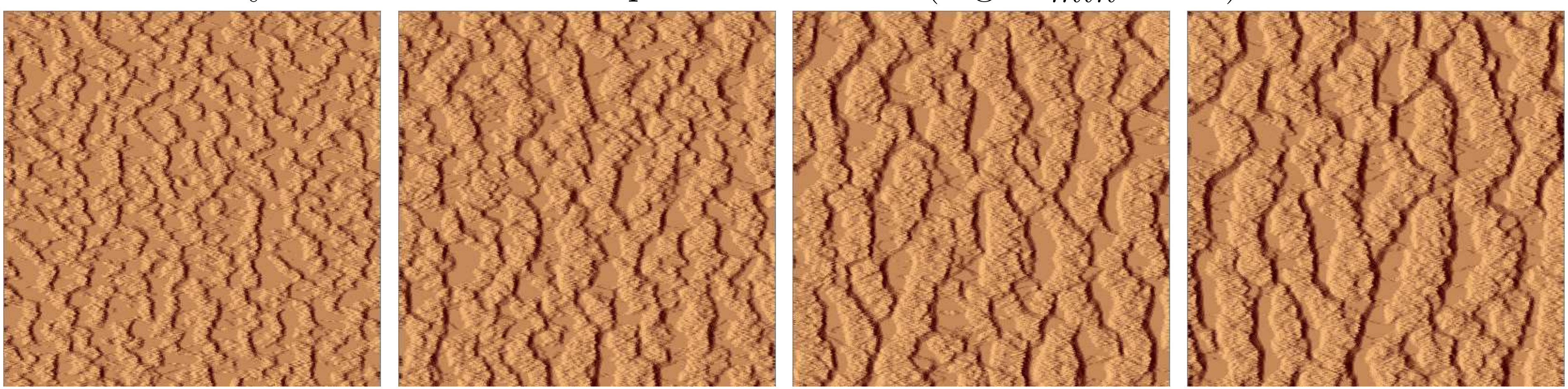
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
5
10
15
20
25
30
35
V/aQ $(1/l_0)$
dune height H (l_0)
$\tau_1 = 0$ (superimposed dune)
$\tau_1 = 0$ (dune)
$\tau_1 = 10\tau_0$ (superimposed dune)
$\tau_1 = 10\tau_0$ (dune)
$\tau_1 = 20\tau_0$ (superimposed dune)
$\tau_1 = 20\tau_0$ (dune)
dune velocity V (l_0/t_0)
dune height H (l_0)

Transverse dunes

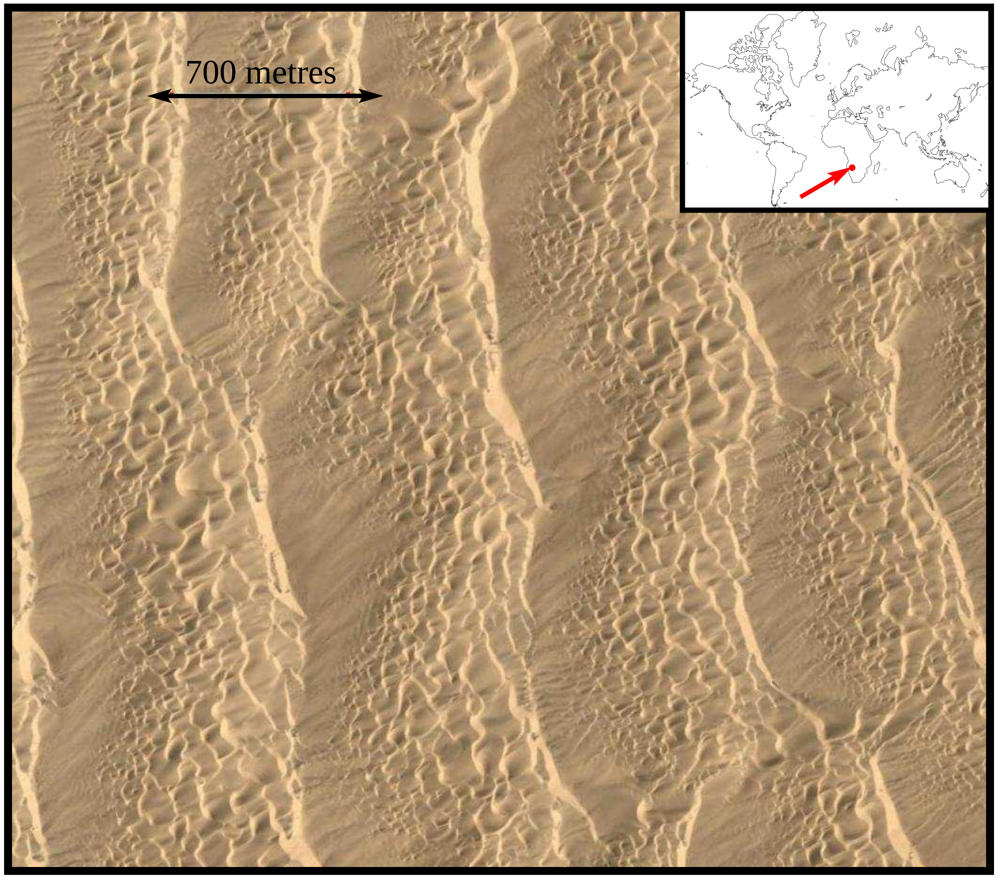
Thick sand layer under high wind speed conditions (low τ_{min}-value)

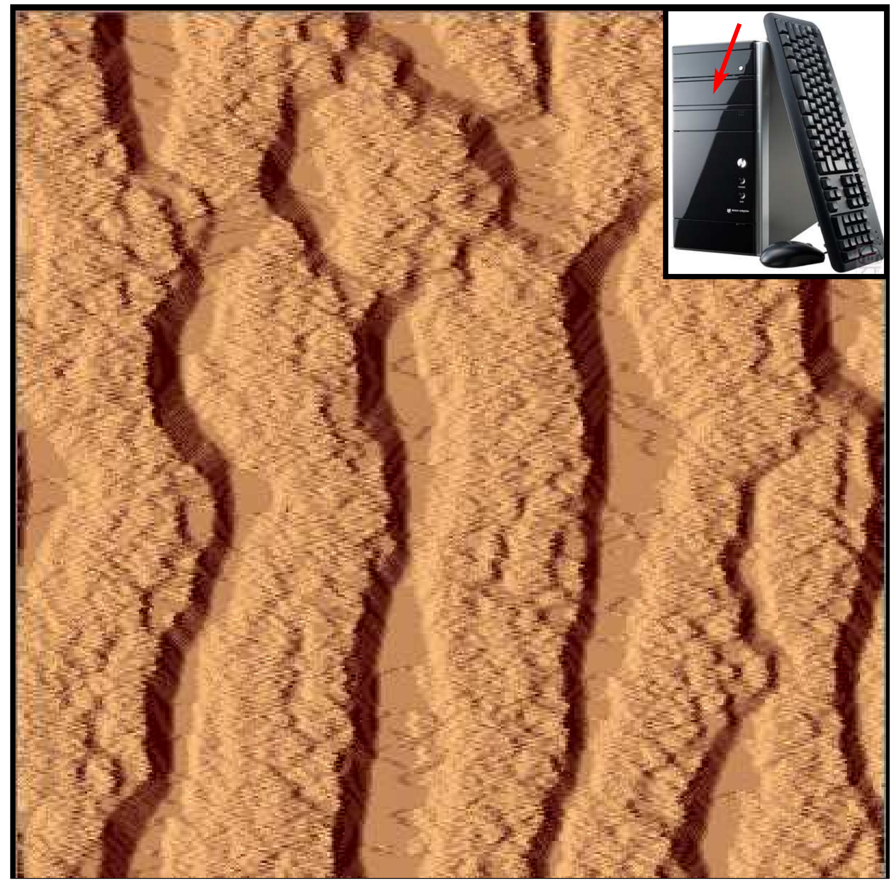


Thick sand layer under low wind speed conditions (high τ_{min}-value)

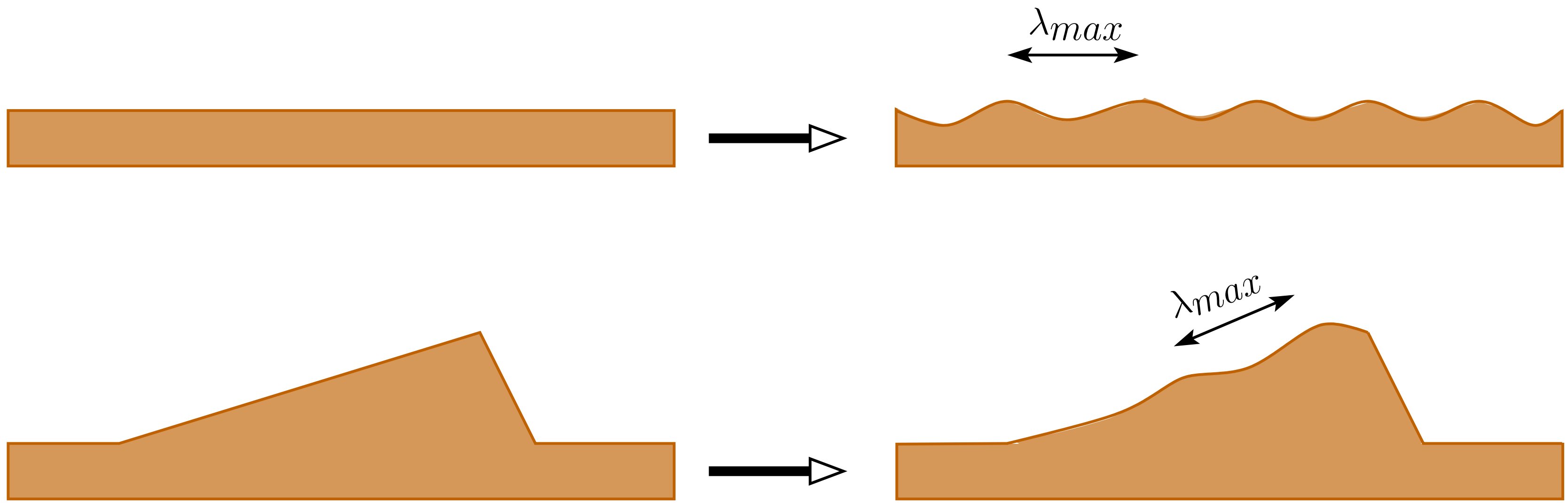


Transverse dunes





Nucleation of dunes patterns in the model

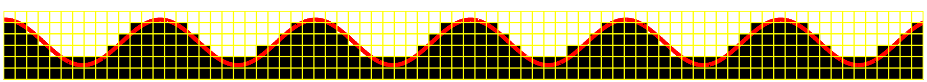


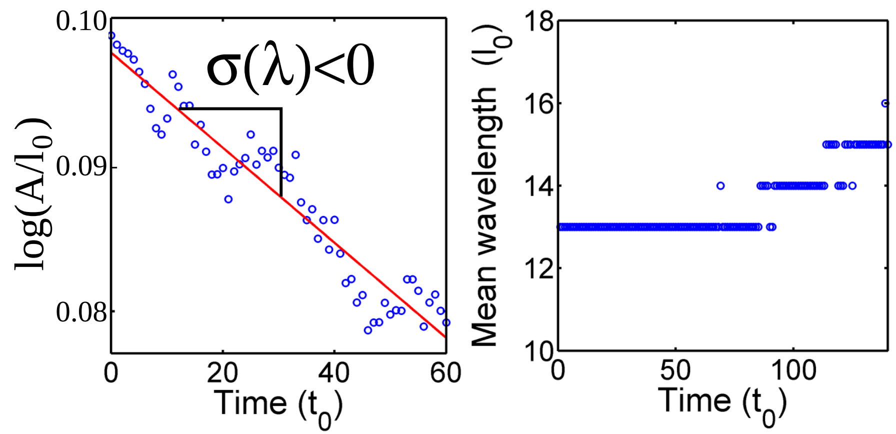
What is the characteristic length scale for the nucleation of dunes in the model ?

$$\lambda_{max} \;=\; ?? \;\; l_0$$

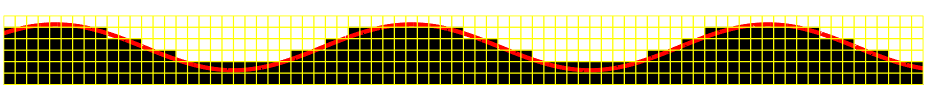
Linear stability analysis of the sand waves instability

