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Emergence of a band-limited power law in the aftershock decay rate of a slider-block model

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[1] In order to elucidate how structural heterogeneities affect the aftershock decay rate, we examine the aftershock sequences produced by a slider-block model of seismicity. In this model, the geometry of the seismic zone is the only free parameter and all aspects of the system are known. The power law aftershock decay rate holds only for smooth faults. A band-limited power law emerges at intermediate fault complexity. For rough faults, only a transient regime toward an exponential decay is observed. In all fault geometries examined, a band-limited power law model fits the synthetic aftershock decay rate better than the Modified Omori Law. Then, as the connected seismic elements form a simpler localised surface, we show that the power law aftershock decay rate extends over longer time, and that the power law exponent increases. These results support the inference that the correlation time of the power law aftershock decay rate increases as the deformation localises along dominant major faults. *INDEX TERMS:* 3299 Mathematical Geophysics: General or miscellaneous; 7230 Seismology: Seismicity and seismotectonics; 7260 Seismology: Theory and modeling. **Citation:** Narteau, C., P. Shebalin, S. Hainzl, G. Zöller, and M. Holschneider, Emergence of a band-limited power law in the aftershock decay rate of a slider-block model, *Geophys. Res. Lett.*, *30*(11), 1568, doi:10.1029/2003GL017110, 2003.

1. Introduction

[2] Following an earthquake, aftershocks are seismic events of smaller magnitude occurring in the neighbourhood of the rupture. *Omori* [1894] first suggests a hyperbolic aftershock decay rate with respect to the time from the mainshock. This precursory work permits a more general description of the temporal clustering of aftershocks through the modified Omori Law [*Utsu*, 1961],

$$\Lambda(t) = \frac{K}{(t+c)^p}, \tag{1}$$

where $\Lambda(t)$ is the aftershock rate at time t, p is a positive power law exponent, K is a constant of proportionality, and c is a time constant essential to define a finite aftershock frequency at $t = 0$, the mainshock time. This empirical law predicts a power law aftershock decay rate over long time and a constant rate over short time. Later, different models of aftershock decay rate have been tested to improve the fit of natural aftershock sequences [*Gross and Kisslinger*, 1994]. For example, an exponential decay replaces the power law decay to account for an acceleration of the decay rate over long time [*Kisslinger*, 1993].

[3] Following *Scholz* [1968], *Narteau et al.* [2002] develop a model of aftershock decay rate based on a Markov process with stationary transition rates. These transition rates λ vary according to the magnitude of a scalar representing the state of stress, and defined as the overload σ_0. Thus, the aftershock decay rate in the model is a weighted sum of independent exponential decay functions with different characteristic times. From different overload distributions $N(\sigma_0)$ and different expressions of the transition rates $\lambda(\sigma_0)$, it has been shown that the aftershock decay rate can be written as

$$\Lambda(t) = \frac{A(\gamma(q, \lambda_b t) - \gamma(q, \lambda_a t))}{t^q}, \tag{2}$$

where $\gamma(\rho, x) = \int_0^x \tau^{\rho-1} \exp(-\tau) d\tau$, is the incomplete Gamma function and where q and A can be expressed according to analytical results. λ_b is a characteristic aftershock rate which corresponds to an upper bound of the overload distribution. λ_a is a characteristic aftershock rate which corresponds to a limit of crack growth where the magnitude of the transition rate jumps from 0 to a finite positive value.

[4] The main characteristic of the aftershock decay rate produced by equation 2 is that, if $\lambda_b \gg \lambda_a$, three major regimes of the aftershock decay rate emerge: (1) A linear decay is observed for $t < t_1^\zeta$ where $t_1^\zeta = x_b/\lambda_b$. (2) A power law decay is observed for $t_1^\zeta < t < t_2^\zeta$ where $t_2^\zeta = x_a/\lambda_a$. (3) An exponential decay is observed for $t > t_2^\zeta$. In this description, x_b and x_a are coefficients defined from the q-value and a threshold of divergence ζ between the aftershock decay rate and a permanent power law decay. Thus a band-limited power law aftershock decay rate results from the upper bound of the overload distribution and a limit of crack growth over short and long time respectively. In the framework of this model, (q, λ_a, λ_b) are the only free parameters, but sometimes it is more convenient to refer to the temporal limits (t_1^ζ, t_2^ζ) that result from these parameters in order to describe the different types of aftershock decay rate produced by equation 2 (Figure 1).