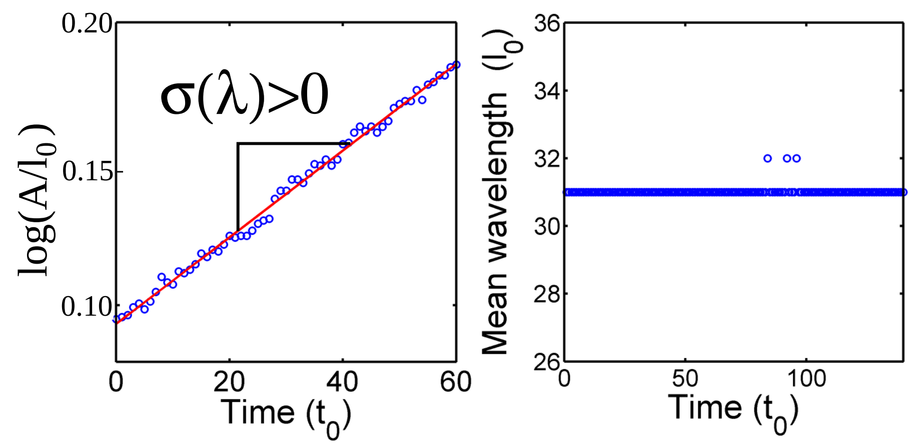
Stable wavelength





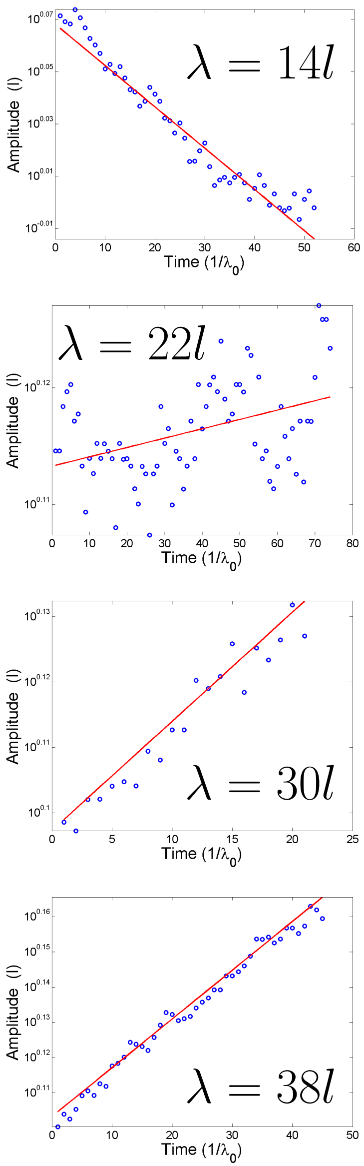
Unstable wavelength

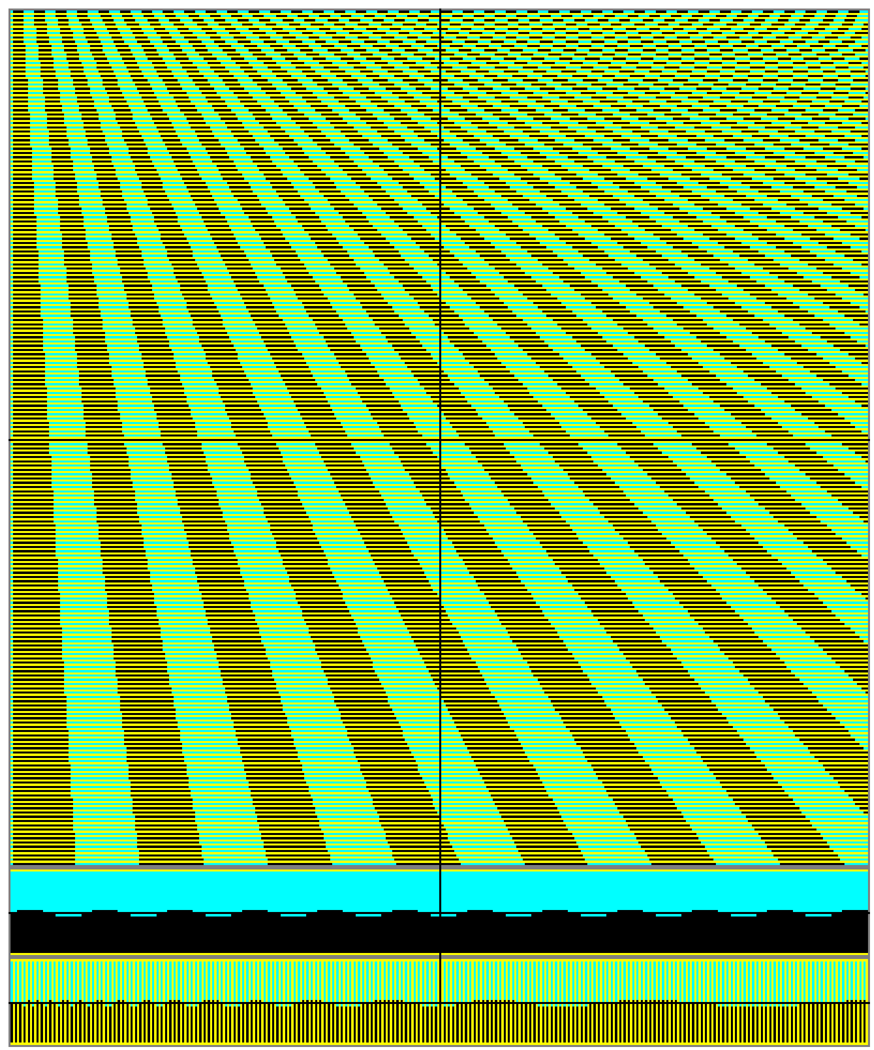


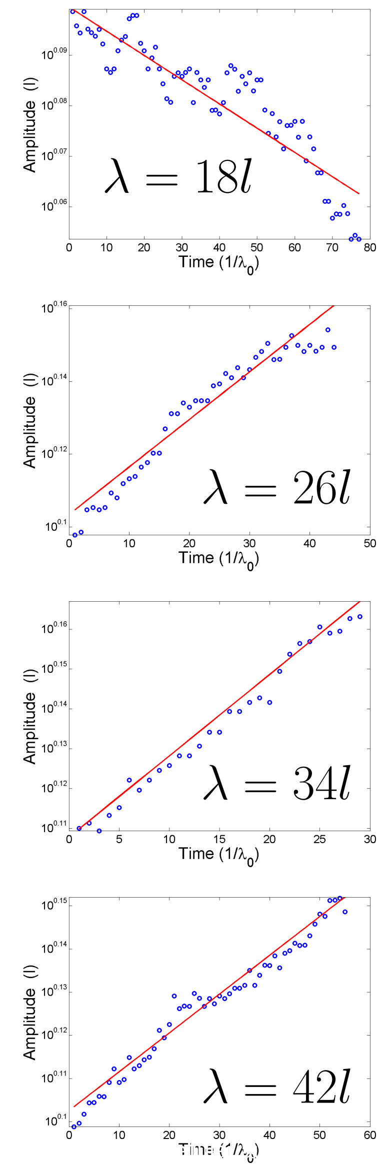


Linear stability analysis of the sand waves instability

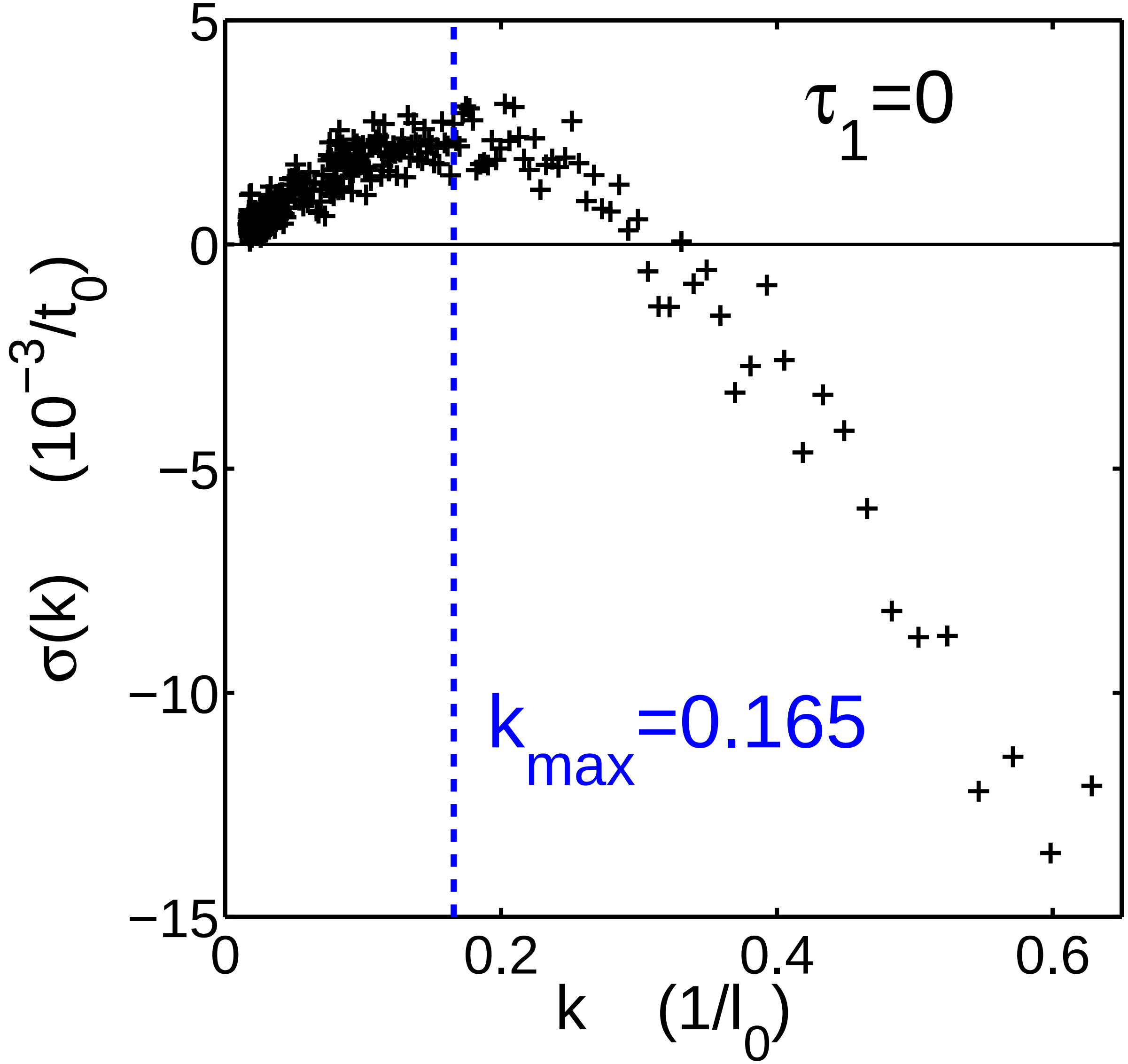
$$A(\lambda, t) \sim \exp(\sigma(\lambda)t)$$







Linear stability analysis of the sand waves instability



Linear stability analysis of the sand waves instability

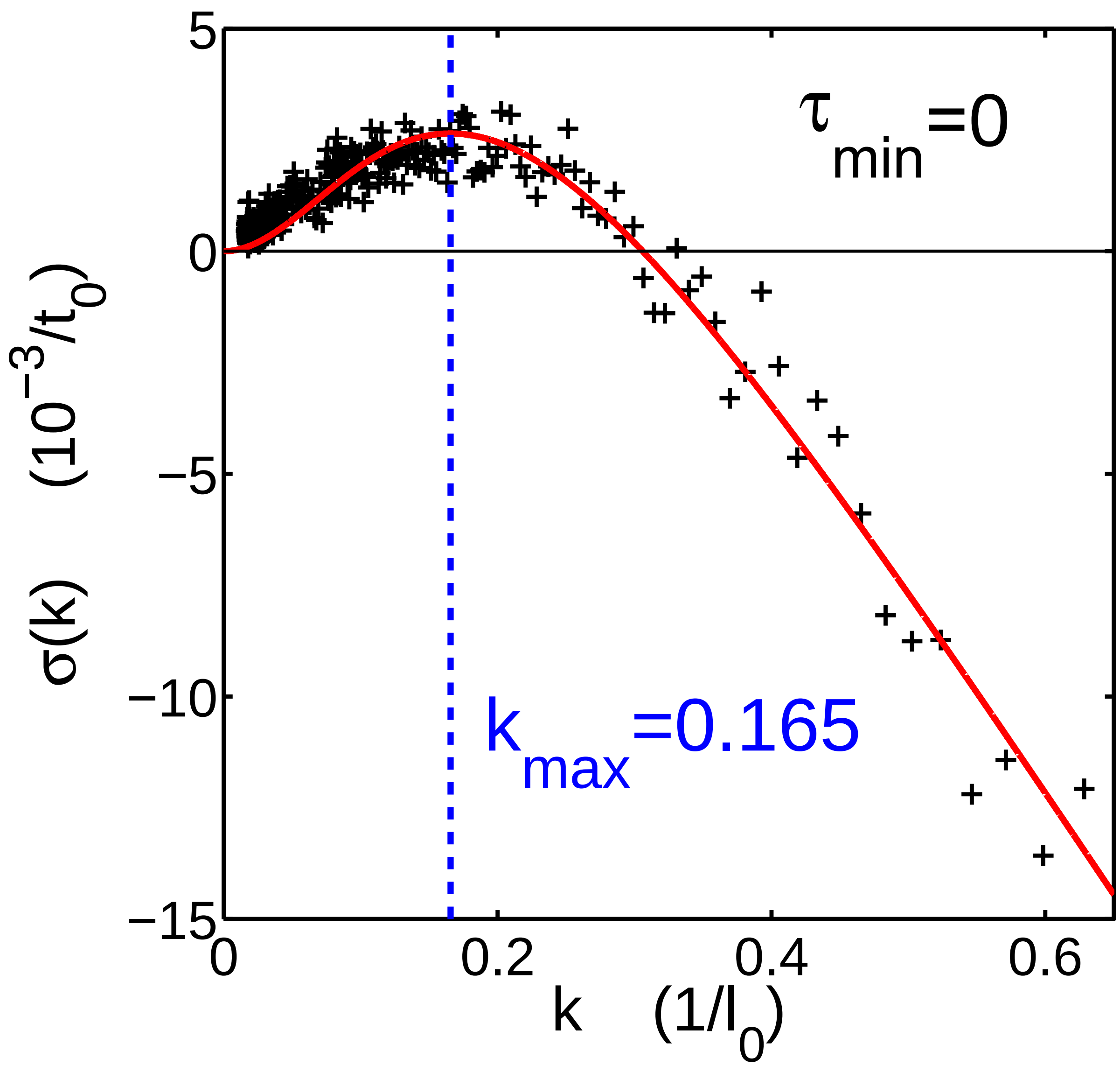
To quantify this fastest growing mode, we fit our data points to the function

$$\sigma(k) = \sigma_0(\alpha k)^2 \frac{1 - \beta(\alpha k)}{1 + (\alpha k)^2},$$

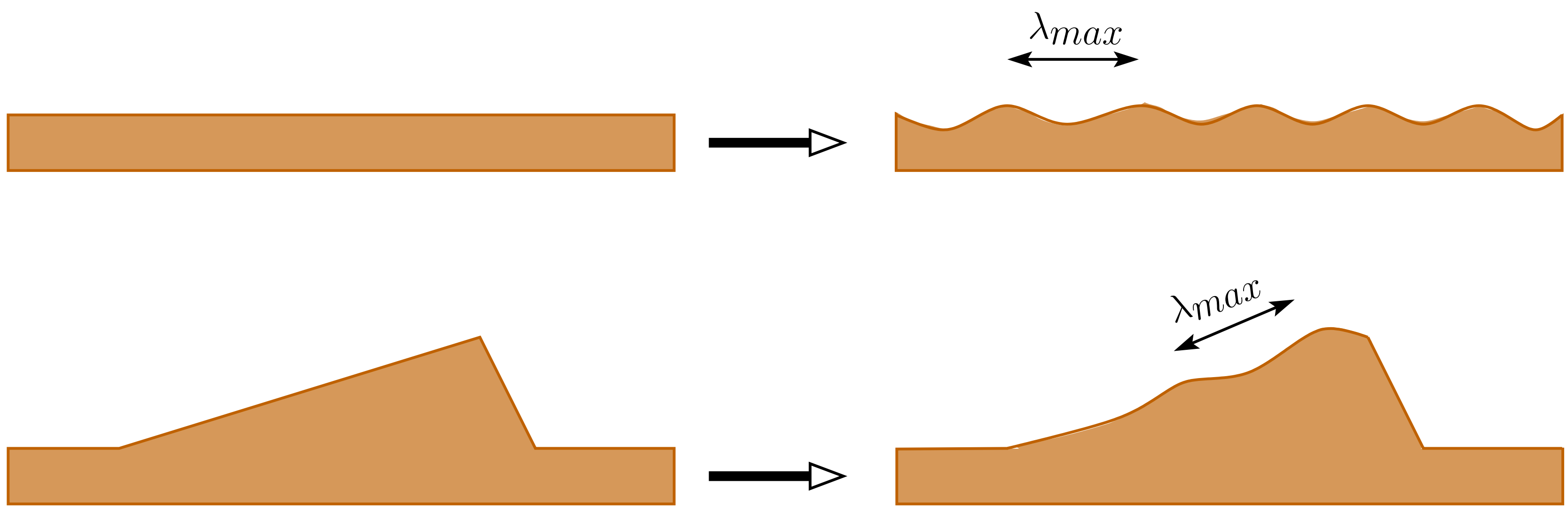
where the three fitting parameters are $\{\sigma_0, \alpha, \beta\}$.

This is the analytical expression of the growth rate proposed by *Andreotti et al. (2002)*.

Linear stability analysis of the sand waves instability



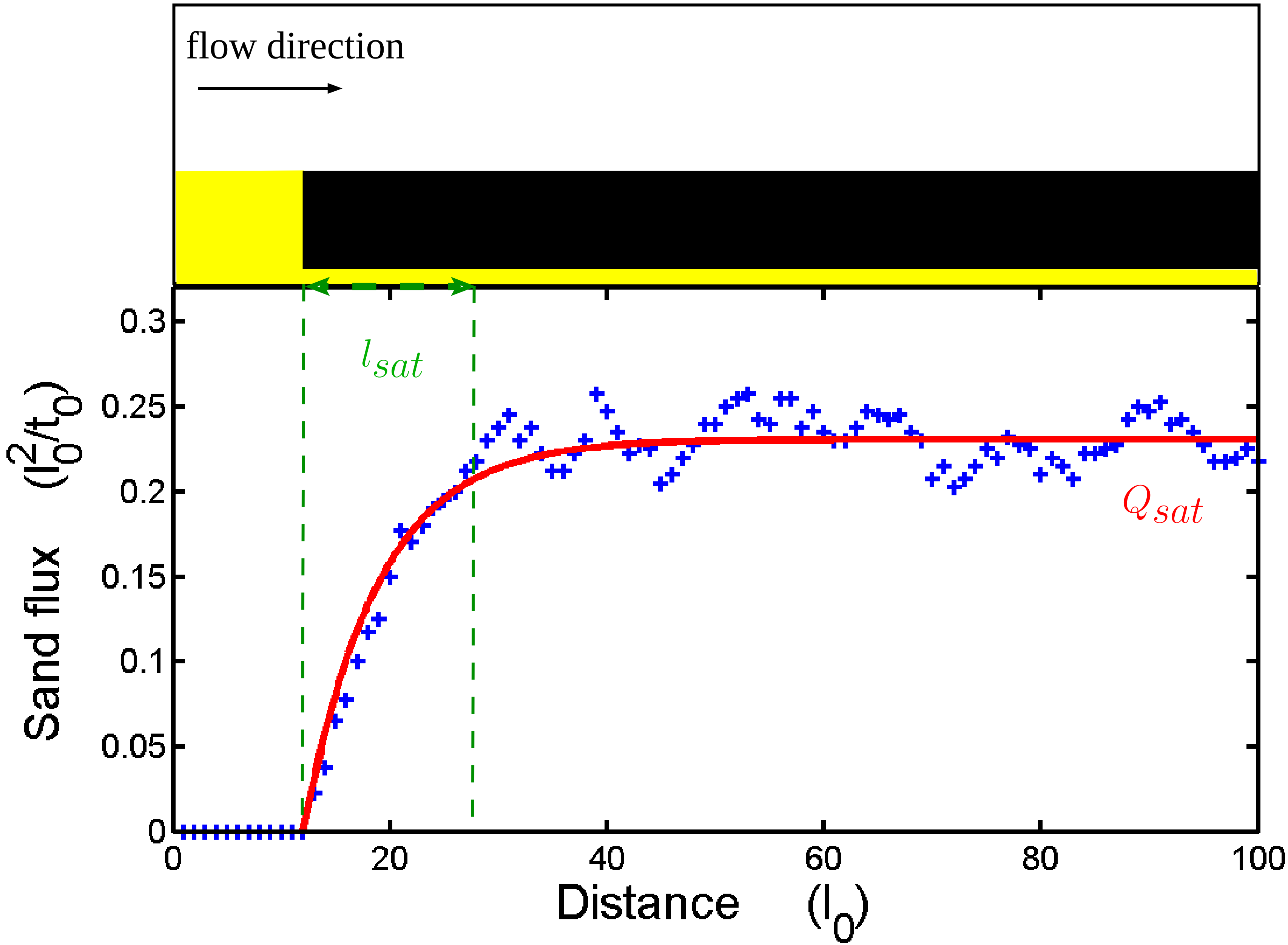
Nucleation of dunes patterns in the model



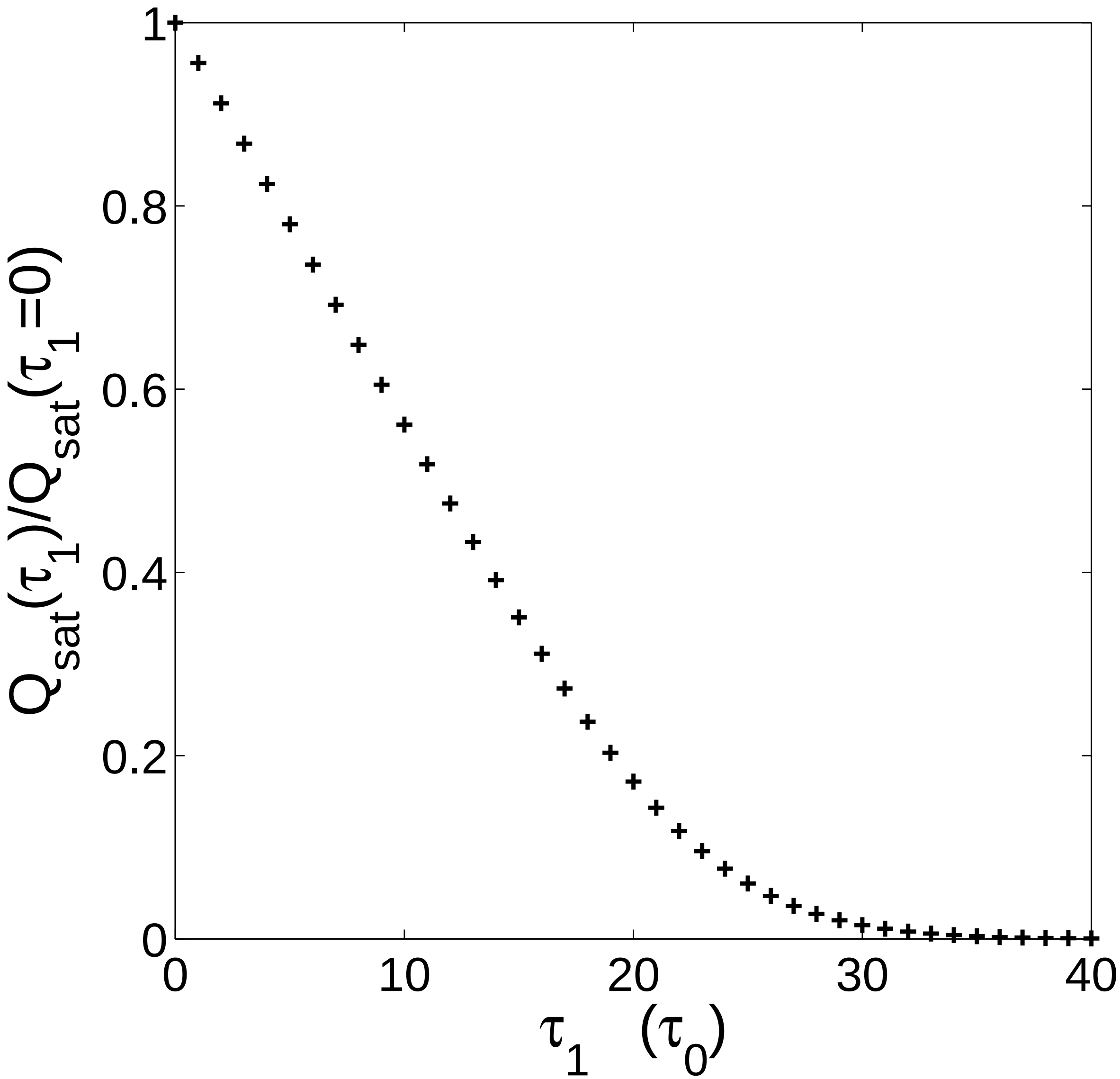
$$\lambda_{max} \approx 40 \ \ l_0$$

How does this length scale relate to the saturated flux Q_{sat} and the saturation length l_{sat}?

Saturation length



Saturated flux



Characteristic scales

$$Q_{sat} = \begin{cases} 0 & \text{if } \tau_s \leq \tau_{th}, \\ \tau_s^{\gamma}(\tau_s - \tau_{th}) & \text{if } \tau_s \geq \tau_{th}. \end{cases}$$

$\Longrightarrow$

$$\frac{Q_{sat}}{Q_{sat}^0} = 1 - \left(\frac{u_{th}}{u_*}\right)^2 .$$

$\Longrightarrow$

$$\langle Q_{sat} \rangle = Q_{sat}(u_*)$$

Characteristic scales

$$l_{sat} \approx 6\ l_0 \qquad \lambda_{max} \approx 40\ l_0$$

$$\lambda_{max} \approx 7\ l_{sat}$$

A characteristic length scale:

$$l_0 = 0.5 \text{ m}$$

A characteristic time scale:

$$t_0 = \frac{Q_{sat}}{\langle Q_{sat} \rangle} l_0^2$$

Conclusion

The model displays

$\longrightarrow$ many types of dune morphologies.

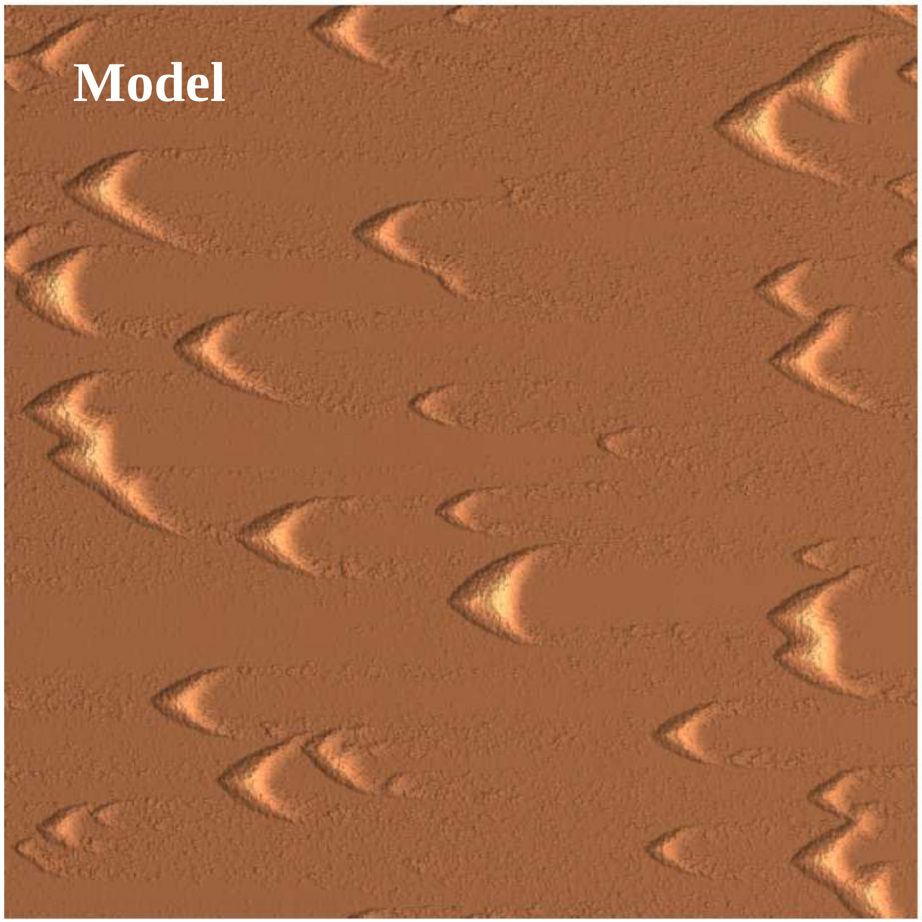
$\longrightarrow$ waves on the surface of dunes.

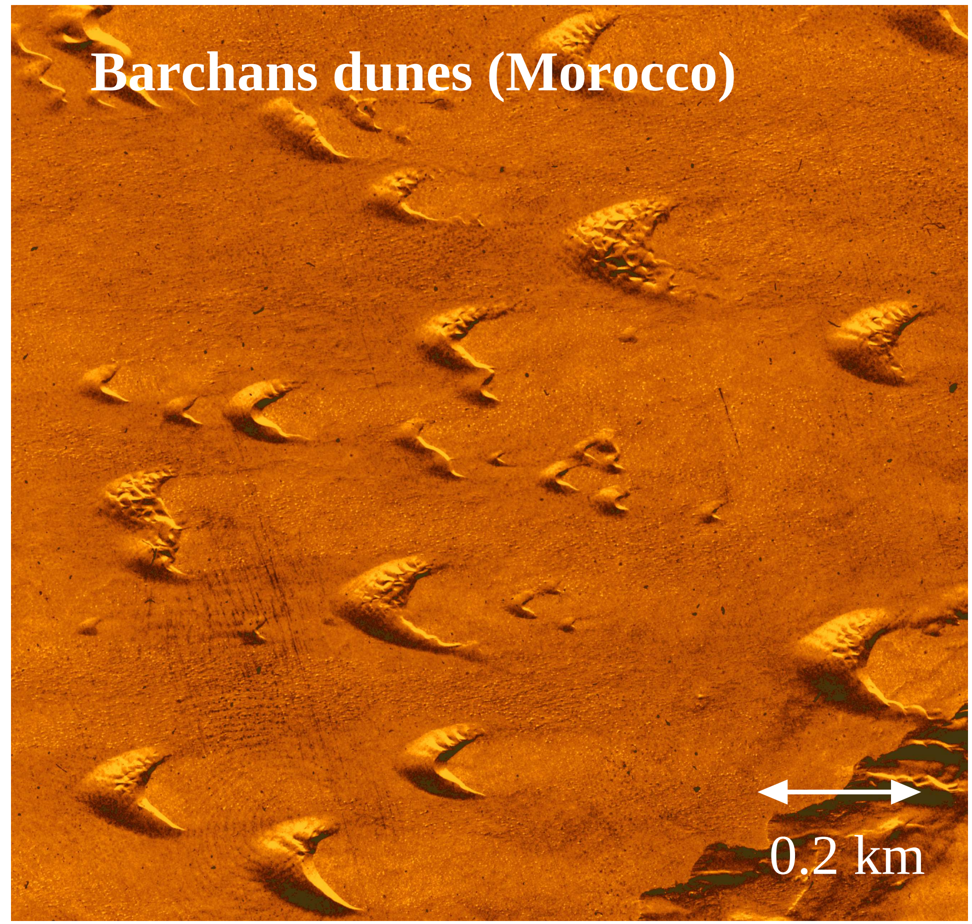
$\longrightarrow$ l_{sat} and Q_{sat}.

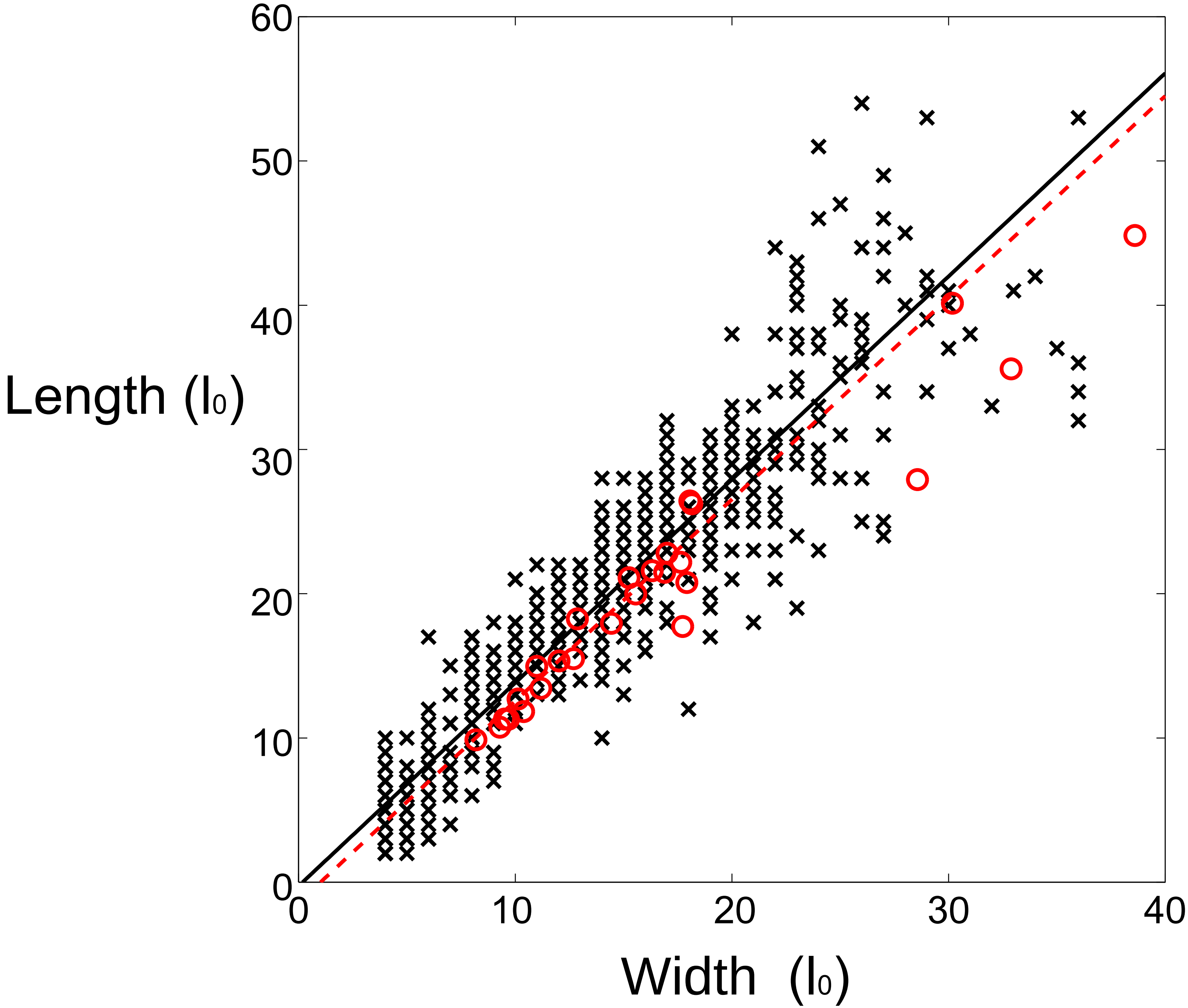
$\longrightarrow$ λ_{max}.

It can be used for systematic quantitative analysis of dune patterns in all physical environments.