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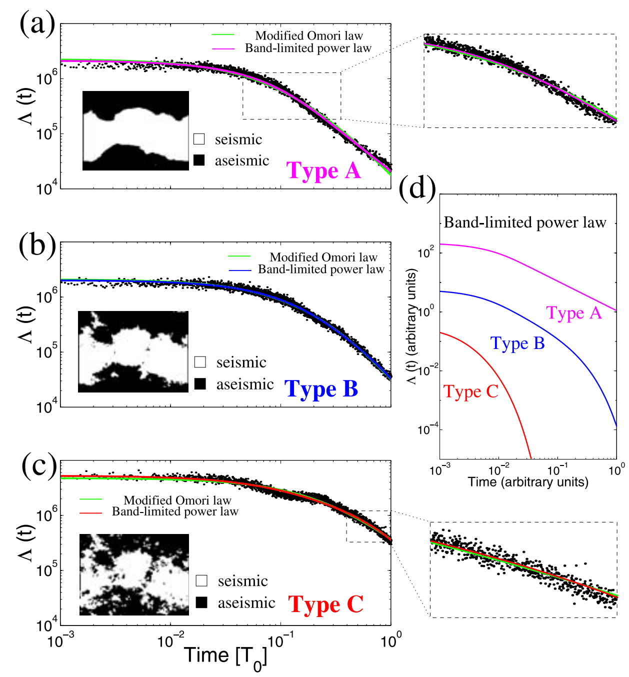

Figure 1. Averaged aftershock activity and best-fits of the band-limited power law and the modified Omori law for different fault geometries of the slider-block model. (a) D_0 = 1.1. (b) D_0 = 1.25. (c) D_0 = 1.4. Enlargements show that the band-limited power law is always closer to the data. (d) Generic behaviours of the band-limited power law. *A*-values are arbitrary, λ_b = 200 and q = 1 (i.e. $t_2^{\xi} = cte/\lambda_a$). Type *A*: power law decay rate over long time ($1/\lambda_a = 10^6$). Type *B*: band-limited power law ($1/\lambda_a$ = 0.2). Type *C*: transient regime toward an exponential decay rate ($1/\lambda_a = 10^{-3}$).

[5] From several well known aftershock sequences in different geophysical environments, *Narteau et al.* [2002] have suggested that the power law aftershock decay rates extends over longer time according to the concentration of the deformation along dominant major faults. In addition to the limit of crack growth, it has been proposed that structural properties control the time of the transition from a power law to an exponential aftershock decay rate (i.e. correlation time of the power law aftershock decay rate). Such conclusions have now to be confirmed by a systematic analysis of a large number of aftershock sequences. In a preliminary study presented in this paper, we test our hypothesis from aftershock sequences generated by a slider-block model [*Hainzl et al.*, 2003]. In this model, the complexity of the seismic zone is the only free parameter. Thus, modifications of the aftershock decay rate only result from changes in the geometry of the system. On the other hand, all the artifacts coming from data acquisition and data selection are avoided in numerical simulations. Our approach may be therefore visualised as an independent procedure for characterising a relationship between the duration of the power law aftershock decay rate and the faulting complexity.

2. The Model

[6] In the modelling of the earthquake phenomenology, slider-block models opened up a variety of new applications, all based on a standard representation of a frictional behaviour along faults. The model to be presented here retains the simplicity of the conventional slider-block model but specifically accounts for stress corrosion cracking [*Lawn and Wilshaw*, 1975]. Then earthquake clustering and, in particular aftershocks, result from short-range interaction over long time in the neighbourhood of ruptures.