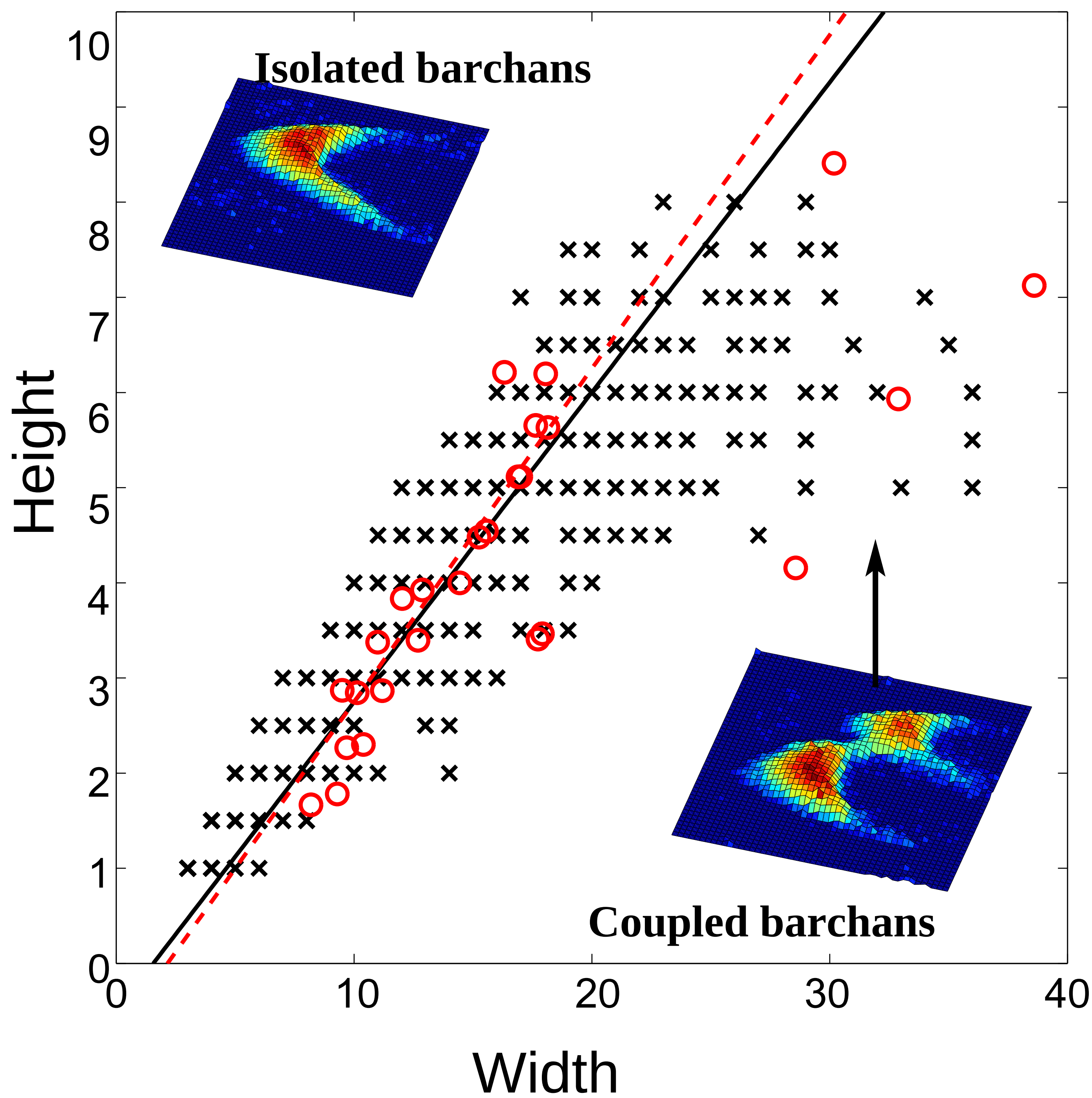
Population of dunes

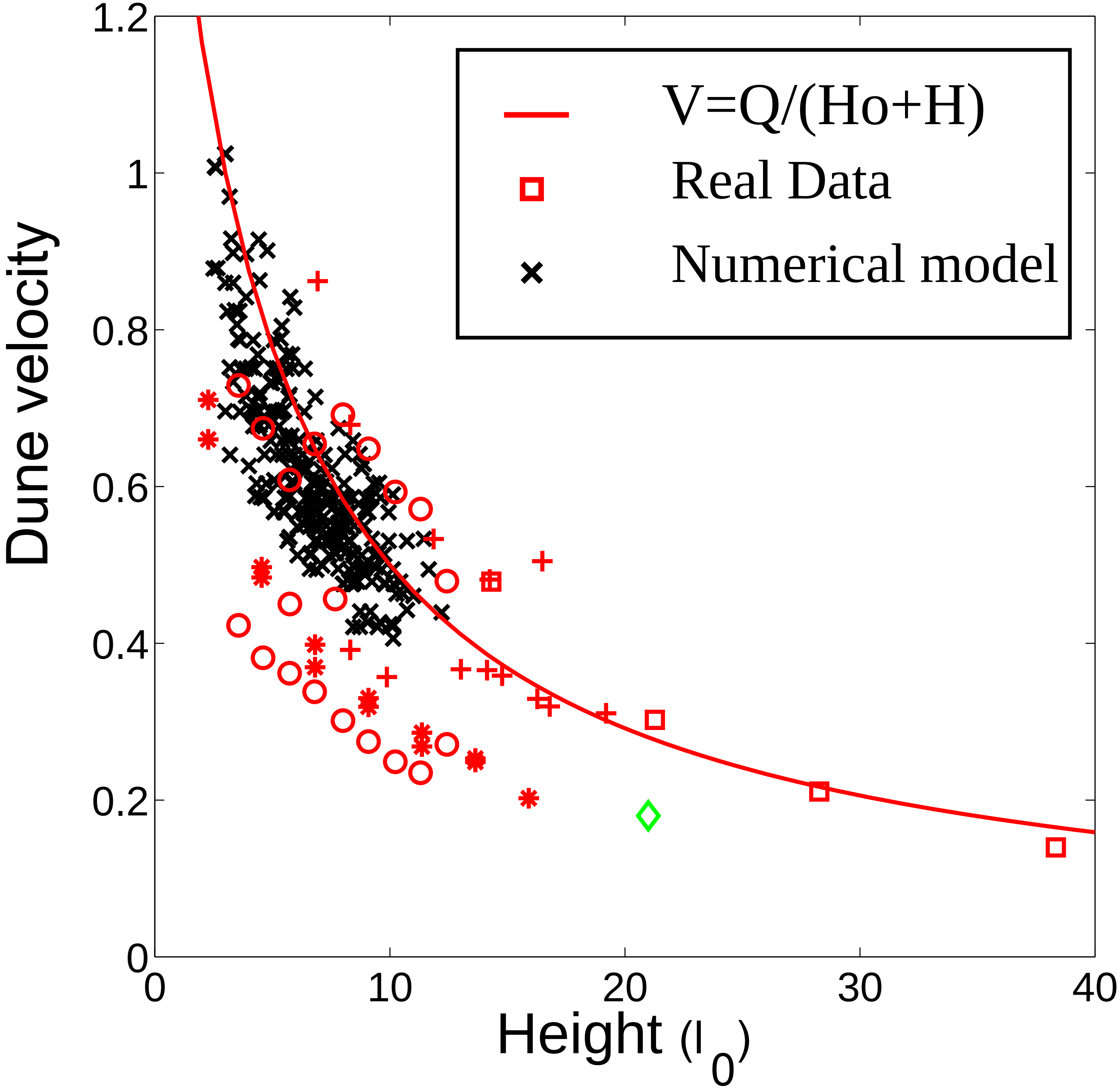






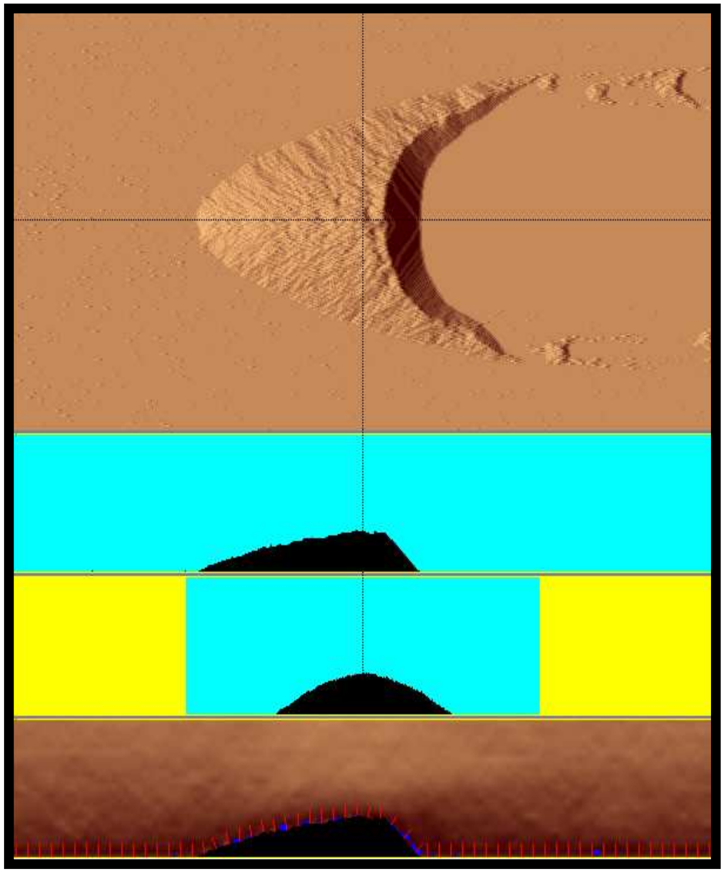
Length (l0)
Width (l0)
0
10
20
30
40
50
60
0
10
20
30
40


Isolated barchans
Coupled barchans
Height
Width
0
10
20
30
40
0
1
2
3
4
5
6
7
8
9
10

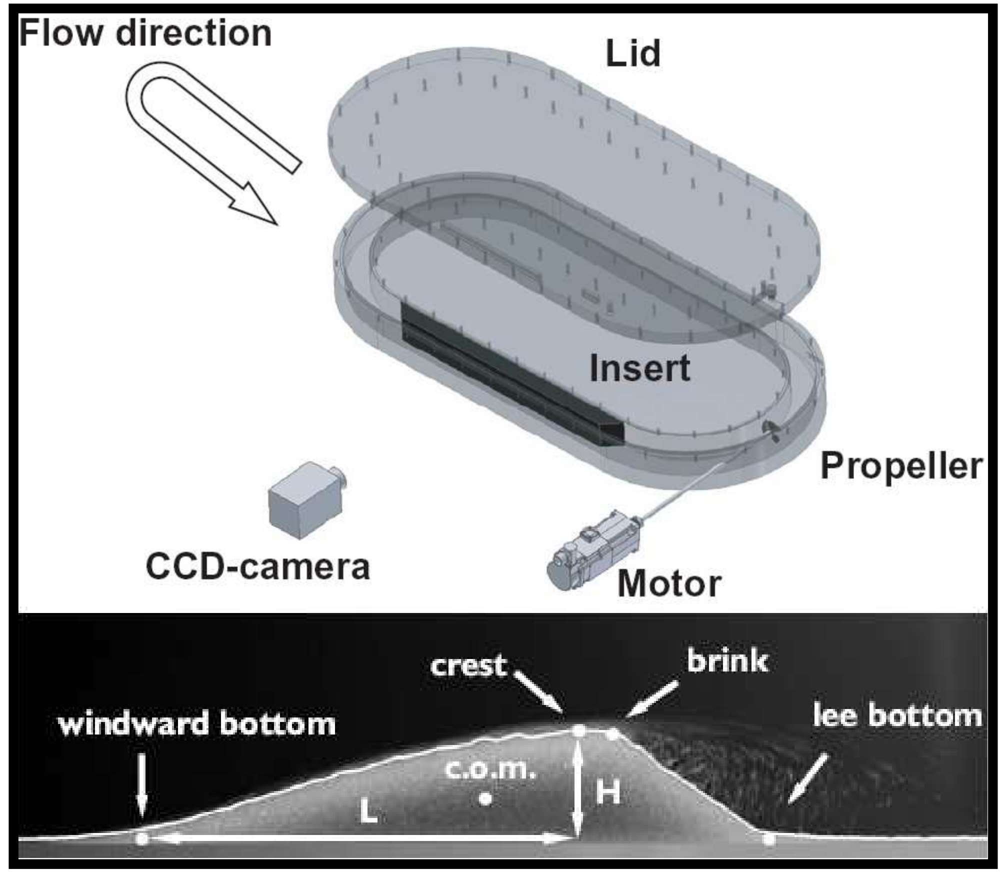


V=Q/(Ho+H)
Real Data
Numerical model
Dune velocity
Height (l 0)
1.2
1
0.8
0.6
0.4
0.2
0
0
10
20
30
40

Comparisons between numerical and experimental data

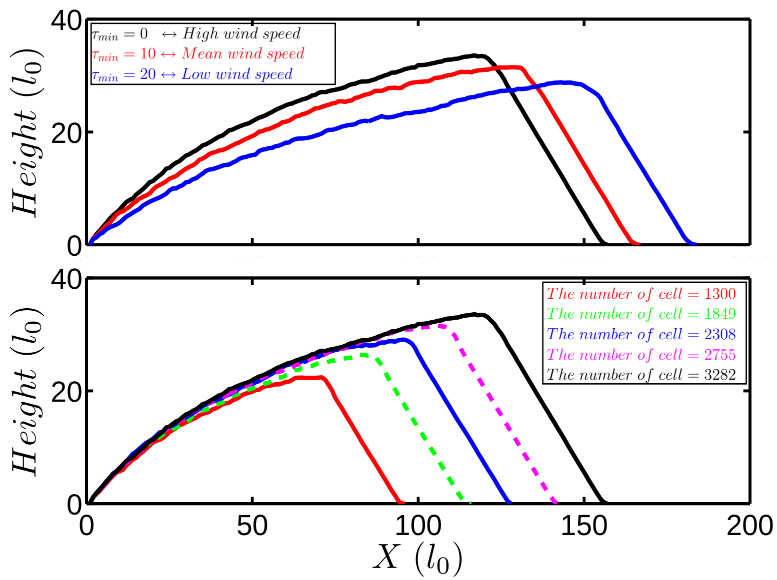


Numerical results

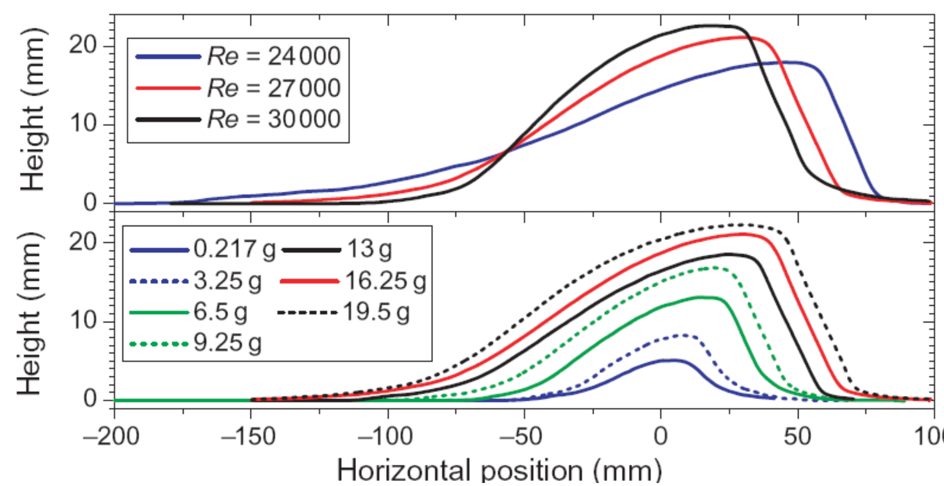


Laboratory experiments (Germany)