[7] In this model previously described by *Hainzl et al.* [2003], the evolution of individual 'earthquakes' is a cellular automaton version [*Olami et al.*, 1992] of the two-dimensional spring-block model originally proposed by *Burridge and Knopoff* [1967]. Elastic interactions are incorporated by nearest neighbour coupling, and the frictional behaviour respects a Mohr-Coulomb criterion with a static failure threshold σ_F. In addition to static stress changes, the strength of the blocks bordering the rupture linearly decreases [*Lee and Sornette*, 2000] to mimic subcritical crack growth by stress corrosion. Then a characteristic time of a chemical reaction rate can be related to the unloading time T_0 which defines the rate of postseismic strength decrease $-\sigma_F/T_0$. Hence the duration of an earthquake is assumed to be few orders of magnitude shorter than T_0. Under this assumption, from a pre-stressed fault patch and a stochastic nucleation, different parts of the fault segment are unloaded by a succession of individual events. In this sequence of events, it is possible to determine foreshocks, mainshock and aftershocks with respect to the time of the events and their sizes.

[8] The model is implemented in the form of a two-dimensional $L \times L$ array of blocks. To account for complex boundary conditions, a seismic zone of $L^2/2$ blocks is embedded within an aseismic region. Practically, different fractal strength fields are generated from different fractal dimensions [*Turcotte*, 1997], and the seismic zone is defined by the blocks with the higher strength. Then we characterise the fault geometry by estimating D_0 the fractal box-counting dimension of the border line between seismic and aseismic zones (i.e. the fault roughness). In addition, we calculate the average number N_c and the correlation length L_0 of the aseismic regions within the seismic zone [*Stauffer and Aharony*, 1994]. These two parameters are negatively correlated to the degree of localisation of the connected seismic elements. Hence, in the following, we consider that the deformation occurs along simpler localised fault as L_0 and N_c decrease.

3. Results

[9] Different aftershock sequences obtained for a given fault complexity are stacked in a unique catalogue with respect to the time from the mainshock [*Davis and Frohlich*, 1991]. Thus we take advantage of the computing capabilities to reduce the statistical bias associated with individual aftershock sequences. Simultaneously, we verify that the temporal properties of a vast majority of individual sequences are the same as for the stacked catalogue. Each sequence initiates at t = 0 and continues at least until t/T_0 = 1. Then, we analyse the aftershock decay rate between 0 and T_0 to avoid finite size effects at $t > T_0$ when individual sequences are more likely to stop. The numerical procedure is the same as in *Narteau et al.* [2002]. We define the parameters of the modified Omori law (p, c) and the parameter of the band-limited power law (q, λ_a, λ_b) using state-of-the art statistical

Table 1. Simultaneous Comparisons Between the Aftershock Decay Rates of the Slider-Block Model With Different Fault Geometries, and Between the Band-Limited Power Law and the Modified Omori Law

Fault geometry				Mod Omori law		Band-limited power law				Temporal limits		
D_0	L_0	N_c	$N(\times 10^4)$	p	c	λ_a	λ_b	q	Δ_{AIC} $(\times 10^5)$	t_1^ζ	t_2^ζ	Type
1.10	2.50	5	3.41	2.18	0.12	10^{-3}	22.8	1.59	−7.04	0.11	394	*A*
1.25	4.52	24	4.37	2.60	0.25	0.84	15.1	1.41	−9.03	0.14	0.54	*B*
1.30	5.22	40	4.22	2.02	0.18	0.75	18.1	1.12	−8.61	0.10	0.38	*B*
1.35	6.89	92	4.97	2.27	0.31	1.28	17.0	0.41	−10.1	0.04	0.012	*C*
1.40	7.73	185	4.69	1.75	0.30	0.86	22.9	0.15	−9.40	0.008	$2.3\ 10^{-5}$	*C*