Comparisons between numerical and experimental data

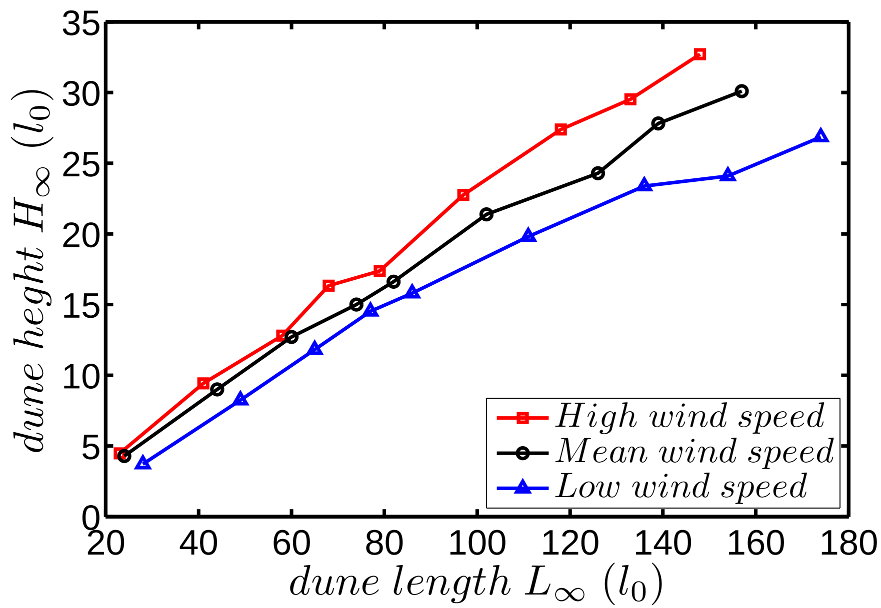


Numerical data

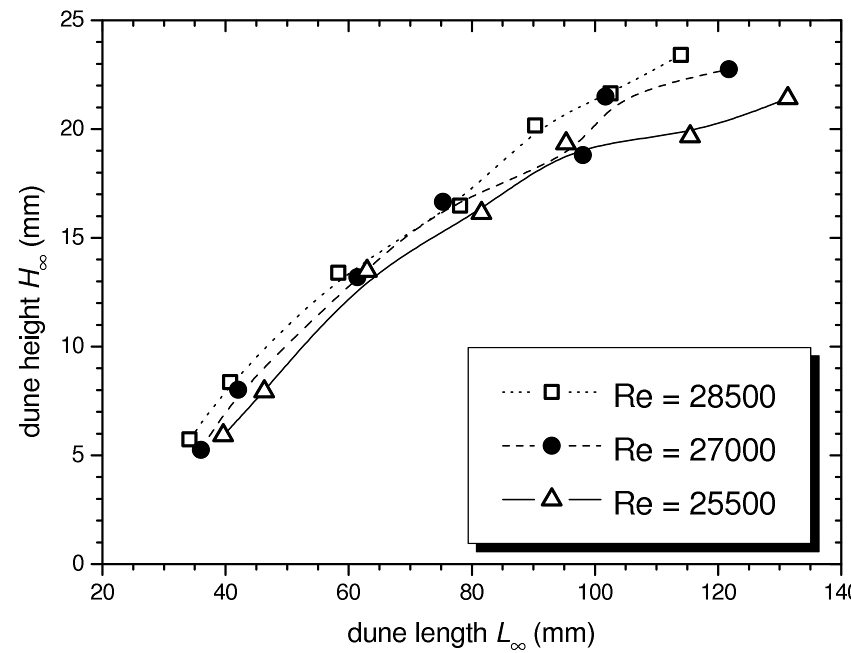


Experimental data

Comparisons between numerical and experimental data

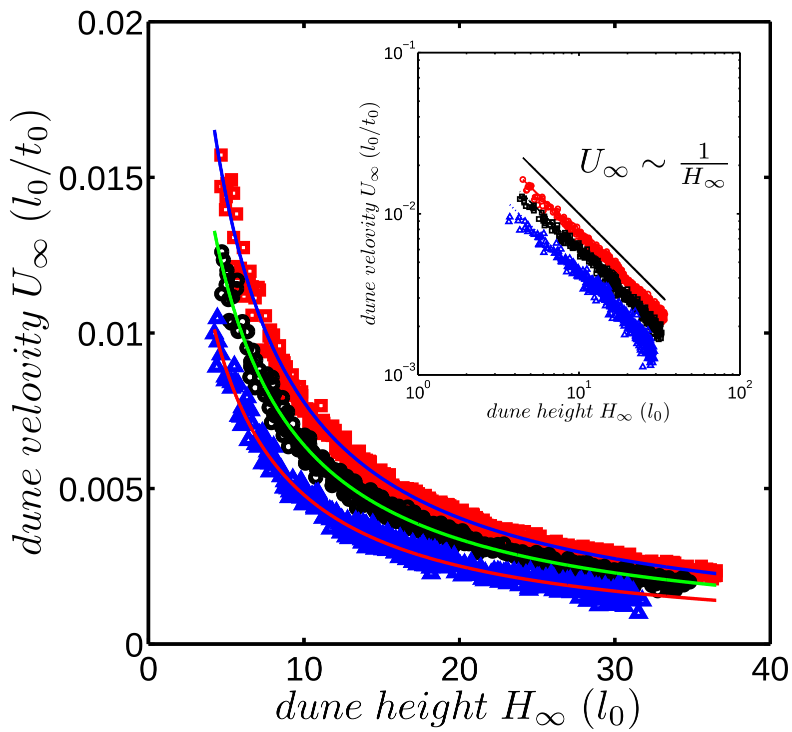


Numerical data

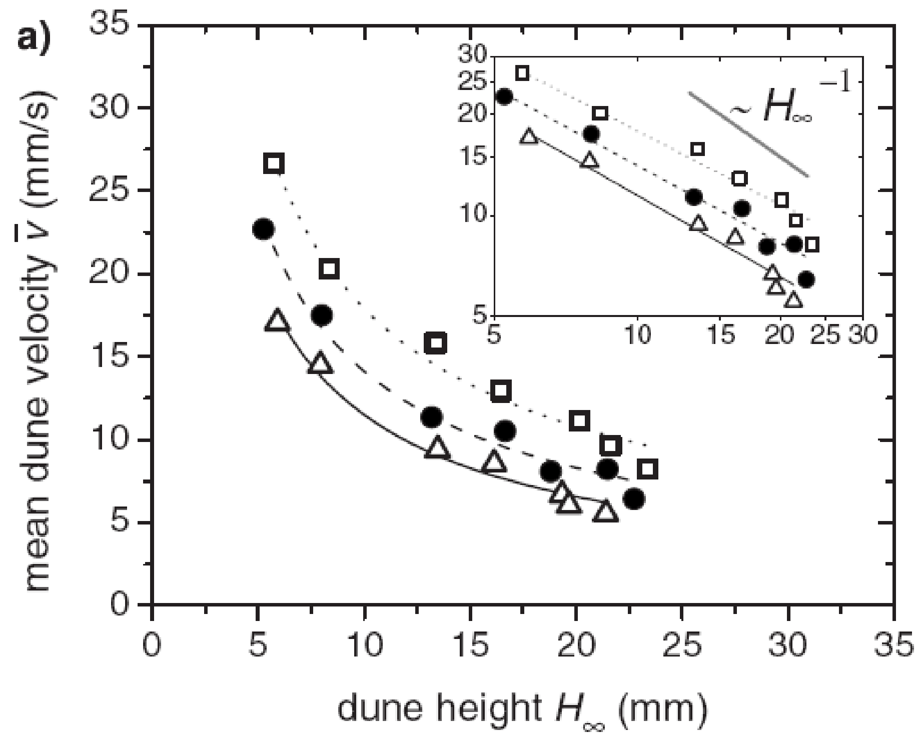


Experimental data

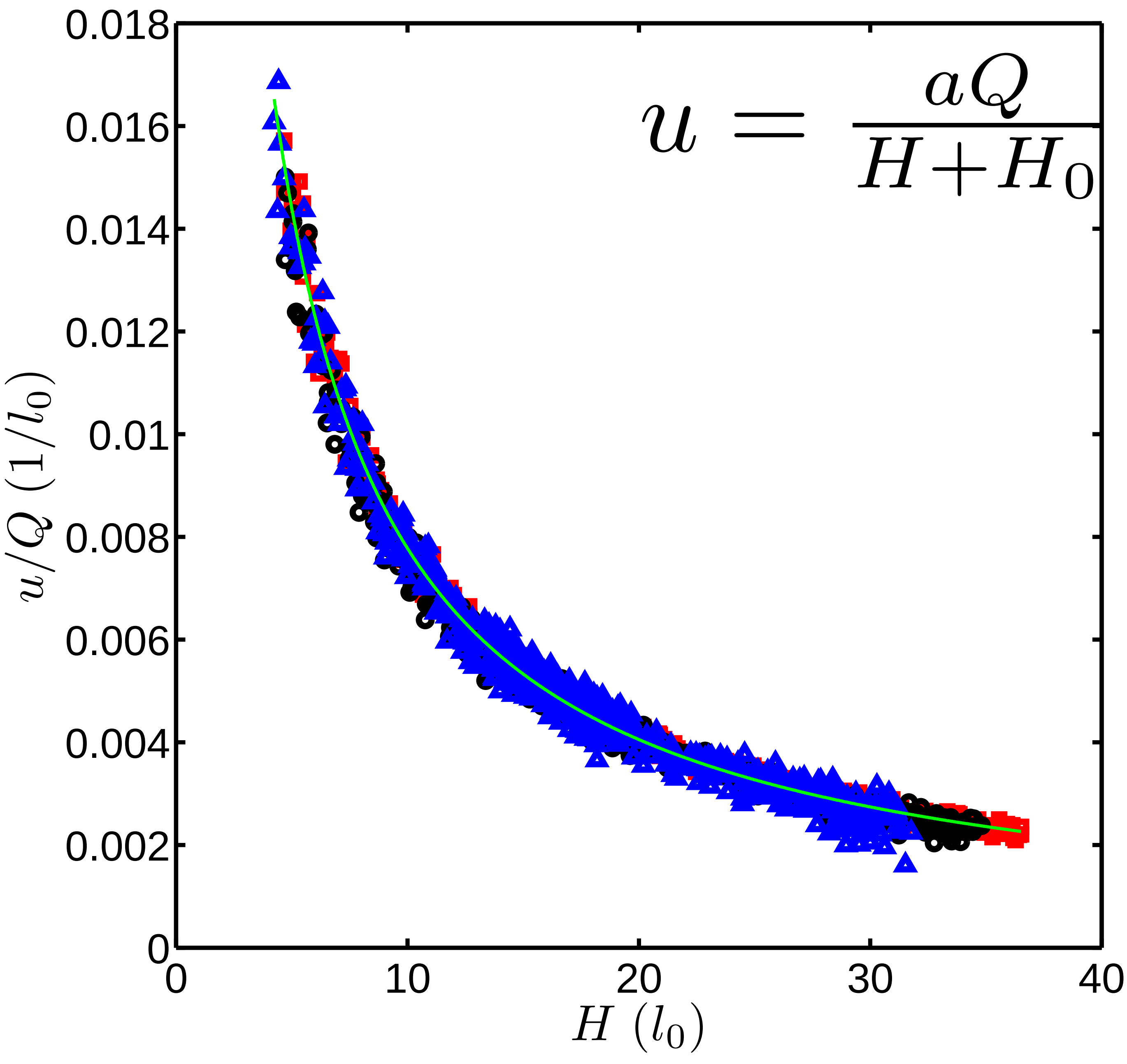
Comparisons between numerical and experimental data



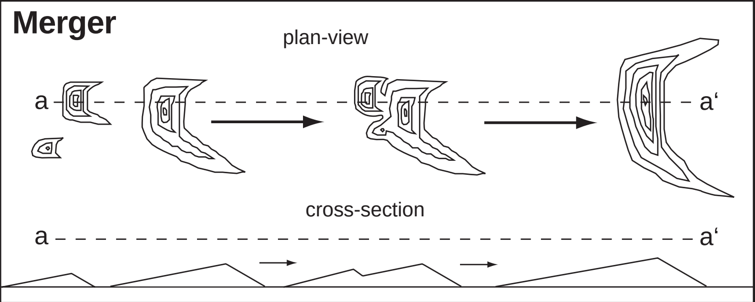
Numerical data

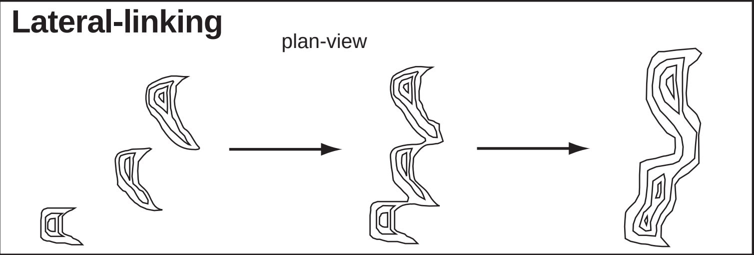


Experimental data

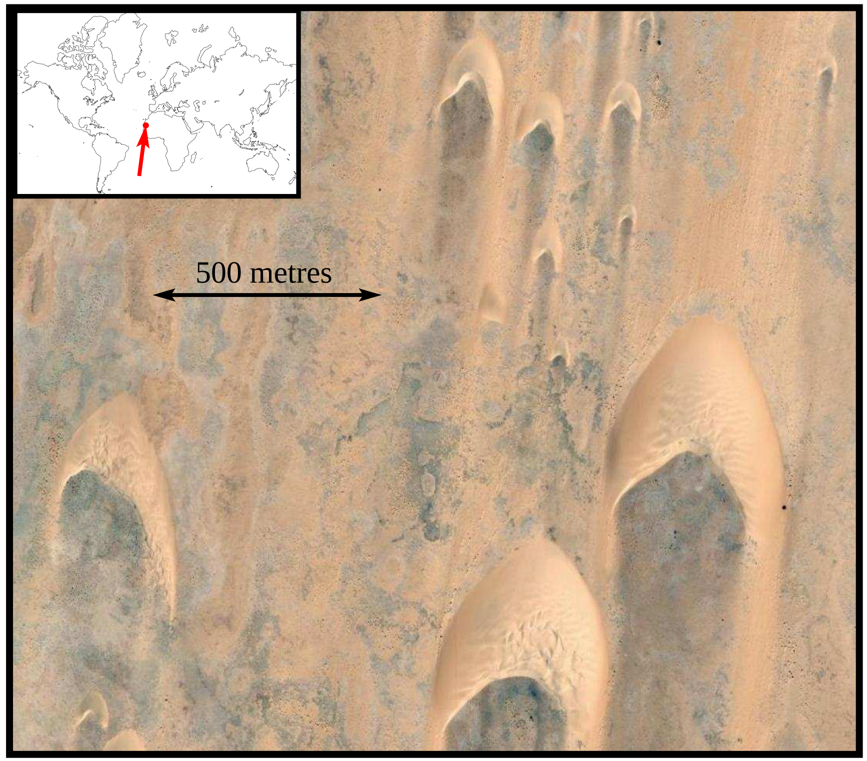


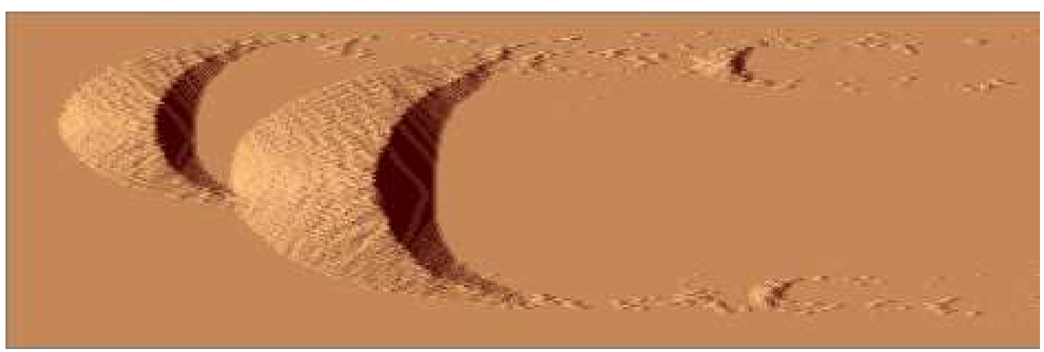
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
0
10
20
30
40
$u/Q\ (1/l_0)$
$H\ (l_0)$
$u = \frac{aQ}{H + H_0}$

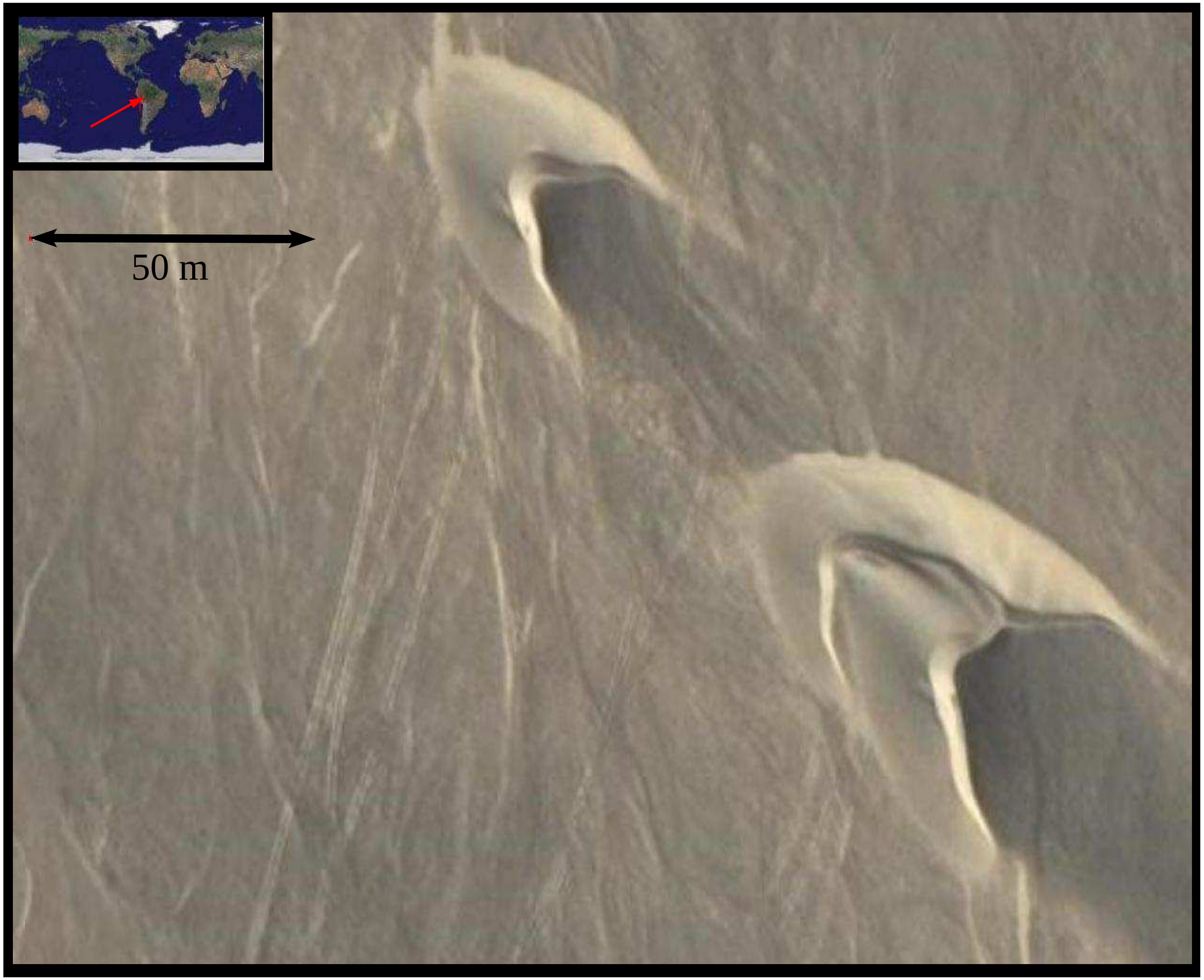

Merger
plan-view
a
a'
cross-section
a
a'

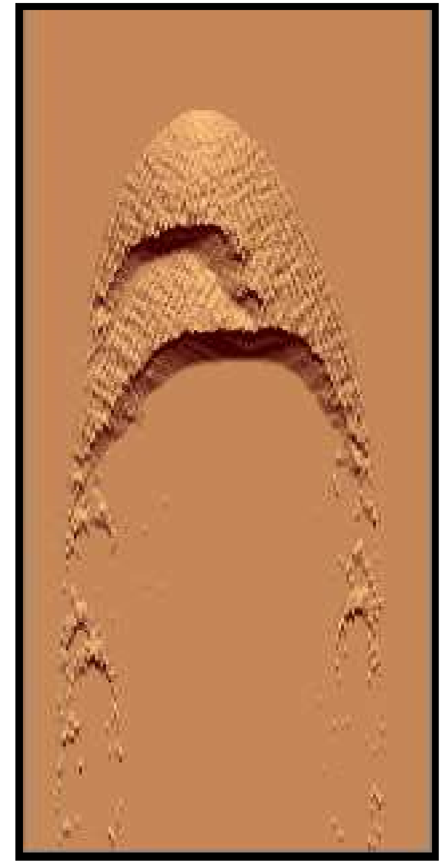

Lateral-linking
plan-view

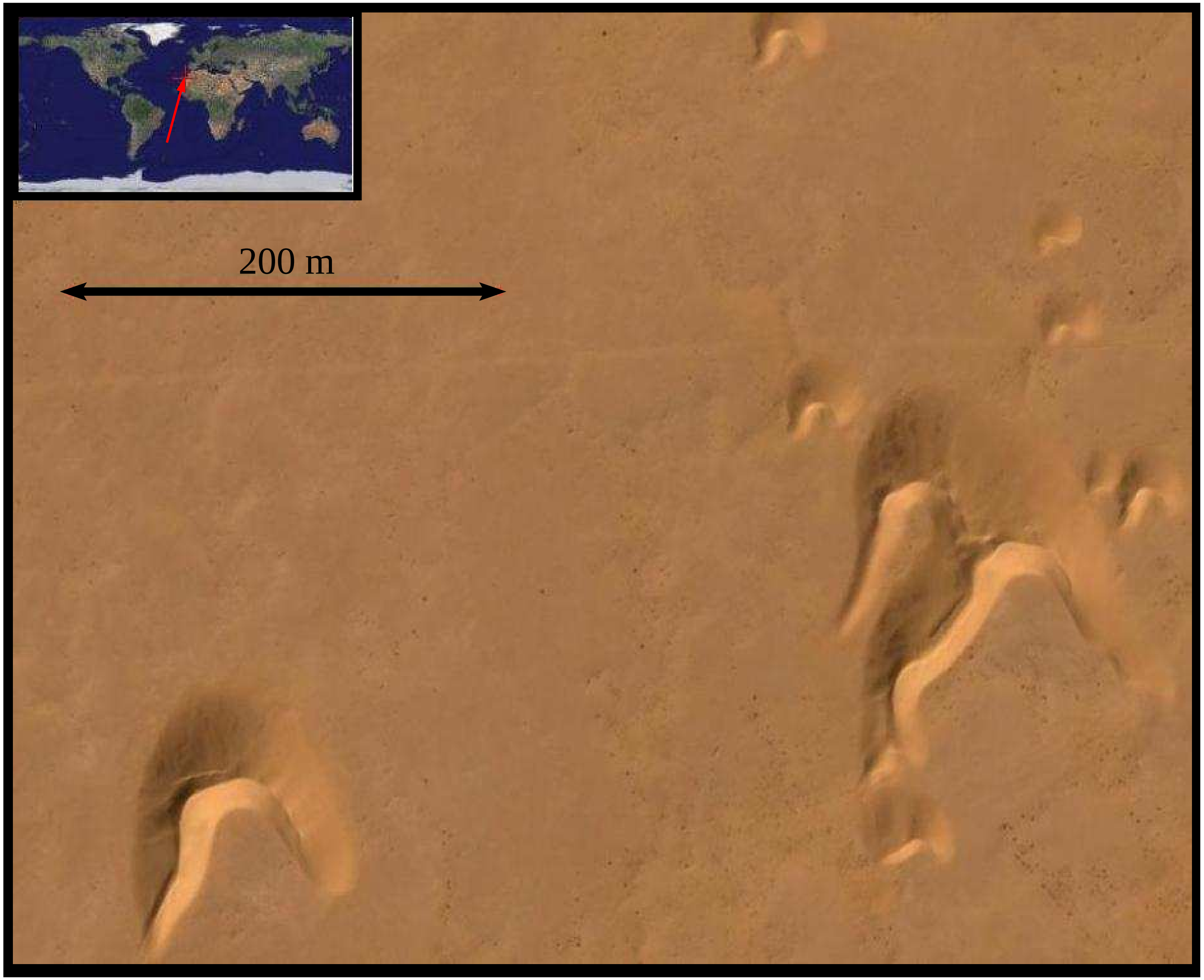
Collision of dunes

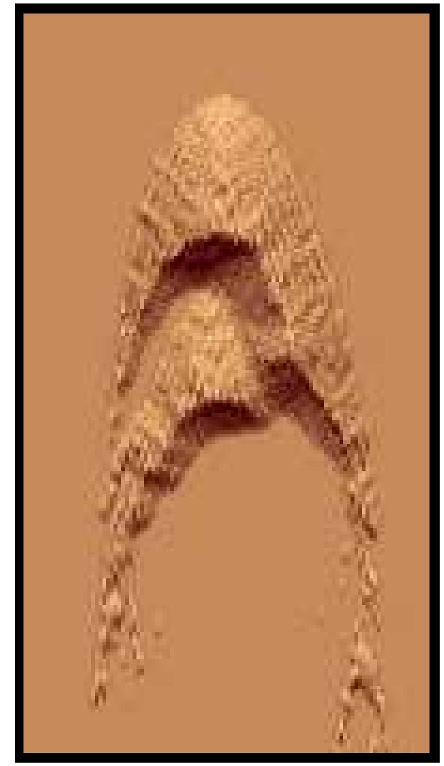




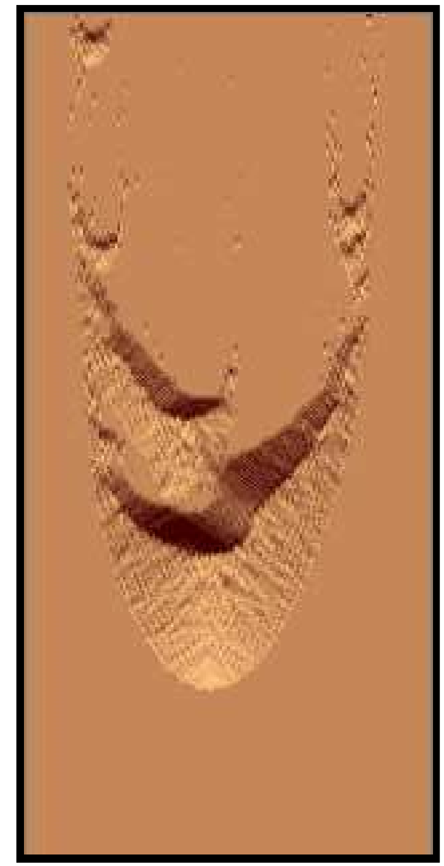

50 m



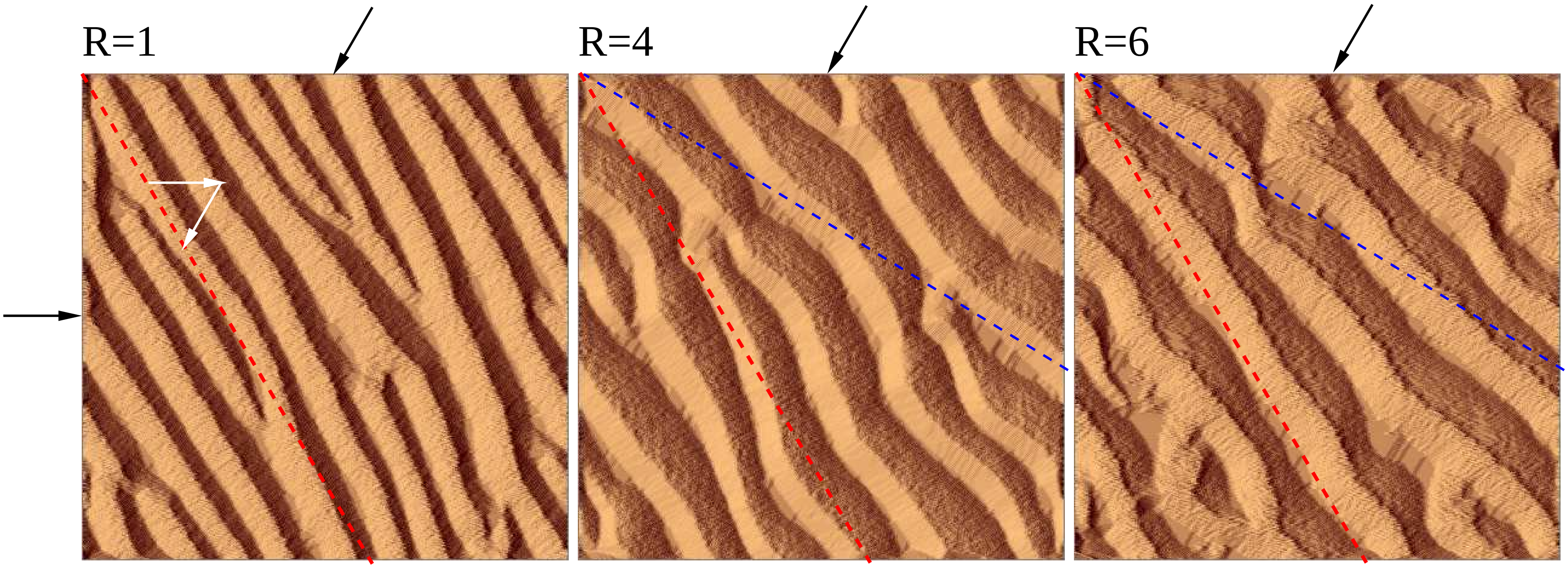

200 m




300 m

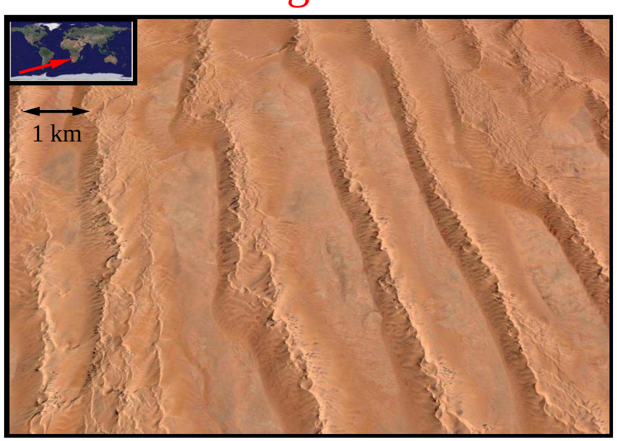


Longitudinal dunes

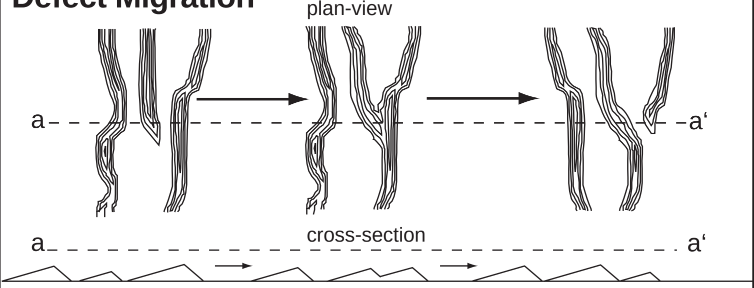


.... Longitudinal

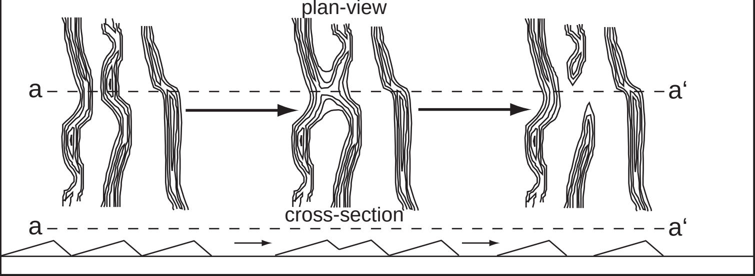
.... Transverse