D_0 is the fractal box-counting dimension of the border line between seismic and aseismic regions. N_c and L_0 are the number and the correlation length of aseismic regions within the seismic zone. N is the number of aftershocks. The time scale is given by the unloading time T_0. $\lambda_{a,b}$ have units of frequency. $t_{1,2}^\zeta$ and c have units of time, while q and p are dimensionless. Δ_{AIC} is the difference between the AIC's of the band-limited power law and of the modified Omori law. $t_{1,2}^\zeta$ are inversly proportional to $\lambda_{a,b}$ and vary according to the q-value and a threshold of divergence expressed as a percentage (ζ = 0.8). As in Figure 1, type *A* corresponds to a power law decay rate over long time $t_2^\zeta \to \infty$, type *B* corresponds to a band-limited power law $0 < t_2^\zeta < 1$, and type *C* corresponds to a transitional behavior toward an exponential decay rate $t_2^\zeta \to 0$.

measures [*Akaike*, 1974; *Ogata*, 1983]. Then we compare the models by calculating the difference Δ_{AIC} between the Akaike Information Criterion (AIC) values obtained from both model. $\Delta_{AIC} < 0$ means that statistically the band-limited power law provides a better fit of the data than the modified Omori law, despite the additional parameter.

[10] Table 1 shows the parameters of the modified Omori law and the parameter of the band-limited power law for five different degrees of fault complexity (L = 128). For three of them, Figure 1 shows the aftershock decay rate and the best fits provided by both laws. In Table 1, the N_c-value and the L_0-value rapidly decrease as the D_0-value decreases. Thus we study the transition from a more distributed to a simpler localised seismic zone. In all fault geometries examined, the best-fit of the aftershock sequence is obtained by the band-limited power law.

[11] For the modified Omori law, the p-value is high and almost constant for the different fault geometries examined. On the other hand, the q-value continuously decreases from 1.6 to 0.15 as the fault complexity increases. In particular, the q-value jumps from 1.1 to 0.4 as D_0 moves from 1.3 to 1.35. Small t_1^ζ-values suggest a rapid onset of the power-law aftershock decay rate. The c-value is always higher than the t_1^ζ-value, and the ratio c/t_1^ζ increases for rougher faults. Thus, the modified Omori law may capture non power law behaviours over long time scales but provides a worst fit to the short-term behaviour.

[12] A rapid decrease of the t_2^ζ-value is associated with an increasing complexity of the border line separating seismic and aseismic regions of the model. For a smooth fault (D_0 = 1.1), $t_2^\zeta \to \infty$ and no transition toward an exponential aftershock decay rate can be observed (Figure 1a). For an intermediate complexity (D_0 = 1.25), the t_2^ζ-value is less than T_0 and a transition toward an exponential decay rate occurs over long time (Figure 1b). From D_0 = 1.1 to D_0 = 1.25, we have therefore the emergence of a band-limited power law, and a lower Δ_{AIC}-value is obtained. For a more complex fault geometry (D_0 = 1.3), the power law aftershock decay rate is still the dominant regime over intermediate time although the smaller t_2^ζ-value. For rough faults ($D_0 \geq$ 1.35), $t_2^\zeta \to 0$ and $t_2^\zeta < t_1^\zeta$. It is impossible to observe a power law decay and only a transient regime from a linear decay to an exponential aftershock decay rate occurs (Figure 1c). As shown by the Δ_{AIC}-value, the modified Omori law is unable to deal with this decay type. From D_0 = 1.35 to D_0 = 1.3, we have therefore the emergence of a band-limited power law. As the connected seismic elements form a simpler localised surface, the power law regime emerges from a linear regime over short time, and extends over longer time to the detriment of the exponential regime.

4. Discussion

[13] In the slider-block model with stress corrosion cracking, the aftershock decay rate results from strength decreases spatially localized at the border of the rupture over a population of blocks with heterogeneous stresses. Hence, the extension of the rupture is essential in determining when aftershocks will be triggered. Aseismic regions included in the seismic zone may inhibit the propagation of an event, and are likely to affect the stress field at the edge of rupture areas. In fact, as the fault becomes smoother, aftershocks are in average triggered at lower stresses, and the macroscopic strength of the fault is decreasing. The controlling properties of the aftershock decay rate in the slider block model and in the model presented by *Narteau et al.* [2002] are therefore the same.