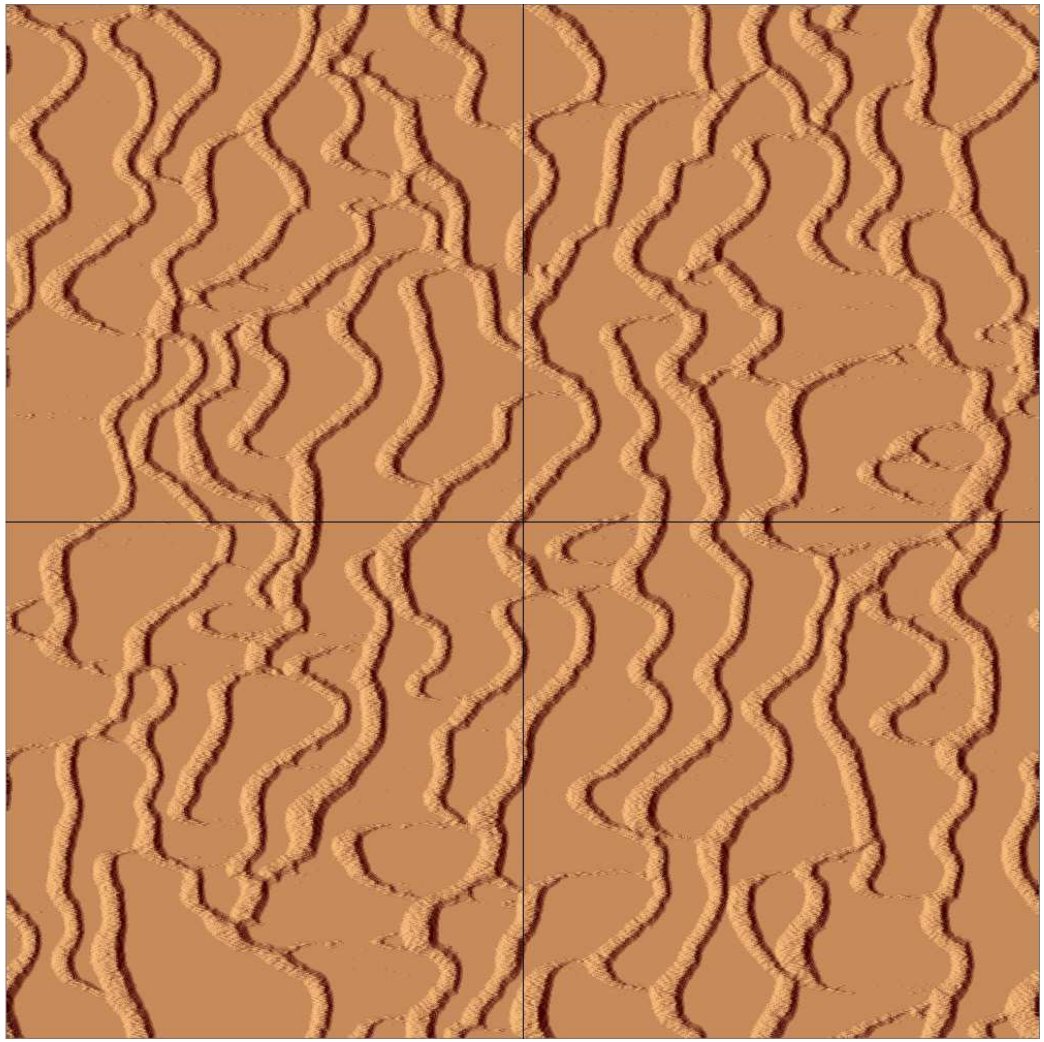
Defect Migration



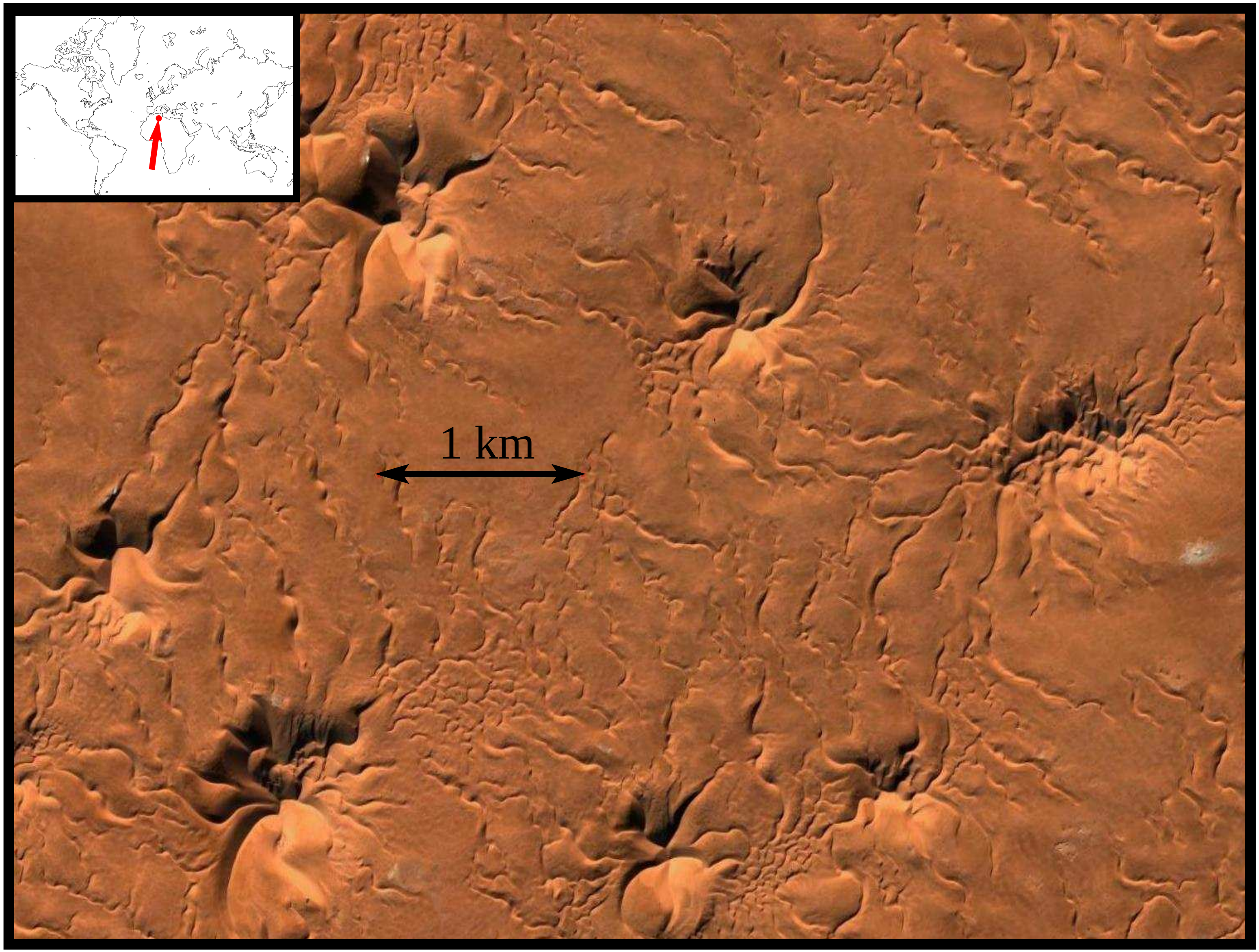
Termination Creation

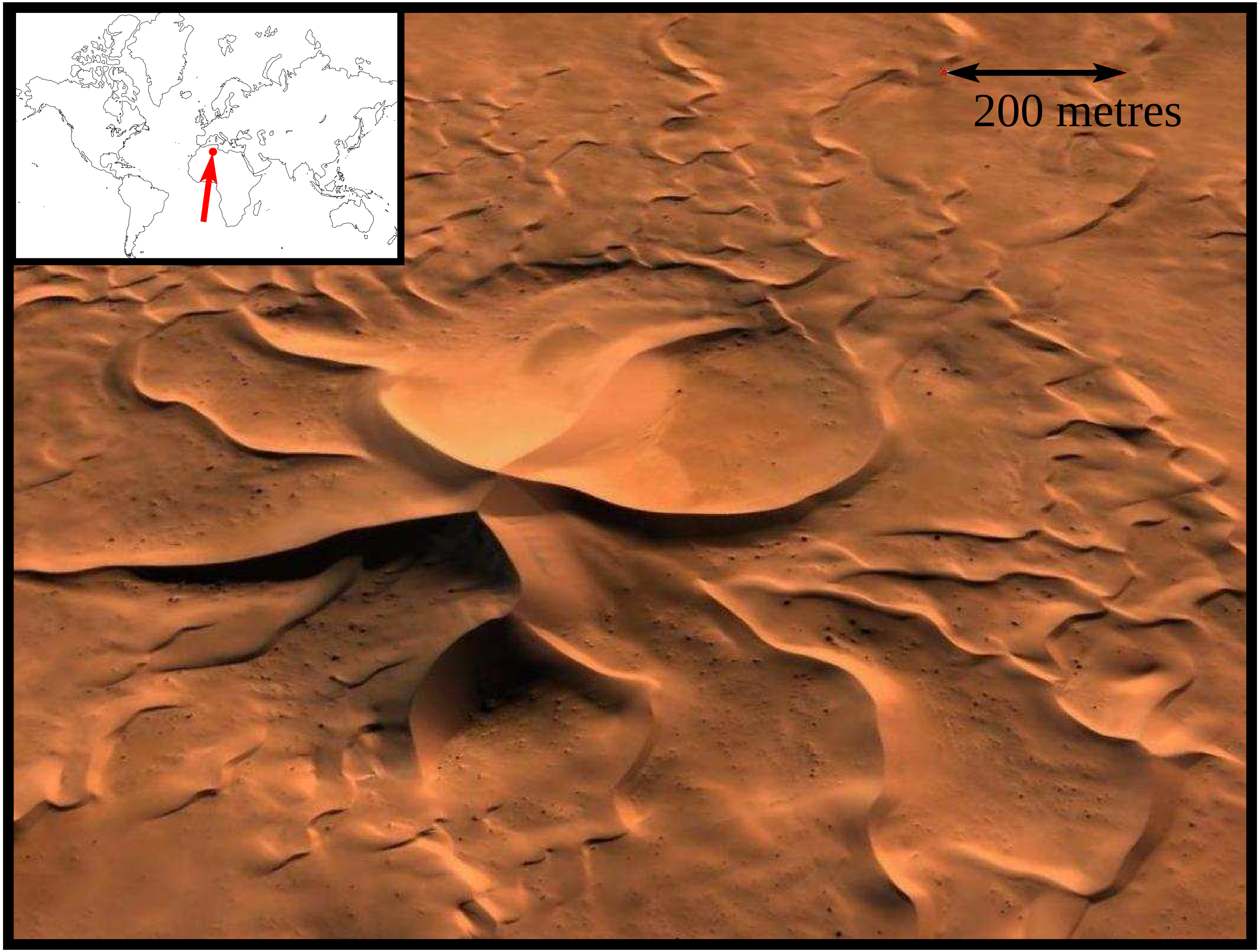


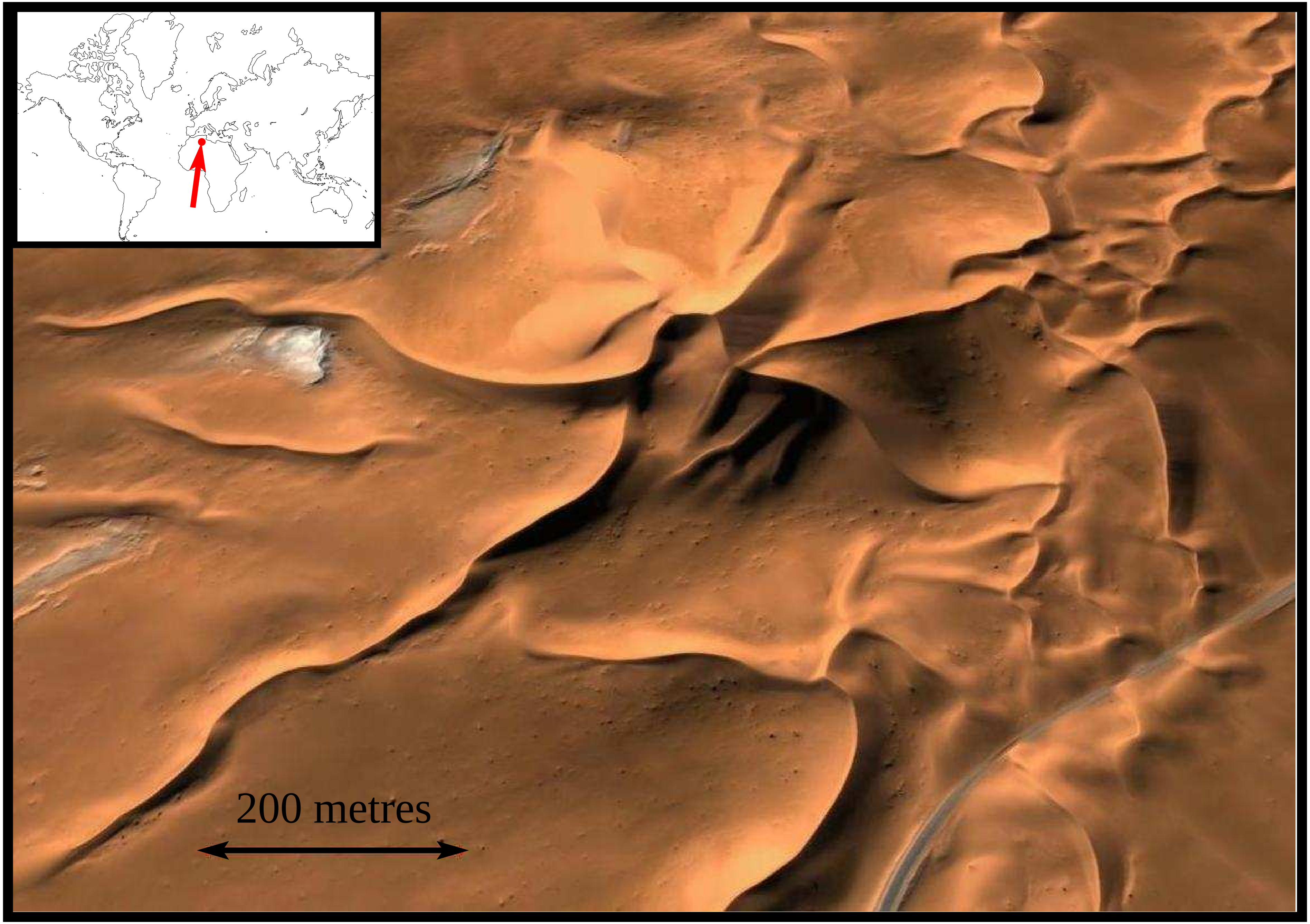
From transverse to barchan dunes



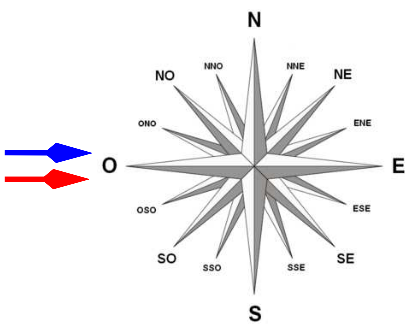
Star dunes



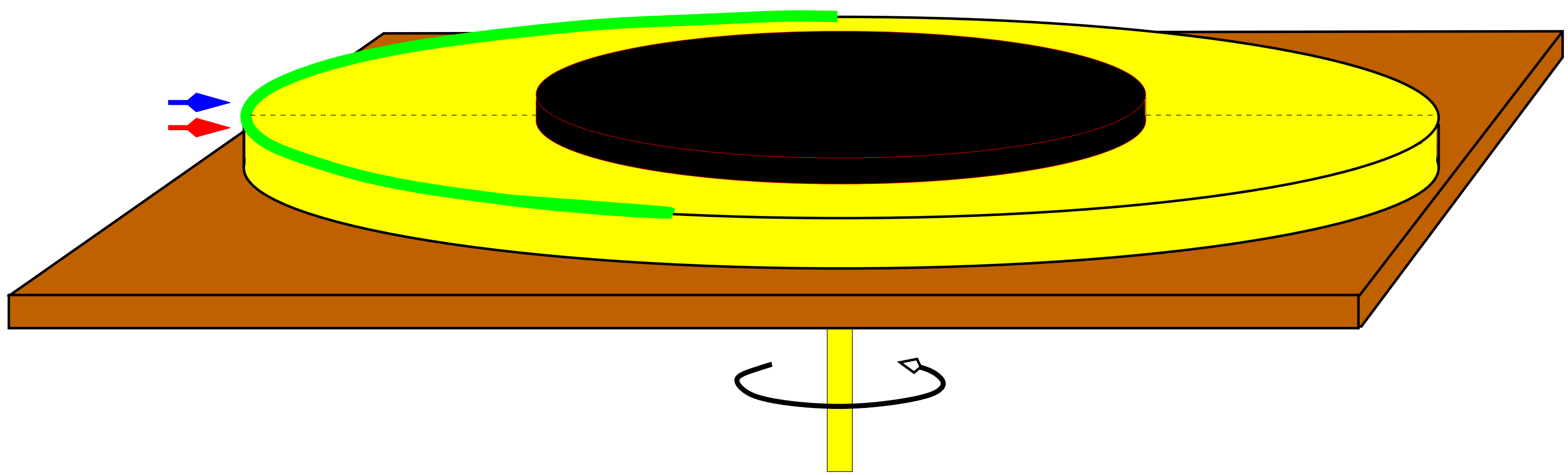

200 metres


200 metres

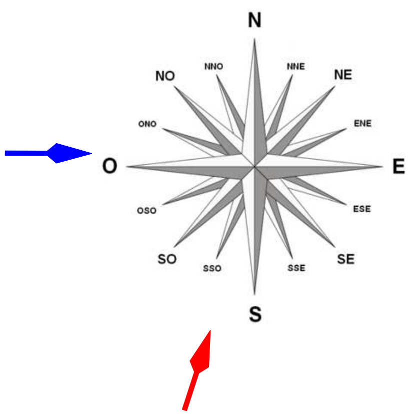
Rotating table for variable wind orientations



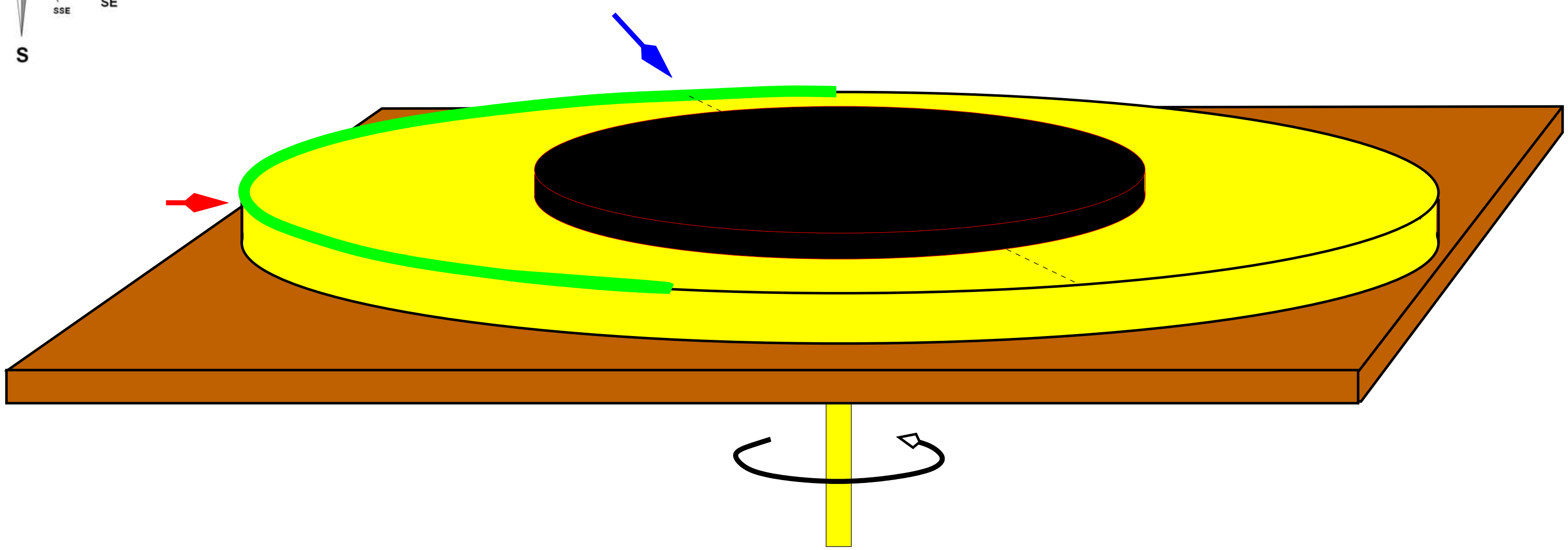
$$\alpha = 0^{o}$$



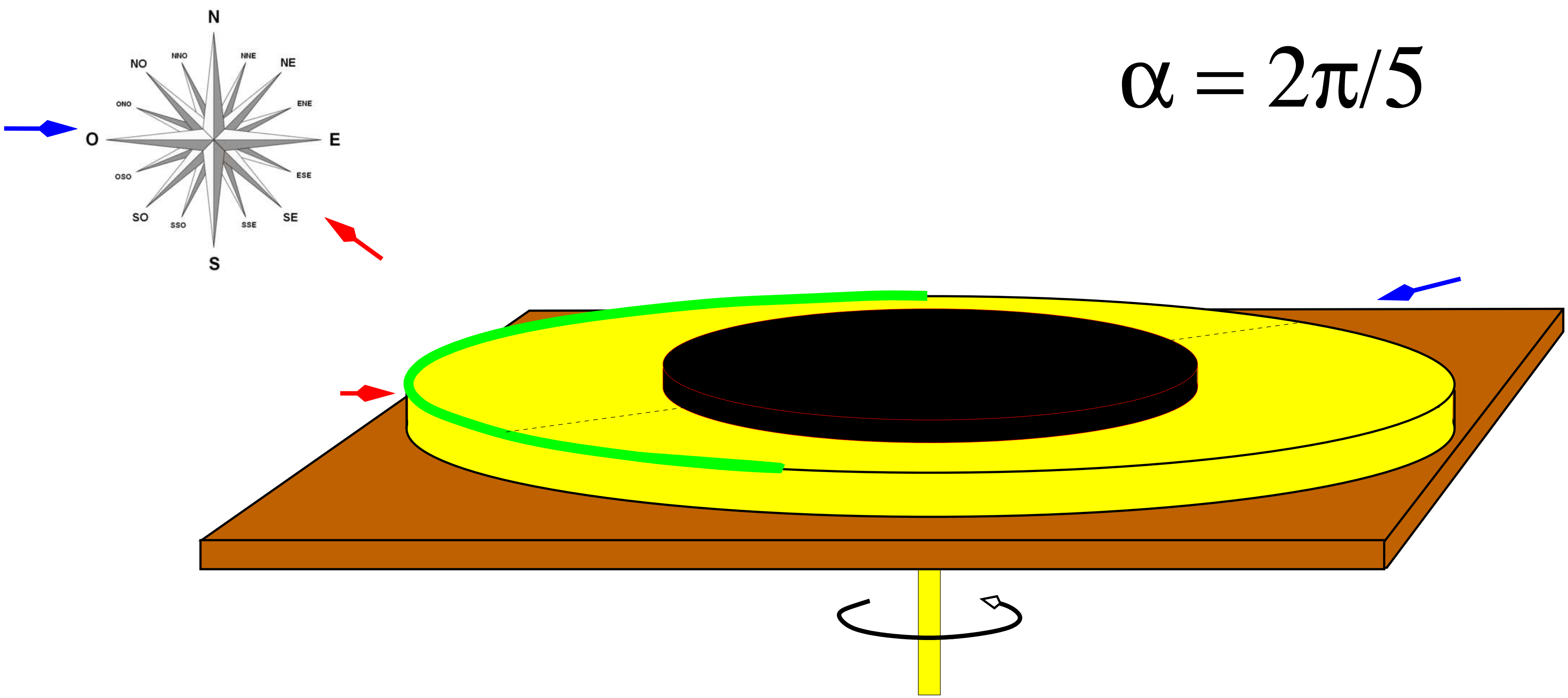
Rotating table for variable wind orientations



$$\alpha = \pi/5$$



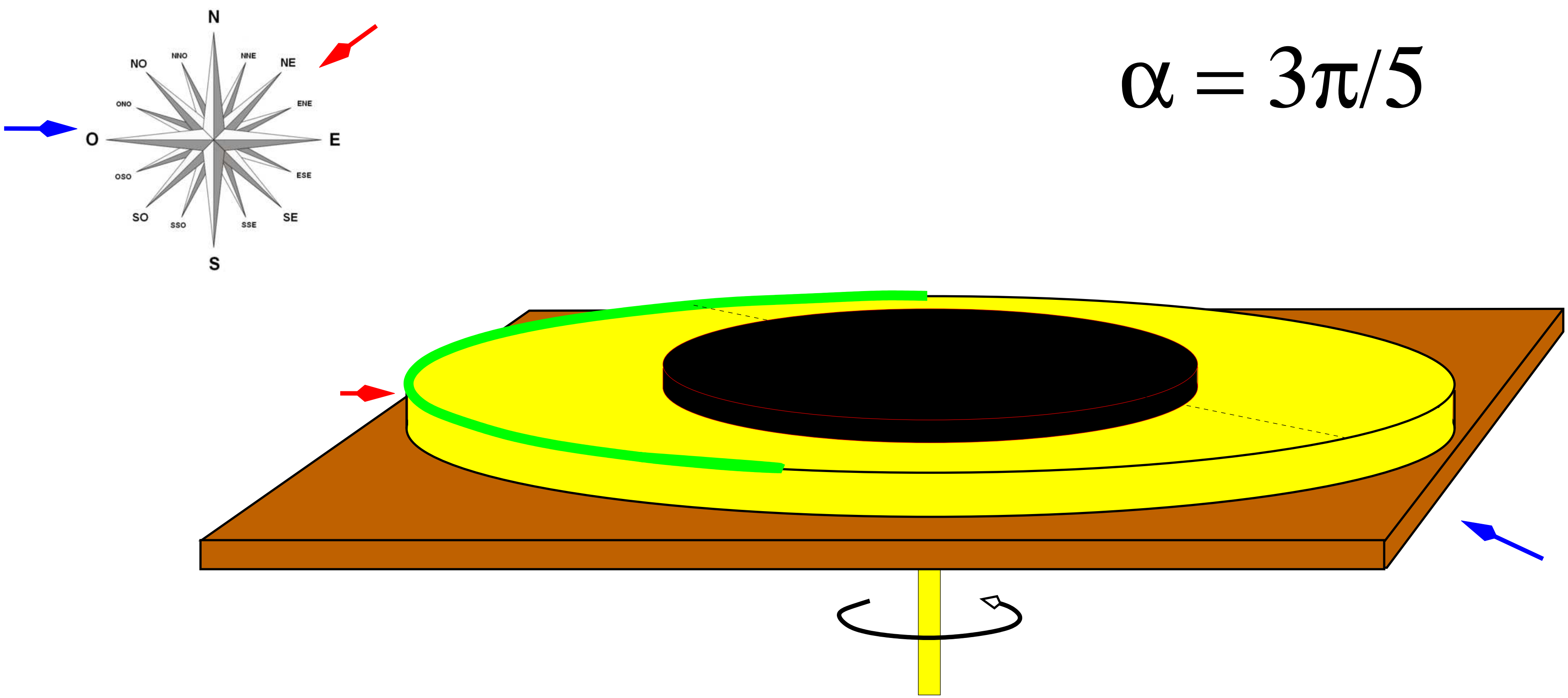
Rotating table for variable wind orientations



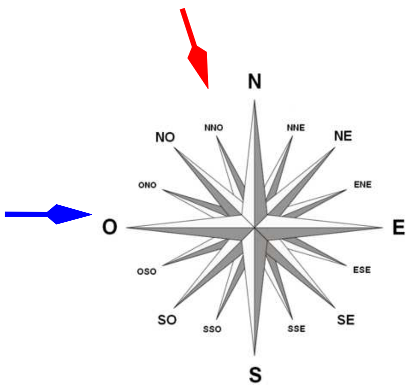
$$\alpha = 2\pi/5$$

Rotating table for variable wind orientations

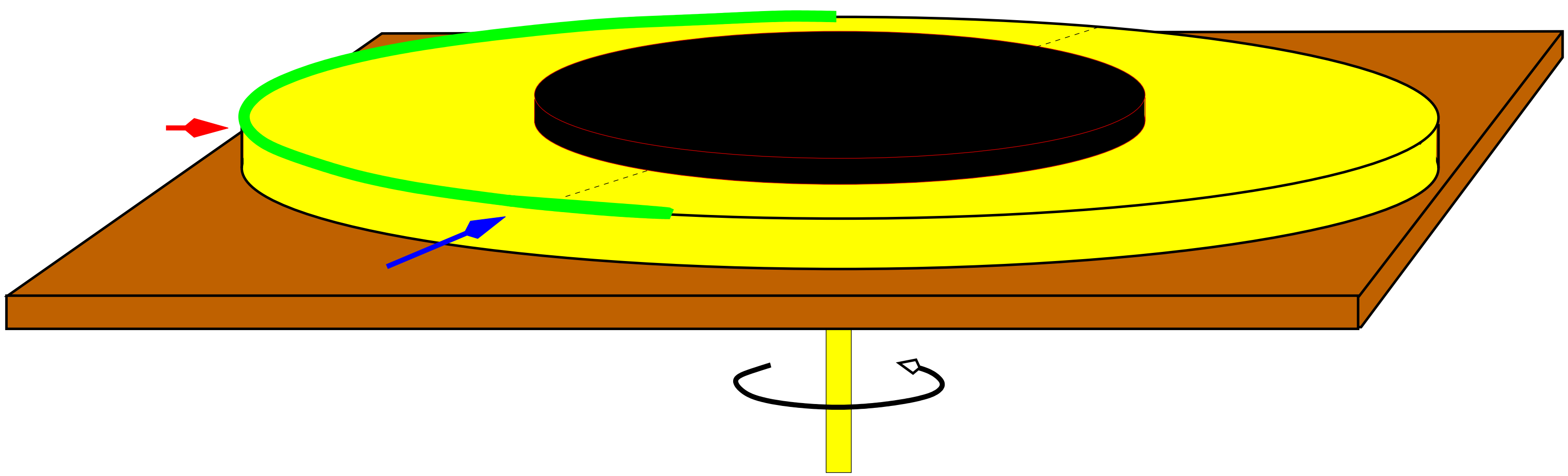
$$\alpha = 3\pi/5$$



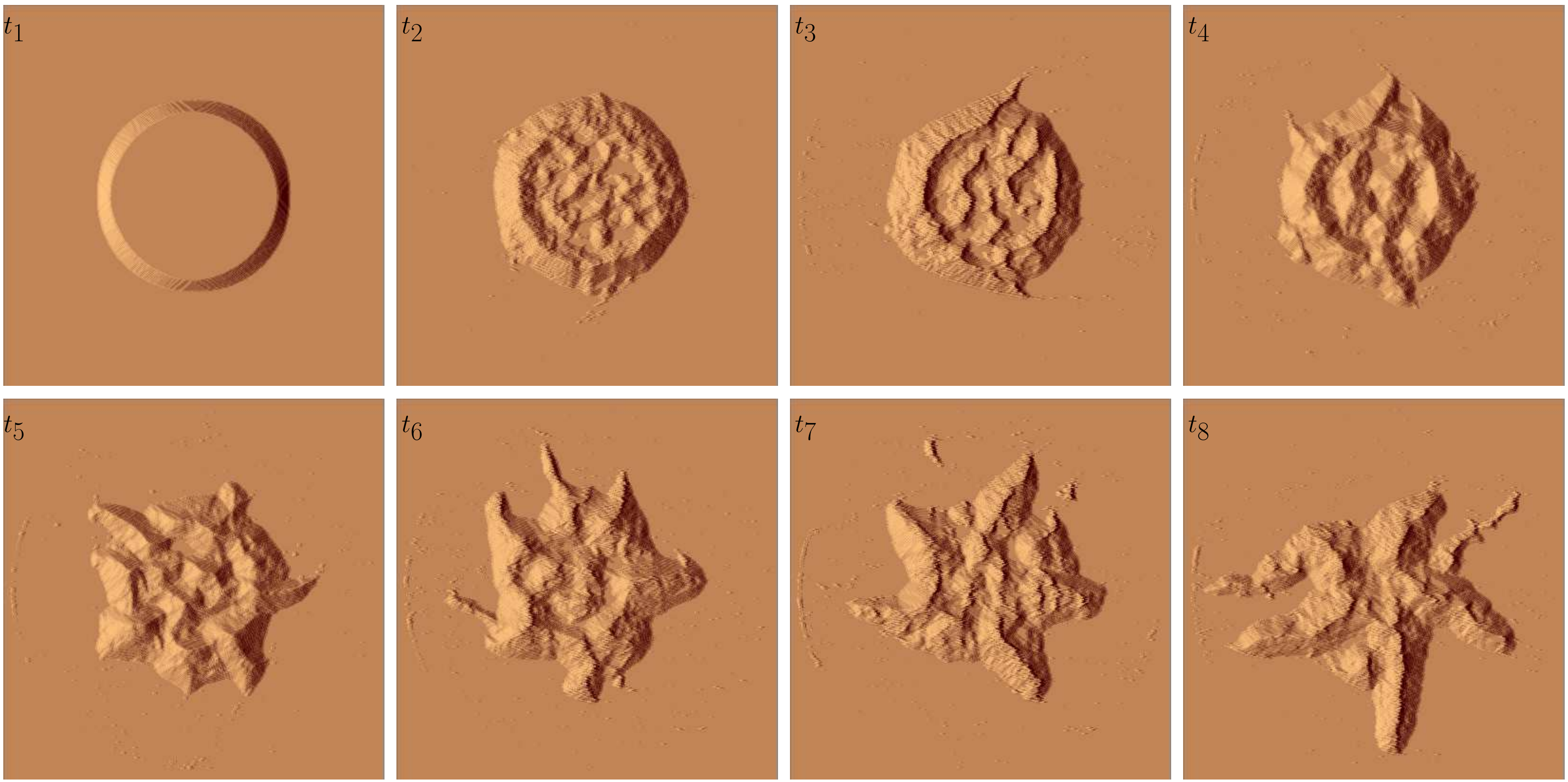
Rotating table for variable wind orientations



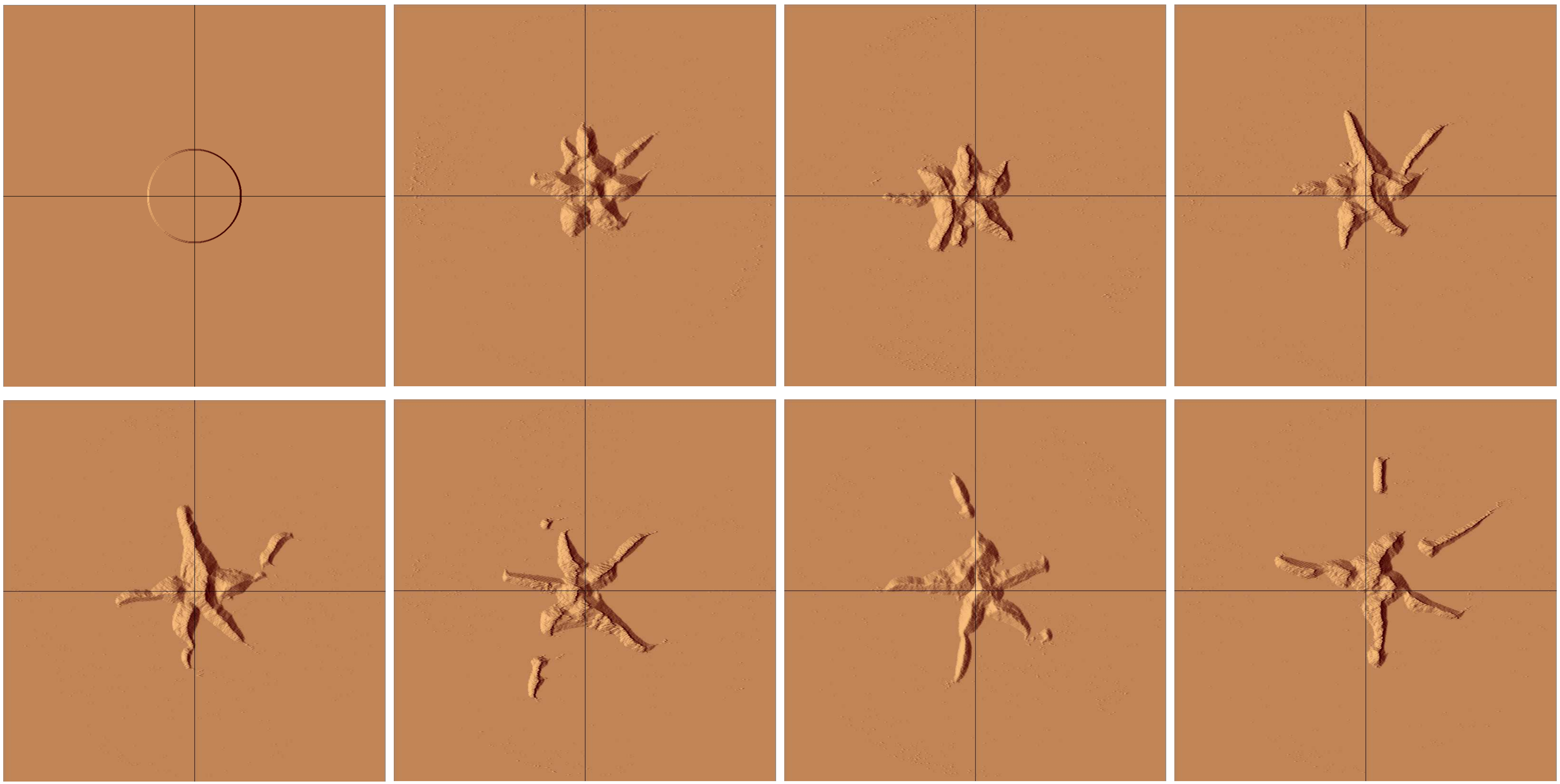
$$\alpha = 4\pi/5$$



Star dunes



Star dunes



A large set of observations ☑

Qualitative descriptions of dune shapes ☑

Elegant physical formalisms ☑

Pattern selection ☑

Population dynamics ☑

A quantitative analysis of sediment transport ☑

$\Downarrow$

We can use dunes patterns

to constrain climatic variations