[14] The nature of the heterogeneity that we introduce in the slider block model might not be the same as the heterogeneity in the real Earth. However, the particular mechanism of the heterogeneity is unlikely to be critical to the broadly averaged statistics of aftershock decay, and the huge AIC values resulting from the large numbers of events being fit is a benefit of studying models.

[15] In this paper, we examine how the geometry of the seismic zone affects the temporal properties of the aftershock sequences produced by the slider-block model. In particular, we determine the parameters of a band-limited power model and of the modified Omori law. The band-limited power law predicts a decrease of the exponent of the power law aftershock decay rate with respect to an increasing fault complexity. From real aftershock sequences, *Nanjo et al.* [1998] already suggest such a decrease of the power law exponent with respect to an increase of the fractal dimension of the fault populations. Nevertheless, this analysis suggests that, if the transition toward an exponential regime rate is not taken into account -as in the modified Omori law- such variation may be more difficult to capture because of a worst fit of the aftershock decay rate.

[16] In the model, the power law aftershock decay rate is permanent along smooth faults. For an intermediate fault complexity, a transition toward an exponential decay rate occurs over long time and a band-limited power law emerges. Finally, for the most complex geometries, only a

transient regime toward an exponential aftershock decay rate may be observed. Therefore there is a rapid increase of the correlation time of the power law aftershock decay rate with respect to a decreasing fault complexity.

[17] Without the restrictions applied to seismological and experimental data analysis, these observations are in good agreement with (1) the experimental results of *Hirata* [1987] that show that an exponential decay is replaced by a power law aftershock decay rate following the advance of the fracturing process, and (2) the inference that have been done from natural sequences [*Narteau et al.*, 2001]. Aftershock sequences produced by the slider-block model along smooth fault correspond to real sequences observed in zones where the strain is concentrated on a major fault and where the aftershocks exhibit a dominant faulting mechanism. In laboratory experiments, they correspond to aftershock sequences observed just before the macroscopic failure (i.e. dynamic stress drop). For an intermediate fault complexity, synthetic aftershock sequences correspond to real sequences observed in zones where the strain is partitioned on smaller faults and where no major structure emerges. Synthetic sequences along rough fault correspond to aftershock sequences observed in zones of distributed damage where the deformation is accommodated by different types of faulting mechanism or where the strain rate is significantly low (i.e. higher crustal strength). In laboratory experiments, such sequences are observed during the preliminary stages of fracture growth.

[18] In the multi-disciplinary science of critical phenomena, a critical behaviour is characterised by an increase of both the correlation time and length in the proximity of a critical point and by a divergence of these parameters at the critical point. During the formation and the evolution of fault populations, individual fault growth, interact, strain localise and structural irregularities are smoothed by the accumulation of the deformation. In a slider-block model, this work suggests that, simultaneously, the correlation time of the power law aftershock decay rate increases. As said above, similar behaviours may have been captured in real sequences and in laboratory experiments. Unfortunately, the selection of only a few aftershock sequences for analysis means that the empirical support provided for such behaviours is still of an anecdotal nature. The statistical analysis presented in this paper is a new and independent contribution about the possible relationship between the correlation time of the power law aftershock decay rate and the faulting. The results support the idea that brittle rocks makes a transition from a subcritical state to a precisely critical state during the localisation process of the deformation.

5. Conclusion

[19] Temporal properties of the aftershock sequences produced by a slider-block model depend on the geometry of the seismic zone. Along smooth fault, the power law aftershock decay rate applies at all time scales, but for complex fault geometries, only a transient regime toward an exponential decay may be observed. Between these two end members, a band-limited power law emerges and extends over longer time as the smoothness of the fault increases. In all fault geometries examined, a band-limited power law model fits the synthetic aftershock decay rate better than the Modified Omori Law, despite the additional parameter. In addition to the duration of the power law aftershock decay rate, we show that the power law exponent increases as the connected seismic elements form a simpler localised surface. Without the usual restrictions applied to geophysical and experimental data analysis, the work presented here supports the inference that the correlation time of the power-law aftershock decay rate increases as the deformation localises along dominant major faults.

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